Brain Ticklers

RESULTS FROM SUMMER

Perfect

*Ammar, Gregory S.	OH	A	'81
Bhatia, Sujata K.	DE	A	'99
*Couillard, J. Gregory	IL	A	'89
Diener, John D.	DE	A	'92
*Gerken, Gary M.	CA	H	'11
*Griggs Jr., James L.	OH	A	'56
*Gulian, Franklin J.	DE	A	'83
Gulian, William F.	Son of member		
Heske III, Theodore	PA	A	'86
Johnson, Mark C.	IL	A	'00
*Kimsey, David B.	AL	A	'71
*Marks, Lawrence B.	NY	I	'81
Marks, Benjamin	Son of member		
*Norris, Thomas G.	OK	A	'56
Richards, John R.	NJ	B	'76
*Schmidt, V. Hugo	WA	B	'51
*Slegel, Timothy J.	PA	A	'80

Other

Alexander, Jay A.	IL	Γ	'86
Bannister, Kenneth A.	PA	B	'82
Bernacki, Stephen E.	MA	A	'70
Brule, John D.	MI	B	'49
Cruise, George W.W.	NM	A	'18
*Dechman, Don A.	TX	A	'57
DeSelms, Bradley C.	MO	A	'82
Edel, Kentworth M.	AZ	A	'83
Ehrgott Jr., Charles	FL	E	'92
Grewal, Rashi	NJ	Γ	'09
Hasek, William R.	PA	Γ	'49
*Janssen, James R.	CA	Γ	'82
Lalinsky, Mark A.	MI	Γ	'77
Marrone, James I.	IN	A	'61
*Mayer, Michael A.	IL	A	'89
Quan, Richard	CA	X	'01
*Riedesel, Jeremy M.	OH	B	'96
*Roche, James R.	IN	Γ	'85
Schweitzer, Robert W.	NY	Z	'52
Sigillito, Vincent G.	MD	B	'58
Spong, Robert N.	UT	A	'58
*Stadlin, Walter O.	NJ	Γ	'52
*Stein, Gary M.	FL	Δ	'04
Summerfield, Steven L.	MO	Γ	'85
Svetlik, J. Frank	MI	A	'67
*Tweeton, Bruce K.	ND	A	'84

* Denotes correct bonus solution

SUMMER REVIEW

It was no surprise that the easiest problem was No. 3, the algebra puzzler taken from the popular movie *Little Man Tate*. Every entry submitted the right answer. Slightly more than 50% of entries correctly answered No. 2, finding the dimensions of a floor in a hemispherical building. It was marginally more difficult than the other three regular problems. The Bonus was most challenging, though the Summer columnist expected it to be harder than it was. Nearly half of all submissions were able to accurately slice the Egyptian loaves of bread.

FALL ANSWERS

1 Ned's house number is 64, Aunt Alice's is 16, X's is 100, Y's is 81, and Z's is 49.

For N to know X, N is 16 or 64 and X is 100 or the other of 16/64. For A to know X, A is the other of 16 or 64; therefore, X is 100.

For N to know Y, Y is 9 or 81. For A to know Y, 9 < A < 81; therefore, Y = 81.

For N to know Z, Z is 25 or 49. Since A < Z (but unknown), it follows that A < 25. Since A guesses 30 < Z, Z = 49. It can be concluded that A = 16 and N = 64.

2 Salem beat Parminster 5-3, lost to Quondam 3-4, and lost to Real 0-1.

For R to have 2 wins with just 2 goals, R's scores are 1,1,0 and opponents' are 0,0,0. There are three ways for R to get 1,1,0, so try all three. First fill in the R row and column.

P must have either 5 or 6 goals scored against. That number of goals is too high for Q to score, hence P-Q has not been played yet. Therefore, P-S is 3-5 or 3-6.

Next fill in the Q-S score of 4-2 or 4-3. Checking for Q having a win and a draw finds that R's draw against S or P produces invalid results. Therefore, R drew against Q.

3 The hands are symmetric about 6 at **08:18:27.6923...** and the hands are in the Golden Mean fraction at **08:18:38.0971...**

Set up equations in terms of fraction of revolution from the 12 o'clock position.

For the symmetric case, the minute hand is M/60; the hour hand is 1 – ((8/12)+(M/60)/12). The hands being symmetric implies M=240/13, or 18 6/13 minutes past 8. For the Golden Mean case, the clockwise angle from the hour hand to the minute hand is the Golden Mean fraction of the whole circle, implying: M/60 + 1 – ((8/12)+(M/60)/12) = (√5 – 1)/2 which leads to M=(360√5-600)/11 or 18 (360√5-798)/11 minutes past 8.

