Winter 2017
The BenT of Tau BeTa pi

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Voellinger, Edward J. Non-member

Summerfield, Steven L. MO

*Spong, Robert N. UT

Sigillito, Vincent G. MD

Schweitzer, Robert W. NY

Riedesel, Jeremy M. OH

Rentz, Mark Member's son

Rentz, Peter E. IN

*Pinkerton, Audrey D. CA

Perfect
*Coillard, J. Gregory IL A '89
*Gei, Albert CA A '79
Gei, Nora W. CA A '79
Gei, Aaron J. CA Ψ '16
*Gerken, Gary M. CA H '11
*Norris, Thomas G. OK A '56
Oliver, Christopher R. AL E '08
*Prince, Lawrence R. CT B '91
Richards, John R. NJ B '76
Seidel, Mark N. MA B '83
Slegel, Timothy J. PA A '80
*Widmer, Mark T. OH A '84

Other
Aron, Gert IA B '58
Ashurst, Bob R. AL A '98
Beaudet, Paul R. Member's father
Berrakci, Stephen E. MA A '70
*Bodhan, Timothy E. IN Γ '85
Demsky, Howard J.
*Griggs Jr., James L. OH A '56
*Gulian, Franklin J. DE A '83
Gulian, William F.
Hays, G. Murray GA A '79
Huftedler, Andrea K. OH B '94
Johnson, Roger W. MN A '79
Jones, Donlan F. CA A '52
Jones, John F. WI A '59
*Jordan, R. Jeffrey CA A '73
*Kimsey, David B. CA A '71
Lalinsky, Mark A. MI Γ '77
Munsil, Wesley E. CA A '71
Parks, Christopher J.
*Pinkerton, Audrey D. TX A '90
Pinkerton, Kate Member's daughter
Rentz, Peter E. IN A '55
Rentz, Mark Member's son
Riedesel, Jeremy M. OH B '96
*Schmidt, V. Hugo WA B '51
Schweitzer, Robert W. NY Z '52
Siggelkow, Vincent G. MD B '58
*Spong, Robert N. UT A '58
Strong, Michael D. PA A '84
Summerfield, Steven L. MO Γ '85
Voel linger, Edward J. Non-member

*Denotes correct bonus solution

SUMMER REVIEW

The most challenging problem by far
was No. 5, about determining cer-
tain digits of a constructed sequence
of numbers. Fewer than half of
submissions contained a fully cor-
rect set of the 13 requested digits.
Contrastingly, No. 3, which required
the formation of a 27 digit sequence,
was easier than anticipated by
the judges. A few readers pointed
out that the problem actually had
186,624 solutions. Everyone who
submitted an answer found at least
one of the valid solutions. We also
acknowledge Wesley E. Munsil, CA
B '77, who gave the exact (9999 deci-
mal place) answer to No. 2.

FALL ANSWERS

1 ONE/FIVE = .TWO translates to 495 / 1875 = .264. For simplicity,
the problem can be restated as FIVE*TWO = ONE,000. O and F
cannot be 0. Observe that O > T * F. TWO is even, so O must be 2, 4, or 6.
For the product ONE,000 to end in 0, either E is 5, or E is 0. Assume
E is 0. The product ONE,000 must be a multiple of 10,000, and therefore
its prime factorization must contain at least 3 2's and 4 5's. None of the
5's can be a factor of TWO (it is even and doesn't end in 0, after all) and
exactly one of the 2's is a factor of FIVE (else FIVE ends in more than
one zero). That is, FIVE is an odd multiple of 5000, and the
FIVE*TWO = ONE,000 product must contain at least 4 2's and 4 5's. None of the
5's can be a factor of TWO and none of the 2's can be a factor of FIVE. That
is, FIVE is an odd multiple of 625 and TWO is a multiple of 8. There are only three four-digit multiples
of 1250 that have four distinct values, and that can meet O
T * F < O. There are fewer than a
dozen TWOs that suffice and have
a leading zero) that is a multiple of
6250. Now, consider TWO, a three
digit number (potentially containing
one zero). That is, FIVE is an odd
multiple of 1250 and TWO is a mul-
the exact (9999 decimal place) answer to No. 2.

The $12,925 the seller of the car
receives over 36 months has a net
present value of $11,443.45. A fair
discount is therefore $12,000 −
$11,443.45 = $566.55.

