Brain Ticklers

RESULTS FROM SUMMER

Perfect

*Couillard, J. Gregory	IL	Α	'89
*Gee, Albert	CA	Α	'79
Gee, Nora W.	CA	Α	'79
Gee, Aaron J.	CA	Ψ	'16
*Gerken, Gary M.	CA	Η	'11
*Norris, Thomas G.	OK	Α	'56
Oliver, Christopher R.	AL	Ε	'08
*Prince, Lawrence R.	CT	Β	'91
Richards, John R.	NJ	Β	'76
Seidel, Mark N.	MA	Β	'83
Slegel, Timothy J.	PA	Α	'80
*Widmer, Mark T.	OH	Α	'84

Other

Aron, Gert	IA	Β	'58
Ashurst, Bob R.	AL	Α	'98
Beaudet, Paul R.	Member's father		
Bernacki, Stephen E.	MA	Α	'70
*Bohdan, Timothy E.	IN	Γ	'85
Demsky, Howard J.	MO	Γ	'88
*Griggs Jr., James L.	OH	Α	'56
*Gulian, Franklin J.	DE	Α	'83
Gulian, William F.	Member's son		
Hays, G. Murray	GA	Α	'79
Hufstedler, Andrea K.	OH	Β	'97
Johnson, Roger W.	MN	Α	'79
Jones, Donlan F.	CA	Ζ	'52
Jones, John F.	WI	Α	'59
Jones, Jeffrey C.	Member's son		
Jordan, R. Jeffrey	OK	Γ	'00
Kimsey, David B.	AL	Α	'71
Lalinsky, Mark A.	MI	Γ	'77
Munsil, Wesley E.	CA	Β	'71
Parks, Christopher J.	NY	Γ	'82
*Pinkerton, Audrey D.	TX	Α	'90
Pinkerton, Kate	Member's daughter		
Rentz, Peter E.	IN	Α	'55
Rentz, Mark	Member's son		
Riedesel, Jeremy M.	OH	Β	'96
*Schmidt, V. Hugo	WA	Β	'51
Schweitzer, Robert W.	NY	Ζ	'52
Sigillito, Vincent G.	MD	Β	'58
*Spong, Robert N.	UT	Α	'58
Strong, Michael D.	PA	Α	'84
Summerfield, Steven L.	MO	Γ	'85
Voellinger, Edward J.	Non-member		

*Denotes correct bonus solution

SUMMER REVIEW

The most challenging problem by far was No. 5, about determining certain digits of a constructed sequence of numbers. Fewer than half of submissions contained a fully correct set of the 13 requested digits. Contrastingly, No. 3, which required the formation of a 27 digit sequence, was easier than anticipated by the judges. A few readers pointed out that the problem actually had 186,624 solutions. Everyone who submitted an answer found at least one of the valid solutions. We also acknowledge **Wesley E. Munsil,** *CA B '71*, who gave the exact (9991 decimal place) answer to No. 2.

FALL ANSWERS

1 ONE/FIVE = .TWO translates to **495 / 1875 = .264.** For simplicity, the problem can be restated as FIVE*TWO = ONE,000. O and F cannot be 0. Observe that O > T * F. TWO is even, so O must be 2, 4, 6, or 8. For the product ONE,000 to end in 0, either E is 5, or E is 0. Assume E is 0. The product ONE,000 must be a multiple of 10,000, and therefore its prime factorization must contain at least 4 2's and 4 5's. None of the 5's can be a factor of TWO (it is even and doesn't end in 0, after all) and exactly one of the 2's is a factor of FIVE (else FIVE ends in more than one zero). That is, FIVE is an odd multiple of 1250 and TWO is a multiple of 8. There are only three four-digit multiples of 1250 that have four distinct values, and that can meet O > T * F: FIVE is one of 1250, 3750, 6250. Now, consider TWO, a three digit number (potentially containing a leading zero) that is a multiple of 8, with no repeated digits, such that T * F < O. There are fewer than a dozen TWOs that suffice and have no conflicting digits, but none meet the TWO * FIVE = ONE,000 criteria. So E is not 0; we conclude E=5.

Using logic similar to above, if E is 5, then the product ONE,000 must be an odd multiple of 5000, and the prime factorization must contain at least 3 2's and 4 5's. None of the 5's can be a factor of TWO and none of the 2's can be a factor of FIVE. That is, FIVE is an odd multiple of 625 and TWO is a multiple of 8. There are only three four-digit multiples of 625 that end in 5, have four distinct digits, and can meet the T * F criteria: FIVE is one of 1875, 3125, and 4375. Again, TWO must be a (odd this time) multiple of 8, with no repeated digits, meeting T * F < O. In the solution, TWO = 33 x 8 = 264 matches with FIVE = 1875 and ONE = 1875 x 8 x 33 / 1000 = 495.