4 The server has a probability $\boldsymbol{w = p_1q_1 + p_2q_2 - p_1p_2q_2}$ of winning a point and has a probability $\boldsymbol{g = w^4 (15 - 34w + 28w^2 - 8w^3) / (2w^2 - 2w + 1)}$ of winning the game. Letting $p_1 = 0.8$, $p_2 = 0.7$, $q_1 = 0.6$, and $q_2 = 0.5$, one gets **w = 11/20 = 0.55** and **g = 50,350,399/80,800,000 = 0.6231485...** The probability of winning a point is $w = p_1 q_1 + (1-p_1)p_2 q_2 = p_1 q_1 + p_2 q_2 - p_1 p_2 q_2$, where p_1 = probability of a successful first serve, p_2 = probability of a successful second serve, q_1 = probability of winning a point if the first serve is good, and q_2 = probability of winning a point if the second serve is good. There are four ways to win a game: the server can win 4 points while the other player wins 0, 1, or 2 points, or they can both win 3 points (in this case, the game goes to deuce, as the score is tied). The probability of winning 4 straight points is w^4; of winning 4 out of 5 points is $C(4, 1)w^4(1-w) = 4w^4(1-w)$; of winning 4 out of 6 points is $C(5, 2)w^4(1-w)^2 = 10w^4(1-w)^2$, where $C(i, j)$ is the number of combinations of i objects taken j at a time. This leaves deuce to be considered. Scores of 5-3, 6-4, ..., require passing through 3-3. For a score of 3-3, the probabilities for the next 2 points are: (1) ww; (2) wL; (3) Lw; and (4) LL, where L = $1-w$. For Case 1, the server wins the game: for Case 4, the server loses; for Cases 2 and 3, the situation reverts to deuce. Since Cases 2 and 3 have no effect on winning or losing the game, we need consider only Cases 1 and 4. Thus, the probability of winning the deuce game is $w^2/(w^2+L^2)$, but there are $C(6, 3) = 20$ ways to get to 3-3, so the probability of winning Case 1 = $20w^3(1-w)^3 w^2/[w^2+(1-w)^2]$. Summing the four cases gives g = $w^4 + 4w^4(1-w) + 10w^4(1-w)^2 +$

$20w^3 (1-w)^3 w^2 /[w^2 + (1-w)^2] = w^4 (15 - 34w + 28w^2 - 8w^3)/(2w^2 - 2w + 1)$.

5 FIVE + FIVE + TEN + TEN + TEN + TEN + THIRTY = EIGHTY
8052 + 8052 + 124 + 124 + 124 + 124 + 190317 = 206917

Either (2E+4N+Y) mod 10 = Y or (2E+4N) mod 10 = 0 or 2(E+2N) mod 10 = 0 or (E+2N) mod 5 = 0.

Since THIRTY and EIGHTY are 6 digits and FIVE and TEN are 4 or fewer digits, E = T+1 implies the need for a carry in of 1. I = H+1 mod 10 with a carry in of 1 and carry out of 1. Therefore, H=9 and I=0. 2F needs to be at least 10 to get a carry out of 1. Either (carry in +2V+4E+T) mod 10 = T or (carry in+2V+4E) mod 10 = 0 implies that the carry in is even, so 2E+4N is 20 or E+2N is 10, which implies E is even and T is odd. We try the few cases that are possible: N=1 => E=8 => T=7 => V=3 => R=7 (failure). N=2 => E=6 => T=5 => V=2 (failure). N=3 => E=4 => T=3 (failure). N=4 => E=2 => T=1 => V=5 => R=3. Since F cannot be 6 or 7, F must be 8, leading to G=6 and Y=7.

Bonus The rectangles with minimum waste are **19x27** for 11^2 and **23x29** for 12^2.

