Ticklers each earned a correct answer on slightly more than half the submissions.

**FALL ANSWERS**

1. The simultaneous cryptic additions:
   
   * * * * * M I R R O R
   
   M I R R O W  I M A G E *
   
   I M A G E * * E G A M I
   
   have the solution:
   
   4 7 2 6 5 2 7 4 4 3 4
   
   2 7 4 4 3 4 7 2 1 6 9 3
   
   7 2 1 6 9 9 9 6 1 2 7

2. The near anti-magic square we wanted (or its transpose) is:

   | 1 | 7 | 2 |
   | 9 | 3 | 5 |
   | 6 | 4 | 8 |

3. Every 19 years, the moon, as observed from Earth, appears in the same place in the sky on the same date. A second full moon in the same month (a Blue Moon) on the same date will occur when an integral number of solar years and an equivalent integral number of lunar months have passed since the last Blue Moon. Let \( y = 365.2425 \) days/year, \( M = 29.53059 \) days/synodic-month. Then, the expression:

   \[ F(n) = nY - M\lceil\text{round}(nY/M)\rceil \]

   gives the relationship between the number of solar years and the corresponding number of synodic months, whose difference (in days) should be very close to 0. Trying various values shows that \( n=19 \) years gives the smallest value for \( F(n) \). This is known as a Metonic cycle, which is 6,940 days (19.0010 solar years or 253.0105 lunar months).

4. The pizza shop has 11 toppings, 1,048,576 factors to 10th, so, each pizza has 10^3 possible combinations of toppings. Use the choose function \( C(m,n) = m!/(n!(m-n)!) \), which is the number of ways of choosing \( m \) objects, \( n \) at a time. Since each pizza can have 0 to 5 toppings, use the choose function 6 times: \( C(N,0) + C(N,1) + C(N,2) + C(N,3) + C(N,4) + C(N,5) \), where \( N \) is the number of toppings from which a selection can be made. Trying various values of \( N \) shows that when \( N=11 \) there are 1+11+55+165+330+462=1024 ways to order one pizza or 1024^2 = 1,048,576 ways to order 2.

5. The order of merit is (first to last) **ABCD**. If E truth \( \rightarrow B \) is 2nd \( \rightarrow B \) lies \( \rightarrow E \) is 1st (and lies) \( \rightarrow B \) not 2nd \( \rightarrow NO \). If E lies (E is 1st or 2nd) \( \rightarrow B \) not 2nd. If B is 1st (E 2nd) \( \rightarrow B \) lies \( \rightarrow E \) 1st (and lies) \( \rightarrow NO \). If B 3rd \( \rightarrow B \) truth \( \rightarrow E \) not 1st \( \rightarrow E \) is 2nd. If C lies then C is last \( \rightarrow E \) not 3rd \( \rightarrow E \) 1st (truth) \( \rightarrow B \) not 2nd \( \rightarrow NO \). So, A lies (is 1st) \( \rightarrow D \) not 3rd (4..5) and truth C is lower than B.

**Bonus.** As the angle of the sector approaches zero, the ratio (area of circles / area of sector) is largest and approaches zero, the ratio (area of circle / area of sector) is largest and approaches zero. Let the circles touch each other and the edges of the sector. For the circle that touches the unit circle, \( \sin \theta = r_j/j(1-r_j) \). Solving for \( r_j \), we get \( r_j = \sin \theta/(1+\sin \theta) \) which we call \( k \). For the next circle, \( \sin \theta = r_j/(1-r_j) \). Solving for \( r_j \), we get \( r_j = k(1-2k) \). For the next circle, \( \sin \theta = r_j/(1-r_j) \). Solving for \( r_j \), we get \( r_j = k(1-2k)^2 \). In general, \( r_j = k(1-2k)^{n-1} \). The area of all the circles is \( \pi(k^2)(1+ (1-2k)^2 + (1-2k)^4 + ... ) = \pi(k^2)(1/(1-2k^2)) \).

**SUMMER REVIEW**

There was a little confusion with #5, the question about poker showdown. Our problem statement did not specify the hierarchy of cards. We thought ace-high was implicit and were looking for an answer of the King of Clubs. However, the judges accepted a different answer, including any that clearly specified the 45th ranked card from the bottom, regardless of order.

