Brain Ticklers

RESULTS FROM SUMMER

Perfect

*Dechman, Don A.	TX	A	'57
*Gee, Albert	CA	A	'79
Gee, Nora W.	CA	A	'79
*Gerken, Gary M.	CA	H	'11
*Griggs Jr., James L.	OH	A	'56
Gulian, Franklin J.	DE	A	'83
Gulian, William F.	Son of member		
Johnson, Mark C.	IL	A	'00
*Norris, Thomas G.	OK	A	'56
*Richards, John R.	NJ	B	'76
*Roche, James R.	IN	Γ	'85
Roche, Kevin M.	Son of member		
Schmidt, V. Hugo	WA	B	'51
*Slegel, Timothy J.	PA	A	'80
*Spong, Robert N.	UT	A	'58
*Strong, Michael D.	PA	A	'84

Other

Alexander, Jay A.	IL	Γ	'86
Aron, Gert	IA	B	'58
Beaudet, Paul R.	Father of member		
Bohdan, Timothy E.	IN	Γ	'85
Budd, Christopher M.	AZ	B	'94
Cohn, Ronald P.	PA	Z	'72
*Couillard, J. Gregory	IL	A	'89
Dahmen, Randall R.	WI	A	'85
Edge, Billy L.	GA	A	'71
Fogel, Arlene Beck	MA	Δ	'77
Handley, Vernon K.	GA	A	'86
Hasek, William R.	PA	Γ	'49
Hawkins, Debbre S.	Wife of member		
Hedegard, Alan H.	IN	A	'64
Janssen, James R.	CA	Γ	'82
Jordan, R. Jeffrey	OK	Γ	'00
Lalinsky, Mark A.	MI	Γ	'77
Mahoney, John	Non-member		
Marrone, James D.	IN	A	'87
Marrone, James I.	IN	A	'61
McCullough, Charles R.	AL	B	'12
Mettler, Kelly M.	CA	Δ	'10
Mettler, Rick A.	WA	B	'81
Rentz, Peter E.	IN	A	'55
Rentz, Mark	Son of member		
Roggli, Victor L.	TX	Γ	'73
Ross, Karen M.	IN	A	'79
Sauer, Daniel M.	MI	B	'05
Schweitzer, Robert W.	NY	Z	'52
Shepperd, Stanley W.	MA	B	'70
Svetlik, J. Frank	MI	A	'67
*Thaller, David B.	MA	B	'93
*Voellinger, Edward J.	Non-member		
Zison, Stanley W.	CA	Θ	'87

*Denotes correct bonus solution

SUMMER REVIEW

Nearly every entry correctly answered problem #3, the algebra problem taken from the 2004 classic cult movie, *Mean Girls*. Admittedly, the columnist expected the problem to be easy, although it did help North Shore HS win the Illinois State Mathletes championship. Tickler #5, concerning choosing pairs of socks from a drawer, was the most challenging regular problem, and nearly as difficult as the Bonus. Only about two-thirds of respondents answered #5 correctly. The problem was computationally intensive and some found it counterintuitive that the two weekdays in the solution were not the same.

FALL ANSWERS

1

```
SIXTEEN + TWENTY + TWENTY + TEN + TWO + TWO = SEVENTY
3092665 + 276528 + 276528 + 265 + 271 + 271 = 3646528
```

Arrange the addends above each other and designate the columns from right to left as C_1 to C_7. Since there is no carry from C_6 to C_7, T must be less than 5. So, start by assigning a value (1, 2, 3, or 4) to T. Note that C_4 consists only of T's and E's, so if we have a value for T and can estimate the carry from C_3, we can calculate E. Now, C_6 consists only of T's, E's, and an I, so we can calculate I. Also, C_3 is composed of only N's, T's, and an E, so we can calculate N and C_1. One other observation that can be made is that X and V are closely coupled (see C_5) and C_2 is made up of only O's, Y's, and N's, so O and Y are also closely coupled. At this point, five of the letters have values assigned and five do not, but we have a relationship between X and V, so if we pick a value for X from the unused values the corresponding value of V must also be one of the unassigned values. Finally, two of the last three values must work for Y and O. S can have any value, so we need not be concerned about S. Care needs to be exercised to carefully keep track of carries. The unique answer presented above results when T = 2. No valid solutions are found for other values of T.

2 Hook (H) caught **16**, Line (L) caught **8**, and Sinker (S) caught **2** fish. For H to know, H has exactly one factorization with two different numbers. So, H is either a product of two different primes, or a cube, or a fourth power of a prime. For L to know, L cannot be a prime larger than 3, but could be a square or cube of either 2 or 3. Since S is an even number, the only choices are: 2×3=6, 2×4=8 and 2×8=16. Since George (G) determines the others, L < G < H, therefore, H=16, L=8, S=2.