1^2 thru 11^2 can fit in a 19x27 with a waste of 7 as:

```
88888888555554444aaaaaaaaaa
88888888555554444aaaaaaaaaa
88888888555554444aaaaaaaaaa
88888888555554444aaaaaaaaaa
888888885555522--aaaaaaaaaa
88888888333-122--aaaaaaaaaa
88888888333666666aaaaaaaaaa
88888888333666666aaaaaaaaaa
bbbbbbbbbbb666666aaaaaaaaaa
bbbbbbbbbbb666666aaaaaaaaaa
bbbbbbbbbbb666666-999999999
bbbbbbbbbbb666666-999999999
bbbbbbbbbbb7777777999999999
bbbbbbbbbbb7777777999999999
bbbbbbbbbbb7777777999999999
bbbbbbbbbbb7777777999999999
bbbbbbbbbbb7777777999999999
bbbbbbbbbbb7777777999999999
bbbbbbbbbbb7777777999999999
```

1^2 thru 12^2 can fit in a 23x29 with a waste of 17 as:

```
cccccccccccc9999999997777777-
cccccccccccc9999999997777777-
cccccccccccc9999999997777777-
cccccccccccc9999999997777777-
cccccccccccc9999999997777777-
cccccccccccc9999999997777777-
cccccccccccc9999999997777777-
cccccccccccc99999999922666666
cccccccccccc99999999922666666
cccccccccccc--1----4444666666
cccccccccccc--555554444666666
cccccccccccc--555554444666666
bbbbbbbbbbb333555554444666666
bbbbbbbbbbb33355555aaaaaaaaaa
bbbbbbbbbbb33355555aaaaaaaaaa
bbbbbbbbbbb88888888aaaaaaaaaa
bbbbbbbbbbb88888888aaaaaaaaaa
bbbbbbbbbbb88888888aaaaaaaaaa
bbbbbbbbbbb88888888aaaaaaaaaa
bbbbbbbbbbb88888888aaaaaaaaaa
bbbbbbbbbbb88888888aaaaaaaaaa
bbbbbbbbbbb88888888aaaaaaaaaa
bbbbbbbbbbb88888888aaaaaaaaaa
```

Computer Bonus. The (decimal) count of and the sum of all palindromic numbers between 1 and $100{,}000{,}000_{base}$, inclusive, are shown in the table for bases 2 through 10:

Base	Count	Sum
2	30	2,632
3	160	327,039
4	510	9,986,436
5	1,248	141,759,070
6	2,590	1,240,259,700
7	4,800	7,770,547,197
8	8,190	38,125,189,144
9	13,120	155,181,063,996
10	19,998	545,045,045,040

WINTER PROBLEMS

1 Hey baby, it's cold outside! What is the solution to the following winter-themed cryptarithm? SLEET + SNOW = FROSTY. Each different letter stands for a different digit, and each different digit is always represented by the same letter; no leading zeros are allowed. Standard rules apply.

—**Jeffrey R. Stribling,** *CA A '92*

2 A high school shop class decides to make dollhouse furniture for a Toys for Tots drive. Each of the five students made only one kind of toy. Altogether, the students made 30 toys, but each made a different number greater than two. Gail made one more than the student who wore lilac and one less than the student who made desks. Hal made chairs. Irene made five toys. The student who wore mauve made the end tables. The student wearing navy made only one third as many toys as Jill. Kyle wore orange; one student wore purple; and one student made beds. There were more floor lamps than any other toy. Match names, colors worn, and toys made.

—*Read Magazine*

3 Provide an exact expression of a nontrivial, continuous, real-valued function $f(x)$ with $f(0) = 1$ possessing continuous derivatives of all orders, and satisfying the infinite-order differential equation $f = f' + 2f'' + 3f''' + 4f'''' + \ldots$

—Adapted from *Technology Review*

4 Construct n equally spaced points on the circumference of a unit circle and label the points from 1 to n. Now, construct chords from point 1 to all the other points (a total of n-1 chords). What is the product of the lengths of these chords? That is, what is the value of $P = \prod_{i=2}^{n} L_i$, where L_i is the length of the chord from point 1 to point i.

—*The Call of the Primes* by Owen O'Shea

5 The monk's abbey has 13 rooms: 12 cells for the monks and one chapel as denoted by the letters A through M in the diagram. Brother Adrian, in cell A, desires to visit the chapel, M, for compline. But he belongs to a stern and silent order, which keeps movement and contact to a minimum. No monk may ever enter an occupied room or stop in a corridor. Only one monk may be in movement at any time. Luckily the order is a bit below strength at present, and there are only Adrian, Bernard, Crispin, Ethelbert, Francis, Hadrian, Imogius, Keith, and Leo, each in the cell of his initial. Call it one move even when a monk legally moves through more than one cell at a time. In how few moves can Adrian get to the chapel and each other monk return to his own cell? Provide a list of moves as a string of letters representing the order in which the monks move.

```
K-L    A
| |    |
| M    B
|      |
I-J  D-C
|  G   |
|  |   |
H—F—E
```

