BRAIN TICKLERS

Results From Summer

Perfect Scores

*Berthold, Kristopher D.	TX	Β	'04
*Bohdan, Timothy E.	IN	Γ	'85
*Capelli, Ronald B.	MI	Γ	'73
*Couillard, J. Gregory	IL	Α	'89
*Gaston, Charles A.	PA	Β	'61
*Kimsey, David B.	AL	Α	'71
*Parks, Christopher J.	NY	Γ	'82
Sylvester, Noah	Son of member		
Voellinger, Edward J.	Non-member		

Other

*Bannister, Kenneth A.	PA	Β	'82
Dechman, Don A.	TX	Α	'57
*Golemme, Steven S.	IL	Α	'20
*Griggs Jr., James L.	OH	Α	'56
Marks, Lawrence B.	NY	Ι	'81
Marks, Benjamin	Son of member		
*McHenry, S. Dale	MO	Β	'81
*Norris, Thomas G.	OK	Α	'56
*Riedesel, Jeremy M.	OH	Β	'96
*Robertson, John A.	IL	Α	'65
Varvel, Virgil	Non-member		
Routh, André G.	FL	Β	'89
Salvia II, Joseph J.	DE	Α	'98
Scott, Darrell J.	NC	Δ	'82
*Slegel, Timothy J.	PA	Α	'80
Summerfield, Steven L.	MO	Γ	'85
Szostek, Renee	MI	Α	'87
Zison, Stanley W.	CA	Θ	'83

*Denotes correct bonus solution

Summer Review

While the total number of entries for summer was relatively modest, the percentage correct for each scored Tickler was very high. The hardest problem was #5 (Toroid Planet), with 78 percent of submitted solutions correct.

The Bonus (Twenty Questions) wasn't as difficult for our readers as the judges anticipated; it drew more correct answers than either #2 (Tennis Rankings) or #5. The Computer Bonus (Seven Stones) inspired 10 responses, but only two could designate a set of stones that maximized the elder's age at 122.

Fall Answers

1: Cleopatra loves her **dad**.

Fall Answers Continued

2: There are **94,589** ascending numbers from 5,402 to 97,543,210, inclusive, that have no repeating digits, nor a 6 or an 8.

3: The chance of getting a perfect bridge deal is **$4!(13!)^4/52! = 1$ / 2,235,197,406,895,366,368,301, 560,000**.

That comes from dealing the cards one by one and keeping track of the probability as (52/52)(39/51)(26/50)(13/49)(12/48)(12/47)(12/46)(12/45)(11/44)(11/43)(11/42)(11/41)(10/40)(10/39)(10/38)(10/37)(9/36)(9/35)(9/34)(9/33)(8/32)(8/31)(8/30)(8/29)(7/28)(7/27)(7/26)(7/25)(6/24)(6/23)(6/22)(6/21)(5/20)(5/19)(5/18)(5/17)(4/16)(4/15)(4/14)(4/13)(3/12)(3/11)(3/10)(3/9)(2/8)(2/7)(2/6)(2/5)(1/4)(1/3)(1/2)(1/1).

4: **16,853 + 68,539 = 85,392** is the solution of ABCDE + BCDEF = CDEFG.

5: Mercury is the closest planet to Pluto about **1.5 percent** of the time. One method to solve this is a computer simulation of the planets in their orbits—this results in a range of answers depending upon how long the simulation is run. The second method is to look at seven pairs of intersecting circles—one circle centered on Pluto with radius of Pluto-sun distance, the other centered on the sun with radius of each sun-planet. Where the circles intersect determines where Pluto-sun distance equals Pluto-planet distance, and hence, what percent of the time the sun is closer than that planet. Since Mercury is so close to the sun, those percentages can be used. The product of those seven percents gives the overall percent for Mercury being the closest.

BONUS:

From basic physics, work done by expanding gas at constant temperature $=nrT*\ln(V_2/V_1)= P_1*V_1*\ln(V_2/V_1)$, where V2 is final volume and V1 is initial volume.

The ship (with mass of 1000 kg) is initially at rest with 1000 kg of water in it. Let the pressure of the nitrogen gas be N_P. The initial pressure is 6 MPa. Let the volume of the nitrogen gas be N_V. The initial volume is 1 m^3. Let the volume of the nitrogen gas increase by ΔN_V as some small amount of water is vented.

The volume of water vented is also ΔN_V. Therefore, the mass of water vented: $\Delta H_2O = 1000. * \Delta N_V$.

The work done by that expansion is: $W = N_P*N_V*\ln((N_V+\Delta N_V)/N_V)$.

The work done by that expansion results in a delta of kinetic energy. Assume all of the kinetic energy is transferred to the water vented.

Since work is a change in kinetic energy which is $0.5*m*v^2$. Therefore, $v = \sqrt{(2*work/m)}$. Hence, the velocity of the small amount of vented water: $vH_2O = \sqrt{(2. * W / \Delta H_2O)}$.The momentum of the vented water: $momH_2O = \Delta H_2O * vH_2O$. That results in the same momentum (opposite direction) being given to the ship.

That results in the ship having a delta velocity of $momH_2O$ / (ship mass + H_2O mass in ship).