Results from Summer

 Month Loan Int.  Princ.  Payment Factor DCF
 1  4999.96  26.39  277.78  305.56 1.1698 294.08
 2  5277.74  27.78  300.00  314.86 1.1598 309.22
 3  5555.52  29.17  301.39  324.29 1.1498 324.34
 4  5833.30  30.56  300.00  333.73 1.1398 339.40
 5  6110.88  32.67  297.78  343.30 1.1298 354.46
 6  6388.66  33.33  295.56  354.03 1.1198 369.52
 7  6666.44  34.00  293.39  365.05 1.1098 384.57
 8  6944.22  34.66  291.39  376.07 1.0998 400.00
 9  7222.00  35.33  289.45  387.09 1.0898 416.43
 10 7499.78  36.00  287.63  398.12 1.0798 433.86
 11 7777.56  36.67  285.81  409.15 1.0698 452.29
 12 8055.34  37.34  284.01  420.18 1.0598 470.73
 13 8333.12  38.00  282.21  431.21 1.0498 490.16
 14 8610.90  38.66  280.41  442.24 1.0398 509.59
 15 8888.68  39.33  278.62  453.27 1.0298 529.03
 16 9166.46  40.00  276.83  464.30 1.0198 549.47
 17 9444.24  40.67  275.05  475.33 1.0098 569.91
 18 9722.02  41.34  273.27  486.36 0.9998 590.36
 19 10000.00 42.00  271.50  497.39 0.9898 610.81

2 The cash discount would be $566.55. One approach to the prob-
lem is to make use of a spreadsheet with the following definitions:

Month Loan Int.  Princ.  Payment Factor DCF

$11,443.45 = $556.55.
**New Winter Problems**

1. Looking for some PUZZLEs to fill up your WINTER? What is the solution to the following cryptarithm? PUZZLE + PUZZLE + PUZZLE = WINTER. Standard rules apply. Each different letter stands for a different digit, and each different digit is always represented by the same letter; no leading zeros are allowed.

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**Jeffrey R. Stribling, Ph.D.**

**CA A ’92**

2. Art, Bob, Cai, Dan, and Eli took a test and were ranked (with no ties) according to their performance. They were each informed of their own ranking, but no one was told details of the others’ ranks. Art said he wasn’t second, and Bob, who had recently claimed himself a psychic, announced confidently he was sure he was two places better than Dan. Cai overheard these remarks and came to the conclusion—for reasons that need not concern us—that one of them was right and the other wrong. After a pause for reflection Cai, who is a Tau Bate and hence highly intelligent, said that he could announce the correct ranking for all five people. But when he did so, he had got everyone in the wrong place except the bottom two. What is the correct order? (It is somewhat satisfying that Art and Bob are placed in the order of their truthfulness.)

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**101 Brain Puzzlers, E.R. Emmett**

3. Find a scalene triangle with integral sides and no right angle, such that at least one of its angles is an integral number of degrees. What are the lengths of the sides of such a triangle with the smallest perimeter?

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**Adapted from Technology Review**

4. The end of my sink faucet bends at a right angle to the horizontal so the opening, which has an inside diameter of 2.0 cm, points vertically down. The water system has a pressure regulator that, with the faucet fully open, produces a steady downward stream with a circular cross section at a rate of 8.328 L/min. I noticed that the stream tapers down to a diameter of 0.9 cm at the point at which it reaches the drain. To the nearest millimeter, how far is the drain from the faucet opening? Use

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**NEW WINTER PROBLEMS**

1. Consider an equilateral triangle of side length 15 whose corners could be truncated (equilateral triangles of side length 2 can be removed) to form the given hexagon. The apothem of this triangle is 15/3\sqrt{3}. Draw a radius r of the circumscribing circle of the hexagon to one of the vertices, creating a right triangle with sides 15/3\sqrt{3}, 11/2, and a hypotenuse of r. Use the Pythagorean Theorem to find r^2 = (675/36) + (121/4) = 49, so r = 7.

2. **Bonus** \(f(n) = \left(2^{n+1} - 2^{n+4} \text{ and } n/3\right).\) It is best to initially proceed by inspection and deduce a pattern:

<table>
<thead>
<tr>
<th>n</th>
<th>f(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>t 1.1 &gt; 0</td>
</tr>
<tr>
<td>2</td>
<td>2: 11 &gt; 01 &gt; 00</td>
</tr>
<tr>
<td>3</td>
<td>3: 111 &gt; 110 &gt; 010 &gt; 011 &gt; 001 &gt; 000</td>
</tr>
<tr>
<td>4</td>
<td>4: 1111 &gt; 1101 &gt; 1100 &gt; 0100 &gt; 0101 &gt; 0110 &gt; 0111 &gt; 0011 &gt; 0001 &gt; 0000</td>
</tr>
<tr>
<td>5</td>
<td>5: 11111 &gt; 11101 &gt; 11100 &gt; 01100 &gt; 01101 &gt; 01110 &gt; 01111 &gt; 00111 &gt; 00110 &gt; 00101 &gt; 00100 &gt; 00010 &gt; 00011 &gt; 00001 &gt; 00000</td>
</tr>
</tbody>
</table>

Continuing in this fashion, it is also seen that \(f(5) = 21 \text{ and } f(6) = 42.\)