—Martin Hollis in *New Scientist*

Bonus In a game of blackjack (Twenty-one) using a single ordinary deck of 52 cards, you hold fifteen (points). The dealer has a nine showing. Looking around the table at your compatriots' hands, and remembering cards which have already been played, you remark on the odd coincidence that there is one of each rank (ace through king) still unaccounted for. Play has made its way to you, and, not wanting to take the risk of drawing a card, you decide to stand pat and pass the action over to the dealer. In blackjack, the dealer must draw until her hand totals at least 17 at which point she must stop; the dealer loses if her point total exceeds 21. Face cards equal 10 points and an ace can take on a value of either 1 or 11 points as decided by the cardholder. What is the exact probability that you win the hand?

—Adapted from *Problem Solving Through Recreational Mathematics* by Bonnie Averbach and Orin Chein

Double Bonus Here is a different type of puzzle for you to consider, made possible by several TBP chapters as illustrated here. As contributions from each school were anonymous, it seems reasonable to acknowledge the school mascots for helping me out. Alas, in my haste to rush this into *The Bent*, I've failed to recognize one individual. Who did I forget? It may seem Greek to you at first, but stick with it and you can sort it out!

—Jeffrey R. Stribling, *CA A '92*

Postal mail your answers to any or all of the Brain Ticklers to **Curt Gomulinski, Tau Beta Pi, P. O. Box 2697, Knoxville, TN 37901-2697** or email to *BrainTicklers@tbp.org* as plain text only. The cutoff date for entries to the Winter column is the appearance of the Spring *Bent* which typically arrives in early January (the digital distribution is several days earlier). The method of solution is not necessary. We welcome any interesting problems that might be suitable for the column. The Double Bonus is not graded. Curt will forward your entries to the judges who are **H.G. McIlvried III,** *PA Γ '53;* **J.C. Rasbold,** *OH A '83;* **F.J. Tydeman,** *CA Δ '73*, and the columnist for this issue,

—J.R. Stribling
CA A '92

LETTERS Continued from page 7

for electric power storage and distribution; hydrogen will not see similar infrastructure investment. Allocation of R&D funding will lead to more rapid maturation of battery storage than hydrogen generation, storage, and energy release technologies.

While articles with exclamation points in their titles are exciting, perhaps *The Bent* should insist on articles that place greater emphasis on safety and economic feasibility.

Steven E. Zalesch, MD B '73

[Arielle Emmett writes: *Any energy source can release its energy in an uncontrolled manner, so safety must be addressed for hydrogen, batteries, diesel, natural gas, and gasoline. The common belief that hydrogen is distinctively hazardous has little basis in fact. In theory, it can escape more easily than other fuels but, as a very light gas, hydrogen dissipates rapidly in air. Its behavior is unlike liquid fuel spills, which can continue to burn for some time and engulf a vehicle after a crash. Even if ignited, a small continuous hydrogen leak will burn with a local, non-radiating flame—dangerous but much less likely to spread than liquid fuel. And hydrogen explosions? Highly improbable since only a confined flame front will transition to explosion, according to Alistair Miller, a research emeritus at Canadian Nuclear Laboratories. Further, carbon fibre-reinforced tanks containing compressed hydrogen at 700 atmospheres are remarkably robust.*]

Statement of Ownership, Management & Circulation

Date of filing: September 29, 2017

The Bent of Tau Beta Pi, ISSN 0005-884X, is published quarterly by The Tau Beta Pi Association, Inc., 508 Dougherty Engineering Building, University of Tennessee, Knoxville, TN 37996-2215. The annual subscription price is $10.00. Editor is David S. Roberts, P.O. Box 2697, Knoxville, TN 37901-2697.

The magazine is owned wholly by The Tau Beta Pi Association, Incorporated, P.O. Box 2697, Knoxville, TN 37901-2697. There are no individual owners, bondholders, mortgagees, or other security holders. Nonprofit postal status has not changed during the past 12 months.

	Average No. Copies Each Issue Preceding 12 Months	Actual No. Copies Single Issue Nearest to Filing Date
Total No. copies printed (net press run)	83,849	82,100
Paid mail circulation	82,452	81,150
Sales through dealers and counter sales	0	0
Free distribution by mail (samples, complimentary)	661	675
Total distribution	83,113	81,825
Copies not distributed	736	275
Total	83,849	
82,100		

I certify that the statements made above are correct and complete.

—David S. Roberts, Editor