BRAIN TICKLERS

Results From Summer

Perfect Scores

*Bohdan, Timothy E.	IN	Γ '85
*Capelli, Ronald B.	MI	Γ '73
*Couillard, J. Gregory	IL	A '89
*Griggs Jr., James L.	OH	A '56
*Johnson, Mark C.	IL	A '00
Marks, Lawrence B.	NY	I '81
Marks, Benjamin	Son of member	
*Marx, Kenneth D.	OR	A '61
*Norris, Thomas G.	OK	A '56
Parks, Christopher J.	NY	Γ '82
*Seidel, Mark N.	MA	B '83
*Slegel, Timothy J.	PA	A '80
Spong, Robert N.	UT	A '58
*Wells, Alan T.	RI	B '16

Other

Alexander, Jay A.	IL	Γ '86
Aron, Gert	IA	B '58
*Berthold, Kristopher D.	TX	B '04
Bertrand, Richard M.	WI	B '73
*Bukowski, Justin D.	OH	A '90
Dangler, Paul E.	NY	Θ '76
*Dechman, Don A.	TX	A '57
Dungan, Michael R.	VA	B '92
Gerken, Jeffrey D.	OH	Δ '76
Glaze, James B.	KS	Γ '75
*Gulian, Franklin J.	DE	A '83
Gulian, Joseph D.	Son of member	
*Hasek, William R.	PA	Γ '49
Hedegard, Alan H.	IN	A '64
*Hosford, Glen A.	OK	A '95
Johnson, Roger W.	MN	A '79
Jordan, R. Jeffrey	OK	Γ '00
Lalinsky, Mark A.	MI	Γ '77
Lamb, James A.	NY	Γ '66
*Luetkemeier, Glenn A.	IN	A '73
Marrone, James I.	IN	A '61
McConnell, James C.	PA	B '82
*McCullough, Charles R.	AL	B '12
Mettler, Rick A.	WA	B '81
Oliver, Christopher R.	AL	E '08
*Panczyk, Mark M.	NJ	Γ '08
Rand, Timothy W.	ND	B '86
*Richards, John R.	NJ	B '76
*Richardson, Casey L.	IN	Δ '19
*Riedesel, Jeremy M.	OH	B '96
Roggli, Victor L.	TX	Γ '73
Rowland, Ralph W.	MD	B '51
Sauer, Daniel M.	MI	B '05
*Schmidt, V. Hugo	WA	B '51
Seaverson, Audrey	Daughter of member	
Sigillito, Vincent G.	MD	B '58
Skinn, Jeffrey R.	OH	Γ '08
Spring, Gary S.	MA	Z '82
*Strong, Michael D.	PA	A '84
*Voellinger, Edward J.	Non-member	
Weinstein, Stephen A.	NY	Γ '96
Zison, Stanley W.	CA	Θ '83

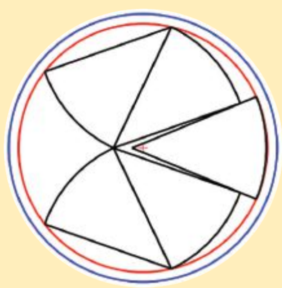
*Denotes correct bonus solution

Summer Review

Problem #4, regarding the distribution of the first decimal digit of 2^N, turned out to be the hardest regular Tickler, drawing a correct answer from less than 50% of all entries. Many respondents approximated the solution with 0.3, but the judges were looking for the more precise $\log_{10}2$.

It was a little surprising that the Bonus sheet-folding question drew exactly the same number of correct answers as #4. The judges were impressed (and occasionally confounded) by the variety of inventive ways respondents used to describe their folding sequences. Kudos to **Jeremy Riedesel,** *OH B '96*, for particularly good artwork.

The pizza slice configuration Double Bonus drew few responses. The problem was open-ended, as the judges were not certain of the optimal diameter values. For the five-slice configuration, **Kris Berthold,** *TX B '04*, improved on the judges' published answer to 27.7997 cm by withdrawing the center slice slightly and rotating the other four, as shown in the layout below. A couple other readers also reported better diameters, but diagrams were not available for verification.



Fall Answers

1: The ranking of the five people (Alf, Bert, Charlie, Duggie, and Ernie) for the three traits is:

	1st	2nd	3rd	4th	5th
Honesty	E	D	B	A	C
Charm	B	D	E	A	C
Intelligence	D	B	C	E	A

If A is truthful, then D is truthful, but since D says A lies, this implies that A must be lying. C is the remaining liar since 4th place for Honesty cannot be truthful. So, C is 5th place for Honesty, and A is 4th.

B has higher Charm than Intelligence and higher Intelligence than Honesty. The lowest (and highest) his Honesty can be is 3rd, so he is 2nd in Intelligence and is 1st in Charm. E is 3rd for Charm, and the sum of D's places must be one less than that of B's which must be 5. This cannot be derived from 1+1+3, since D cannot be 1st in Honesty, so it must be derived from 1+2+2. This implies that D is 1st in Intelligence, 2nd in both Charm and Honesty. From here, E must be 1st in Honesty. A's Charm is higher than his Intelligence, so they are 4th and 5th, respectively. This forces C to be 5th in Charm, and given that in one test E must be lower than C, C is 3rd in Intelligence while E is 4th.