3 **4562** is number of ways (order does not matter) that a \$100 bill can be changed using \$1, \$2, \$5, \$10, \$20, and \$50 bills. The bills \$1, \$2, and \$5 must sum to a multiple of 10, such as 10m, for m in the range of 0 to 10. Now, one can find that the number of ways of getting 10m using 1, 2, and/or 5 is $(4+(10m+4)^2)/20$. On the other hand, the problem of breaking $100-10m = 10(10-m)$ into 10's, 20's, and 50's is equivalent to that of writing $10-m$ as the sum of 1's, 2's, and 5's. That is $\text{floor}((4+(10-m+4)^2)/20) = \text{floor}((4+(14-m)^2)/20)$. The two parts can be combined via a dot product to get: 1×10 + 10×8 + 29×7 + 58×6 +97×5 + 146×4 + 205×3 + 274×2 + 353×2 + 442×1 + 541×1 = 4562.

4 **Fourteen** queens can be placed on a chess board so that each queen could capture exactly two other queens. There are two basic arrangements, as shown below:

QQQ-Q-Q-	QQ-Q----
-------Q	----Q--Q
-------Q	Q-------
Q-------	-------Q
-------Q	Q-------
Q-------	--Q----Q
-----Q--	Q----Q--
Q--Q-Q--	--Q---Q-

5 **Three** weighings are all that are needed to find which (if any) of the 12 bottles have pills that are heavy. Weigh 1+2+4+8 pills from four bottles. Repeat for next four bottles and last four bottles. Each weighing will determine which (if any) of the four bottles are heavy.

Bonus A has **32** on his forehead and B has **530** on his forehead. If the sum of squares can be done in only one way, such as $13=2^2+3^2$, then A will know the two numbers used to form the sum of squares and, therefore, the sum. So, the sum of squares must be done in two different ways. Any group with the sum that has only one sum of squares being two ways (such as 9: $5^2+4^2=41$, $7^2+2^2=53$, $8^2+1^2=65=7^2+4^2$) will be solved by B. So, we need the sum having two ways to do sum of squares in two ways. We need to alternate between those two as follows:

```
    m+n    m^2+n^2   m   n
{   16,    130,  9,   7 }
              |              130 = 11^2+3^2 = 9^2+7^2
{   14,    130, 11,   3 }
     |                       14 = 13+1 = 11+3
{   14,    170, 13,   1 }
              |              170 = 11^2+7^2 = 13^2+1^2
{   18,    170, 11,   7 }
     |                       18 = 17+1 = 11+7
{   18,    290, 17,   1 }
              |              290 = 13^2+11^2 = 17^2+1^2
{   24,    290, 13, 11 }
     |                       24 = 23+1 = 13+11
{   24,    530, 23,   1 }
              |              530 = 23^2+1^2 = 19^2+13^2
{   32,    530, 19, 13 }
```

This is the only chain that is this long.

Another way to look at this is as follows. Create a list L of ordered pairs (m,n) such that integers m and n are relatively prime. (The logicians are smart enough to track an infinite number of pairs in L, but the judges found $m < n < 40$ to be sufficient.) As a consequence of A's first statement, remove pairs (m,n) from L where m^2+n^2 is unique. B's first statement makes us then remove pairs (m,n) from the updated L, where $m+n$ is unique. Repeat three more times each, corresponding to subsequent statements by A and B, updating L each time. At the point when A makes their fifth statement, L contains exactly one pair with a unique sum of squares, $13^2+19^2=530$, from A which can determine his number, 32.

Computer Bonus **7442**, **28658**, **148583**, **177458**, and **763442** are five different positive integers such that the sum of any two of them is a perfect square.

NEW WINTER PROBLEMS

1 My elder son has just entered as a freshman at Cornell University (NY Δ). Being a California native, I was naturally concerned how he would adjust to his first chilly winter there in Ithaca. Find a unique solution to the following cryptarithm to alleviate my fears: CORNELL + STUDENT = ENDURES. Standard rules apply: each different letter stands for a different digit and each different digit is always represented by the same letter; leading zeros are not allowed.

—Jeffrey R. Stribling, *CA A '92*

2 In the World of Bonkers, the ditty for remembering the numbers of days in their months runs as follows:

'Nineteen days hath Cucumber,
Strawberry too, and the number
of days there are in Tiddleywinks
is just the same as 'tis in Jinks,
which is seven whole days more
than the ten there are in Pinafore.
With the month of Collywobble,
diary makers have some trouble;
thirteen days if year is even,
forty-four if odd. And Stephen
quite the least of Bonkers' months,
in fact it's sometimes called the
runt, as you can easily understand,
since it hath but four days
and another two in years of Heaven
(that's if the year's date ends in
seven).'

There are five days in each week —Joyday, Funday, Laughday, Blissday, and Workday, in that order.