Problem #3, asking the number of ways to walk a flight of stairs, was the easiest of Ticklers, drawing a correct answer on about ¾ of the entries. The other four regular
= \((\pi/4)(k/(1−k))\) = \((\pi/4)\sin\Theta\). The area of the sector is \((2\Theta/(2\pi))\pi Y^2\) = \(\Theta\). So, the ratio is \((\pi/4)\sin\Theta\), which has a maximum as \(\Theta\) approaches zero, and that maximum is \(\pi/4\) (equal to the ratio of a circle and its enclosing square).

**Computer Bonus.** 5,536,785,000/369,119 = 15,000 is the third largest prime such that it divides the sum (5,536,785,000) of all the 31,463 primes smaller than itself.

**NEW WINTER PROBLEMS**

1 Former Brain Ticklers’ judge Don A. Dechman (TX A ’57) astutely pointed out that our illustrious head judge Howard G. McIlvried III (PA Γ ’53) has just celebrated an amazing sixty years supporting this column. In commemoration of this event, I thought I would throw together some accolades for him. Find a unique solution to the following cryptarithm: PROPS + KUDOS = HOWARD. Not one to miss an opportunity, Howard replied, “it has been a LABOR + OF + LOVE + FOR + OVER = 60 + YEARS, and they have been prime YEARS.” Solve the two cryptics individually. Standard rules apply: each different letter stands for a different digit, and each different digit is always represented by the same letter; no leading zeros are allowed.

—J.R. Stribling, CA A ’92 and H.G. McIlvried III, PA Γ ’53

2 Adam, Brad, Chet, Doug, and Evan went abroad with their respective wives on vacation last summer to five different destinations. Three of the pairs ventured into Asia with trips to Agra, Bali, or Doha, while the other two couples traveled to Europe and visited either Cork or Elba. I knew the names of the five wives were Anne, Beth, Cate, Dawn, and Emma, but the only information I had about who was married to whom was that for each pair the names of the husband, the wife, and last year’s holiday location all began with different letters.

In an attempt to discover more details, I had some conversation with three of the ladies. Beth told me she was not married to Adam and she had heard from Evan that Chet went to Doha last year. Dawn, however, firmly informed me that Chet went to Elba and Beth went to Doha. “Unlike some people I could mention,” she added darkly, but rather irrelevantly, “Adam always tells the truth.” Cate said when her husband was asked whether Emma was married to Chet he replied ‘No.’ She went on to tell me that Doug went to Bali.

Given that a reliable source divulged to me the curious fact that of each of these married couples, one member always told the truth and the other never did, deduce the name of each man’s wife and where they all went for their vacations.

—Brain Puzzler’s Delight by E.R. Emmet

3 For a spherical Earth in a circular orbit around the sun with the Earth axis tilted at an angle of 23 degrees, find an exact trigonometric expression to give the solar noon latitude as a function of time \(t\) in an Earth year of duration \(Y\). You may assume the time of vernal equinox \(t_{\text{eq}}\) is known to use as a reference point.

—Allan Gottlieb’s Puzzle Corner in Technology Review

4 Yesterday, I rode my bicycle to work, a straight-line distance of 10 kilometers over level ground. In the morning the trip took 30 minutes with a direct headwind; but in the afternoon the trip took only 20 minutes with the same wind (now a tailwind). Assume the only force I overcame was aerodynamic viscous drag, which is proportional to the square of my speed relative to the air. Assuming all other forces (friction, etc.) are negligible and that I generate constant power, calculate yesterday’s wind speed (to the nearest 0.01 km/hr). If the wind is calm today, determine how many minutes the one-way trip will take (to the nearest 0.01 minute).

—J.L. Griggs Jr., OH A ’56

5 Consider a rectangular strip of leather, white on one side and black on the other. Cut two long slits parallel to the sides of the leather strip, but do not cut through the ends of the leather strip. The result is three leather strips joined at both ends, as shown in the following picture:

The objective is to braid the strips without cutting or altering the leather in any way except folding. Find the minimum non-zero number of strip crossings required to ensure that each side of the completed braid is a uniform color. Assume that only two strips cross at a given point and that no crossing is wide enough to hide a twist in the strip.