2: The lily pad is **100cm** in diameter. As the snake moves along the log, the center of mass of that grouping remains stationary. The same is true for the newt on the lily pad as it moves. If x is the distance from the center of the log to the center of mass, then $7(50-x) = 25x$ implying $x = 10.9375$cm, so the log moves away from the lily pad by $2x = 21.875$cm. If the radius of the lily pad is R, and y is the distance from the center of the lily pad to the center of mass after the newt has moved, then $.02(R-y) = 0.3y$ implying $y = 0.0625R$, so the lily pad moves away from the log by

0.0625*R*cm. We then have 75cm + 21.875cm + 0.0625*R* = 100cm, resulting in *R* = 50cm. Thus, the diameter is 2*R* = 100cm.

3: F and **S** are the two swapped letters. Note that all loops require an even number of moves. F and S are the only letters that have an odd distance between their two instances. The layout is:

```
M--D--N--H
|  |  |  |
O  A--L  R
|  |  |  |
I--P  Q--J
|  |  |  |
C  S--E  B
|  |  |  |
T--F--K--G
```

4: 39 × 402 = 15678 is the solution to AB × CDE = FGHIJ with no zeros in the five digit number. This one likely required some number crunching with the aid of a computer!

5: $256^3 + 64^4 = 32^5$ is the smallest positive solution to $a^3 + b^4 = c^5$. Assume $a^3 = b^4$, then the left-hand side needs to be multiple of 12 for the exponents, and the right-hand side needs to be multiple of 5 for the exponent and needs to be one higher than the left side. So, $2^{24} + 2^{24} = 2^{25}$ implies $256^3 + 64^4 = 32^5$.

BONUS: 20 cards (out of the 81 cards) is the maximum number of cards that can be used and still not have a set. The SET cards can be mapped into 4-dimensional lattice space with three axis points for each dimension. In that space, any three points on the same line will be a set. First, consider a one property card deck. It has $3^1 = 3$ cards. This maps to 1-dimensional space. It is obvious that one can have two cards and not have a set. For example, take cards 0 and 2 as:

```
012
* *
```

Next, consider a two property card deck. It has $3^2 = 9$ cards. This maps to 2-dimensional space. Four cards can be taken and still not have a set. One way is:

```
 012
a* *
b
c* *
```

If a 5th card is added to that, a line with three cards will be formed and it will be a set. Notice that a one property solution is a subset of the two property solution. Another way to do four cards is:

```
 012
a *
b* *
c *
```

Now, it appears that adding a 5th card in a corner will not be a line. But, lines in this space loop around. So,

```
 012
a**
b* *
c *
```

has the line a0-c1-b2, which is a set (all properties different). Next, consider a three property card deck. It has $3^3 = 27$ cards. This maps to 3-dimensional space. Nine cards can be taken and still not have a set. One way is:

```
I012      J012      K012
a* *      a         a *
b         b *       b* *
c* *      c         c *
```

Notice that two property solutions are subsets of the three property solution. Adding a 10th card to either the I or K planes makes an obvious line of three (which is a set). Adding a 10th card at Ja0 gets the line: Ic0-Ja0-Kb0. Adding a 10th card at Jb0 gets the line Ia2-Jb0-Kc1. Now, finally, consider a four property card deck. It has $3^4 = 81$ cards. This maps to 4-dimensional space. Twenty cards can be taken and still not have a set. One way is:

```
 I012      J012      K012
 a* *      a         a *
Xb         b *       b* *
 c* *      c         c *

 I012      J012      K012
 a         a         a
Yb *       b         b *
 c         c         c

 I012      J012      K012
 a* *      a         a * *
Zb         b *       b
 c* *      c         c * *
```

Notice that three property solutions are subsets of the four property solutions. Adding a 21st card to any of XJ, YI, YK, or ZJ gets the same line as the three property case. Adding a 21st card to YJb1 gets the line YIb1-YJb1-YKb1. Adding a 21st card at YJa0 gets the line XIa0-YJa0-ZKa0. Adding a 21st card at YJa1 gets the line XKa1-YJa1-ZIa1. These maximum subsets that do not contain a set are called "cap sets" in the literature (Ramsey theory). Results are known for five properties (45) and six properties (113), but nothing higher—it remains an open question.

COMPUTER BONUS:

Start with a 4×4 matrix with fixed relationships between rows and columns:

1	2	5	10
3	6	15	30
4	8	20	40
12	24	60	120

Then, use a double-Latin square "Euler (Graeco-Latin) square" to rearrang to:

1	**30**	**8**	**60**
24	**20**	**3**	**10**
15	**2**	**120**	**4**
40	**12**	**5**	**6**

which is a "gnomon magic square." The magic product is 120^2.

New Winter Problems

1: Cryptic Multiplication

John D. Owens, *CA A '95*, a close friend of the columnist, recently shared this Tickler:

```
      * * *
x       * *
-----------
    * * * *
  * * * *
-----------
  * * * * *
```

BTs continue on page 33.