For \(N\), even, \(f(N) = 2^{N}f(N−1).\) For \(N\) odd, \(f(N) = 2^{2}f(N−1)+1.\) This can be combined into a single recurrence relation \(f(N) = 2f(N−1) + \left(-1\right)^{N−1}/2 + 1/2.\) A particular solution to the above equation can be guessed of the form \(f(N) = A\left(-1\right)^{N}+C,\) and plugging this into the recurrence relation gives \(B\left(-1\right)^{N}+C = 2A\left(-1\right)^{N}+2C + \left(-1\right)^{N}/2 + 1/2.\) Combining and equating like terms gives \(B = -1/6\) and \(C = -1/2.\) A homogeneous solution to the recurrence relation \(f(N) = 2^{2}f(N−1)\) is clearly of the form \(A\left(-2\right)^{N}.\)

The constant \(A\) can be found by taking the sum of the particular solution \(-1/6 \left(-1\right)^{-1/2}\) and the homogeneous solution \(A\left(-2\right)^{N-1}\); plugging in the initial condition \(f(1) = 1,\) giving \(A = 4/3.\)

Thus \(f(N) = \left(4/3\right)\left(-2\right)^{N−1}−1/6 \left(-1\right)^{-1/2} + 1/2.\) Note the last two terms equal either \(-2/3\) or \(-1/3,\) depending on whether \(N\) is even or odd, respectively. These two terms can be combined into one term by taking advantage of modulo arithmetic; this is equivalent to \(-1/3\left(2^{N−1}\right)\) and leads directly to the simplified form stated above.

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**Winter 2017**

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and a value of 9.807 m/s² for the standard acceleration due to gravity.
—Adapted from The Chicken from Minsk by Yuri Chernyak and Robert Rose

Allen and Bill were nominated for president of the Golf Club. The club has 950 members, not all of whom voted. The tellers removed the votes from the ballot box and counted them one at a time. Allen not only won, but he was ahead of Bill throughout the entire count. If the probability of this occurring was exactly one in 100, how many votes did Allen and Bill each receive?
—New Scientist: Colin Singleton

**Computer Bonus** The prime-counting function, usually denoted as \( \pi(x) \), returns the number of primes less than or equal to a positive integer \( x \). Similarly, the semi-prime-counting function, usually denoted as \( \pi_2(x) \), returns the corresponding number of semi-primes (integers with exactly two prime factors, not necessarily distinct). For example, \( \pi(20) = 8 \) and \( \pi_2(20) = 6 \). What is the minimum value of \( x \) such that \( \pi_2(x)/\pi(x) > \pi(\text{the mathematical constant } 3.1415…..)\)?
—Jeffrey R. Stribling, Ph.D., CA '92

**LETTERS TO THE EDITOR**

(Continued from page 9)

and were divided into groups of three. Each group was given sealed instructions and informed that we were to open and follow them and to provide a report by the following morning. Our team’s instructions started with observing the Red Cedar River. Fortunately, I had a car so we pushed it until it started and took off on our assignment.

With complicated instructions involving the quantity of water flowing and the heat loss through the windows of the bars in East Lansing, etc., etc., we settled in an apartment and provided the requested information by morning.

A few days later we were invited to attend a dinner with our wives or girlfriends, during which we were given back our unopened reports and informed that we were now officially members of TBP!

Fun Times!

John D. L'Hote, MI '49

**Cartoon Protest**

- I unintentionally opened the Spring 2016 *Bent* to the last page first. I was upset when I read the PC Weenies cartoon belittling ethics as “boring” and “fiction.” This is an affront to the many engineers that I’ve worked with who work hard at conducting their business ethically and encouraging others to do the same. It is a serious disservice to the profession to encourage members who may consider ethics as abstract or idealistic and therefore discredit the profession as a whole.

John R. Smith, P.E., TX '71

[Artist response: My intent in the referenced comic was not to belittle ethics, but rather draw attention to the fact that ethical standards have decreased across many companies.]

**“Pseudo-Profession” Fear**

- I believe you, your team, and Dr. Gray are doing great work. Based on President Blackford’s Council’s Corner (Spring 2016 *Bent*), I guess I am not the only person that has gained an alarming amount of appreciation of Tau Beta Pi and The Bent as my career—and age—has progressed.

- As J.P. wrote, “The first time I heard about Tau Beta Pi … I received my letter to join.” I didn’t even know why I should join but I did—thankfully. Now, after multiple decades, I’ve become convinced that Tau Beta Pi may be the number one catalyst to stimulate and transform engineering from a “pseudo-profession” to a fully honored, respected, and trusted profession.

I could write pages of testimony why I begrudgingly use the term “pseudo-profession” but that will come at a later time as I witness an increasing rate of poor performances blamed on “engineering” where degree and/or Professional Engineers were NOT involved—the Flint, MI, water crisis, the General Motors ignition switch scandal, etc.

We really need to ELEVATE the profession into the fabric of our “public”—I guess through government … but hopefully without politics. Is that possible?

Randy E. Smith, PA '84

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