Iterate until all of the water has been vented. That leaves the ship with a **final velocity of 61.7 m/s**.

COMPUTER BONUS:

$639{,}172^2$ = 408,540,845,584 is the only other six-digit number with all different digits where the square has none of the digits of the number.

New Winter Problems

1: University Cryptarithm

My younger son is now at UC Berkeley (*CA Alpha*), and one may recall from previous columns my elder son is at Cornell (*NY Delta*). At dinner one night before term, both boys were extolling the virtues of their respective universities and regaling the family with their experiences. As they traded stories, they were surprised to realize just how similar the schools appeared on the surface. Slightly perplexed by this, they asked me what I thought the difference was between BERKELEY and CORNELL. I replied that it was simply the very ESSENCE of the university. Even more befuddled, they began to dismiss my answer as frivolous and without merit until I showed them: BERKELEY - CORNELL = ESSENCE, and they finally understood what I said was true. Find a unique solution to the above cryptarithm. Standard rules apply: each different letter stands for a different digit, and each different digit is always represented by the same letter; no leading zeros are allowed.

—**Jeffrey R. Stribling,** *CA A '92*

2: Finding the Integer

Let N be a positive integer such that $N/2$ is a perfect square, $N/3$ is a perfect cube, $N/5$ is a perfect fifth power, and $N/7$ is a perfect seventh power. What is the smallest such N?

—Unknown

3: New Student Habits

The principal of the girls' school was interested in the habits of her new students, so she sent her assistant to make some notes as they left their rooms. The assistant handed her the following tabulation:

Time Left for Class	Name	Nationality	Sweater Color
9 A.M.	Ann	English	Ivory
10 A.M.	Beth	French	Jade
11 A.M.	Carol	German	Khaki
12 Noon	Doris	Honduran	Lavender

Unfortunately, only one item in each of the last three columns is correctly positioned against time. What the assistant actually observed was as follows. At 10 A.M., either Carol or Doris left but wore neither an ivory nor jade sweater. At one girl's departure time—it was either Ann or Beth—she was not wearing a khaki sweater and was not English nor French. One hour later, the girl who left wore neither a khaki nor lavender sweater. Carol did not leave at Noon, and the Honduran girl left at 9 A.M. Sort out the name, nationality, and sweater color of each girl and the time she left for class.

—*Brain Busters! Mind Stretching Puzzles in Math and Logic* by Barry R. Clarke

4: Unique Passcodes

You wish to construct a 16-digit passcode (in base 10) which uses all ten digits from 0-9 at least once within the string. How many such unique passcodes can be generated, assuming order matters?

—Reddit posting

5: Red Rug with a Glitch

A man has a large square rug, an integral number of feet on a side, the design of which is a blue square with a red border. However, due to a manufacturing glitch, the blue square is off-center, although its sides are parallel to the sides of the rug. Because all four corners have become ragged, it is decided to trim the rug by cutting off a triangle at each corner by making four straight cuts, each just touching a corner of the blue square, with the cuts made in such a way as to maximize the remaining area of the rug. When this was done, it was found that the total area discarded was an integral number of square yards that was exactly 10 percent of the original red area. What was the smallest possible original length of the rug's sides?

—*New Scientist:* Susan Denham

BONUS: Consider a uniform rod of mass, m, and length L, sliding lengthwise on the level surface of an ice rink. Initially, the rod slides on a frictionless patch of ice, but it encounters an area where the surface is rougher and has a kinetic coefficient of friction μ. The tip of the rod comes in contact with the rougher surface at time zero. Find the initial velocity, v_0, such that the rod stops precisely when the trailing end reaches the rough surface and the time t it takes the rod to come to rest.

—Luke B. Stribling

COMPUTER BONUS: A positive integer $x > 1$ with prime factors $p_1p_2p_3...p_i$ that satisfies the relationship $1/p_1 + 1/p_2 + 1/p_3 + ... + 1/p_i - 1/x = k$, where k is a positive integer, is known as a Giuga number. The first few Giuga numbers are 30; 858; 1,722; and 66,198. Let us define an n-th cousin of Giuga to be a positive integer $x > 1$ with prime factors $p_1p_2p_3...p_i$ that satisfies the relationship $1/p_1 + 1/p_2 + 1/p_3 + ... + 1/p_i - 1/x = k/(n+1)$, where $1 \leq k \leq n$. Find the first ten 6th cousins of Giuga.

—**Jeffrey R. Stribling,** *CA A '92*

BTs continue on page 48.

In Memory of Don Dechman

Don A. Dechman, *TX A '57*, TBΠ Brain Ticklers judge from 1996-2016, passed away on September 3, 2021, at the age of 86. Born on June 5, 1935, in Ft. Worth, TX, Don received his bachelor's and master's degrees in chemical engineering from the University of Texas at Austin and enjoyed a 30-year career with Union Carbide.

Don had a passion for math and solving complex math puzzles, playing poker & bridge, and was one of the first people to solve the Rubik's Cube.

In honor of Don's 80th birthday, his three sons: David, *VA B '82*, Ken, a Tufts graduate, and Jim, *TX A '89*, endowed the Tau Beta Pi Don A. Dechman Scholarship.