—Adapted from Professor Stewart’s Hoard of Mathematical Treasures by Ian Stewart

**Computer Bonus.** Consider sequences of exactly thirteen consecutive primes, such that the difference between any two adjacent primes is at most 8. The first such sequence is 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, and 41; there are other examples with small primes — in fact, there is even a sequence.

Brain Ticklers continue on page 36.
Convention

There is a lot of important work to be accomplished in the committees and we rely on your dedication to the tasks at hand.

Also, similar to recent years, we have an excellent line-up of Professional Development Sessions to attend.

Last year, we received feedback that Convention attendees were interested in having more corporations at the Career Fair. We listened. This year, we worked hard to recruit companies and to grow participation in the event. We have 22 companies and 56 total recruiters — our biggest Career Fair yet. Please take advantage of this impressive list.

Finally, I want to mention the photo opportunity on Friday. This was a popular activity last year and we received very favorable feedback from the Convention in Denver; again we listened.

Candidates

For most of the year, the Executive Council governs the operations of Tau Beta Pi. Having capable, professional, and dedicated Councillors is critical to pursuing our professional, and dedicated Councillors is critical to pursuing our operations of Tau Beta Pi. Having capable, professional, and dedicated Councillors is critical to pursuing our goals; and we rely on your dedication to the tasks at hand.

I want to mention one particular Tau Beta who joined our Chapter Eternal this year. Lido ‘Lee’ Iacocca, PA A ’83, at Lehigh University, passed away on July 2, 2019. Former Chairman of Chrysler, Mr. Iacocca spoke at Convention in 1985 when we celebrated 100 years of Tau Beta Pi.

Iacocca always applied the “Nine Cs” of leadership to his executive career and I thought it appropriate to share a couple of them with you as you embark on your careers. I’ve applied them to my own career.

A leader has to be able to communicate effectively. A leader has to have charisma to inspire others and cultivate trust. And a leader has to be a person of character to have the guts to do the right thing in the face of adversity.

WE are the ones who will continue to solve the world’s most daunting challenges and change life in profound ways.

WE are the ones who will be responsible for advancing the standard of living and developing things thought impossible.

As Orville Wright stated, “If we all worked on the assumption that what is accepted as true is really true, there would be little hope of advance.”

So, let’s relentlessly question the status quo. After nearly 20 years as a lawyer, that’s why I remain committed to my engineering roots.

It has been a privilege serving as president of Tau Beta Pi this year, but I have most enjoyed simply serving as a volunteer and helping to advance the goals of our Association.

I look forward to meeting many of you these next few days.

Neil Armstrong once said “I believe every human has a finite number of heartbeats. I don’t intend to waste any of mine.” Don’t waste any of yours.

Thank you and have a great Convention.

Wayne B. Paugh, LL.M., JD, Florida Gamma ’93, has served as an Executive Councillor since 2017, Vice President for 2018, and President for 2019. He has served as a TBIU Engineering Futures Facilitator for 6 years. Wayne earned a B.S. (mechanical engineering) and B.A. (communications) from the University of South Florida. He was an applications engineer until earning a graduate degree in the management of technology. He then pursued a career change by attending law school in Washington, DC, and specialized in intellectual property. He spent 13 years in DC and worked primarily in the federal government in executive and managerial roles. During this time, Wayne served on Capitol Hill and was the Chief of Staff of the U.S. Patent and Trademark Office. He was also appointed U.S. coordinator for international intellectual property enforcement by President George W. Bush. He now serves as special assistant U.S. attorney in the major crimes division.

BRAIN TICKLERS

Continued from page 27.

Postal mail your answers to any or all of the Brain Ticklers to Dylan Lane, Tau Beta Pi, P.O. Box 2697, Knoxville, TN 37901-2697 or email to BrainTickers@tbp.org as plain text only. The cutoff date for entries to the Winter column is the appearance of the Spring Bent which typically arrives in late March (the digital distribution is several days earlier). The method of solution is not necessary. We welcome any interesting problems that might be suitable for the column. The Computer Bonus is not graded. Dylan will forward your entries to the judges who are H.G. McIlvried III, PA F ’53; F.J. Tydeman, CA A ’73; J.C. Rasbold, OH F ’95; and the columnist for this issue.

—J.R. Stribling, CA A ’92