Winter Problems *Continued*

Given the fact that each * independently represents a prime digit, solve the cryptic multiplication.

—*The Moscow Puzzles* by Boris A. Kordemsky

2: Spy Aliases

I have four friends: Anna, Beth, Cate, and Dawn who are spies. As is often the case in the espionage profession, they each assume three unique aliases of different word lengths to suit their need in the moment. Their five-letter aliases are (in some order) Alice, Betty, Carol, and Debra. Their six-letter aliases are (in some order) Astrid, Bianca, Claire, and Dharma. Their seven-letter aliases are (in some order) Allison, Barbara, Cristin, and Darlene. The other day in the spy lounge, I overheard each friend make two statements.

ANNA: 1. Beth is Betty.
2. Cate is "perfect."

BETH: 1. Bianca is "perfect."
2. Anna is Carol.

CATE: 1. Anna is "imperfect."
2. Beth is not Betty.

DAWN: 1. Carol is not Astrid.
2. Dharma is not Darlene.

It is well understood that in the spy world such declaration pairs, if and only if they are both true, indicate the speaker is a "perfect" spy with name and aliases all beginning with unique letters. It is equally known that such declaration pairs, if and only if they are both false, indicate the speaker is an "imperfect" spy with name and aliases all beginning with the same letter. Of course, no spy would ever consider having exactly three of their four monikers begin with the same letter. Find all aliases associated with each spy.

—*Brain Puzzler's Delight* by E.R. Emmet

3: Bubble Experiment

A cylinder which is completely full of water, except for a bubble of air held at the bottom, is sealed at both ends. There is no air gap between the top of the water and the top of the cylinder. The pressure the water exerts on the top of the cylinder is zero. Suppose the bubble is now allowed to rise to the top of the cylinder. Find the percent pressure change experienced (if any) at the bottom of the cylinder. Assume that water is incompressible and that the volume of the cylinder does not change.

—Daryl Cooper

4: Mirror Triangles

Given five identical 3:4:5 triangles, construct a plane figure with mirror symmetry. The triangles may not overlap.

—**Fred J. Tydeman,** *CA Δ '73*

5: Shatter Testing

Al is back on the job testing a new batch of bowling balls. He now has three identical bowling balls and is to test their impact resistance by dropping them out of windows on various floors of a 100-story building. He is to determine the lowest floor from which a dropped bowling ball will shatter on impact with the pavement below. Al knows nothing about the strength of the balls. They may shatter when dropped from the first (non-ground) floor or not until dropped from the 100th floor. Balls that do not shatter may be dropped again and all balls may be destroyed during the test. Find the minimum number of ball drops needed to guarantee that Al can uniquely determine the lowest floor from which the balls will shatter. If Al wants to establish the minimum number of drops as 14, find the maximum number of stories the building can have.

—Adapted from *How to Ace the Brain Teaser Interview* by John Kadorew

BONUS:

A roller coaster at rest drops from an initial height of 100m at which time it begins a loop-de-loop. The loop is formed from an initially horizontal segment of track that bends upward with a radius of curvature of 40m until it becomes vertical, at which point there is a 10m section of vertical track. From this point, the track continues the loop with a reduced radius of curvature of 30m until it is horizontal and at the apex of the loop. The descent of the loop is symmetric to the ascent. Find the time required for the coaster to complete the entire loop, to the nearest hundredth of a second. The coaster can be assumed to maintain motion in a single vertical plane; ignore any effects of friction and drag. Assume $g = 9.81$ m/s^2.

—**Jeffrey R. Stribling,** *CA A '92*

COMPUTER BONUS:

Define SD(n) as the sum of digits of n reduced to a single digit, where n is a positive integer. For example, SD(25) → 2 + 5 = 7, and SD(2598) → 2 + 5 + 9 + 8 = 24 → 2 + 4 = 6. For some n, n/SD(n) is an integer, e.g., 81/SD(81) = 9. For other n, it is not, e.g., 85/SD(85) = 85/4 = 21.25. Find the longest possible sequence of integers for which n/SD(n) is an integer and give the starting integer of the first such sequence.

—Allan Gottlieb's Puzzle Corner in *Technology Review*

Email your answers (plain text only) to any or all of the Winter Brain Ticklers to *BrainTicklers@tbp.org* or by postal mail to **Dylan Lane, Tau Beta Pi, P.O. Box 2697, Knoxville, TN 37901-2697**. The method of solution is not necessary. The Computer Bonus is not graded. Where possible, exact answers are preferable to approximations. The cutoff date for entries to the Winter column is the appearance of the Spring *Bent* which typically arrives in late March (the digital distribution is several days earlier). We welcome any interesting problems that might be suitable for the column. Dylan will forward your entries to the judges who are **J.C. Rasbold,** *OH A '83;* **F.J. Tydeman,** *CA Δ '73;* **G.M. Gerken,** *CA H '11;* and the columnist for this issue,

— **J.R. Stribling,** *CA A '92*