In the year 17 a.b. (after bliss) the King of Bonkers went off for his holiday on Funday, Strawberry 17th and returned, just over 7 weeks later, on Blissday, Tiddleywinks 8th. In the year 19 a.b., there were exactly 10 weeks from Collywobble 41st to Pinafore 7th. Note that Collywobble is the first and most important month of the seven which comprise the year.

What is the order of the seven months of the Bonkers' year? Given Strawberry 13th was a Workday in the year 21 a.b., what day of the week was Collywobble 2nd?

—Brain Puzzler's Delight by E.R. Emmet

3 Find the smallest rational x for which $x–5$, x and $x+5$ are all squares of rational numbers (that is, each number can be written as some p^2/q^2).

—Mathematical Tournament from 1225

4 Primeministan County has a cluster of small towns interconnected by a series of roads, such that there is only a single route between any two towns. Interestingly, when sorted by distance, the ten routes in the county (measured in km) are unique consecutive primes. What is the minimal total road length in Primeministan? How many towns are there and how are the roads connected in this topology?

—Hunter W. Hagadorn, *PA E '59*

5 Start with a 10 cm square piece of paper and color in a 3 mm wide red border on both sides along and including all four edges. Fold the paper so that no red shows, not even a corner or an edge. Paper must either fully wrap around an edge or the edge must be recessed at least 3 mm from the outer perimeter of the folded shape (and be covered) to be considered hidden. If the paper has been folded and a new fold makes more than one crease, the new fold is considered as only one fold. What is the minimum number of folds required?

—The Man in the Milk Carton by Stephen Barr

Bonus Frank has a tablecloth covered with 1 cm diameter black polka dots. The polka dots form a square grid with 2 cm between neighboring centers. He takes a flat 4 cm by 4 cm piece of clear plastic and paints four 1 cm diameter circles on it in the same pattern as on the tablecloth. After laying the tablecloth on a flat surface, he randomly tosses the piece of plastic onto it. To three significant figures, what is the probability that none of the polka dots on the plastic will overlap any portion of any of the polka dots on the tablecloth?

—Don A. Dechman, *TX A '57*

Computer Bonus In the range from 1 to 1,000,000,000, what is the longest string of consecutive integers without a semiprime? A semiprime is an integer that is the product of exactly two (not necessarily different) primes. As an example, 96 to 105 is a string of 10 integers without a semiprime.

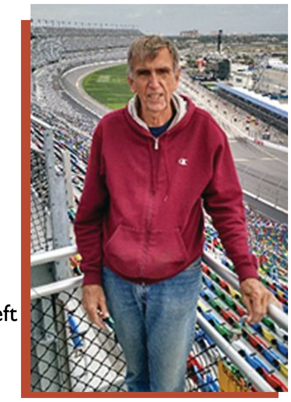
—Howard G. McIlvried III, *PA Γ '53*

Postal mail your answers to any or all of the Brain Ticklers to **Tau Beta Pi, P. O. Box 2697, Knoxville, TN 37901-2697** or email to *BrainTicklers@tbp.org* as plain text only. The cutoff date for entries to the Winter column is the appearance of the Spring *Bent* which typically arrives in late March (the digital distribution is several days earlier). The method of solution is not necessary. The Computer Bonus is not graded. We additionally welcome any interesting problems that might be suitable for the column. Entries will be forwarded to the judges who are **H.G. McIlvried III,** *PA Γ '53*; **F.J. Tydeman,** *CA Δ '73*; **J.C. Rasbold,** *OH A '83;* and the columnist for this issue,

—J.R. Stribling, *CA A '92*

Stories of "Gratefulness"

Tau Beta Pi Major Gifts Officer, **Sherry Jennings-King**, *TN A '93*, shares stories of "gratefulness" during this holiday season. Contact Sherry for more information on the Chapter Endowment Initiative at *sherry.jenningsking@tbp.org*.



Sherry,
Very nice article on the Chapter Endowment Initiative in the Spring 2018 issue of *The Bent*. You look good in a hard hat. The last sentence in the Randy Alexoff interview expresses my feeling: It's nice to know you've left a legacy for future engineering students. Thanks for making that happen for me.

—**Steve Ricks,** *IN A '63*
Chapter Endowment Initiative donor, pictured at Daytona International Speedway.

Note from CA Alpha at UC Berkeley students to Paul Cocotis, CA A '90, Chapter Endowment Initiative donor:

Dear Paul,

This spring, ΤΒΠ HQ informed us that you had given to our Chapter Endowment Fund. Our officers would sincerely like to thank you for your contribution. The endowment fund will greatly help our chapter continue its mission of excellence in academics and contributions to the community.

CA Alpha holds a variety of professional workshops and industry info-sessions and provides an exam database, resume critiques, and mock interviews free to all Berkeley students. Again, thank you so much for your contribution. It gets us one important step closer to attaining our goals as an honor society.

—**Laura E. Brandt,** *CA A '18*
Chapter President