2 The cash discount would be **$556.55.** One approach to the problem is to make use of a spreadsheet with the following definitions:

a [n] = 1 + a[n-1] = month
b[n] = b[n-1] - d[n] = loan balance
c[n] = b[n-1]*.06/12 = interest payment
d[n] = 10000/36 = principal payment
e[n] = c[n] + d[n] = monthly payment
f [n] = $(1+.10/12)^n$ = discount factor
g[n] = e[n]/f[n] = Discounted Cash Flow

A	B	C	D	E	F	G
Month	Loan	Int.	Princ.	Payment	Factor	DCF
0	10000.00	6.0%	2000.00	2000.00	10.0%	2000.00
1	9722.22	50.00	277.78	327.78	1.0083	325.07
2	9444.44	48.61	277.78	326.39	1.0167	321.02
3	9166.66	47.22	277.78	325.00	1.0252	317.01
4	8888.88	45.83	277.78	323.61	1.0338	313.04
5	8611.10	44.44	277.78	322.22	1.0424	309.121
6	8333.32	43.06	277.78	320.84	1.0511	305.26
7	8055.54	41.67	277.78	319.45	1.0598	301.42
8	7777.76	40.28	277.78	318.06	1.0686	297.63
9	7499.98	38.89	277.78	316.67	1.0775	293.88
10	7222.20	37.50	277.78	315.28	1.0865	290.17
11	6944.42	36.11	277.78	313.89	1.0956	286.50
12	6666.64	34.72	277.78	312.50	1.1047	282.88
13	6388.86	33.33	277.78	311.11	1.1139	279.29
14	6111.08	31.94	277.78	309.72	1.1232	275.75
15	5833.30	30.56	277.78	308.34	1.1326	272.25
16	5555.52	29.17	277.78	306.95	1.1420	268.78
17	5277.74	27.78	277.78	305.56	1.1515	265.35
18	4999.96	26.39	277.78	304.17	1.1611	261.96
19	4722.18	25.00	277.78	302.78	1.1708	258.61
20	4444.40	23.61	277.78	301.39	1.1805	255.30
21	4166.62	22.22	277.78	300.00	1.1904	252.02
22	3888.84	20.83	277.78	298.61	1.2003	248.78
23	3611.06	19.44	277.78	297.22	1.2103	245.57
24	3333.28	18.06	277.78	295.84	1.2204	242.41
25	3055.50	16.67	277.78	294.45	1.2306	239.28
26	2777.72	15.28	277.78	293.06	1.2408	236.18
27	2499.94	13.89	277.78	291.67	1.2512	233.12
28	2222.16	12.50	277.78	290.28	1.2616	230.09
29	1944.38	11.11	277.78	288.89	1.2721	227.10
30	1666.60	9.72	277.78	287.50	1.2827	224.14
31	1388.82	8.33	277.78	286.11	1.2934	221.21
32	1111.04	6.94	277.78	284.72	1.3042	218.32
33	833.26	5.56	277.78	283.34	1.3150	215.46
34	555.48	4.17	277.78	281.95	1.3260	212.63
35	277.70	2.78	277.78	280.56	1.3370	209.84
36	0.00	1.39	277.70	279.09	1.3482	207.01
Totals:		925.00	12000.00	12925.00		11443.45

The $12,925 the seller of the car receives over 36 months has a net present value of $11,443.45. A fair discount is therefore $12,000 - $11,443.45 = $556.55.

3 **Algernon is English, Conservative and Catholic; Basil is English, Conservative and Protestant; Clar-**

ence is Irish, Liberal and Protestant. For Basil to know anything about Clarence, both Basil and Algernon must be the same in nationality and politics (which means Clarence is the opposite of Basil for those two traits). For Basil to know Clarence's religion, Clarence must be an Irish Liberal which makes Clarence's religion Protestant. It follows that both Basil and Algernon are English and Conservative. For Basil to know Algernon's religion, Basil must be Protestant and Algernon must therefore be the only Catholic.

4 Let N equal the number of copies, with each copy initially having 36 chapters. Each chapter can be missing from at most 2 copies, for otherwise there would be a choice of 3 copies that was not complete. Furthermore, no two copies can have the same 2 missing chapters, for otherwise there would be a violation of the requirement that for any pair of chapters, there is at least one copy missing only one of them. The number of ways of picking two copies is $C(N, 2)$, where $C(i, j)$ is the number of ways of picking i objects, j at a time. In addition, there can be N chapters that are missing from only one copy without violating the above principle. Therefore, we have $C(N, 2) + N = 36$, which has a solution of $N = 8$. The following array shows one possible solution. Since $C(9, 2) = 36$, $N = 9$ also works, but we asked for the smallest N.

```
01111111 1 11 111 1111 11111 111111 0000000
10111111 1 11 111 1111 11111 000000 1111110
11011111 1 11 111 1111 00000 111110 1111101
11101111 1 11 111 0000 11110 111101 1111011
11110111 1 11 000 1110 11101 111011 1110111
11111011 1 00 110 1101 11011 110111 1101111
11111101 0 10 101 1011 10111 101111 1011111
11111110 0 01 011 0111 01111 011111 0111111
```

5 ***r* = 7.** Consider an equilateral triangle of side length 15 whose corners could be truncated (equilateral triangles of side length 2 can be removed) to form the given hexagon. The apothem of this triangle is $15\sqrt{3}/6$. Draw a radius r of the circumscribing circle of the hexagon to one of the vertices, creating a right triangle with sides $15\sqrt{3}/6$, $11/2$, and a hypotenuse of r. Use the Pythagorean Theorem to find $r^2 = (675/36) + (121/4) = 49$, so $r = 7$.

Bonus $\mathbf{f(N) = (2^{N+1} - 2^{(N+1) \bmod 2})/3}$. It is best to initially proceed by inspection and deduce a pattern:

```
f(1) = 1: 1 -> 0
f(2) = 2: 11 -> 01 -> 00
f(3) = 5: 111 -> 110 -> 010 -> 011 ->
          001 -> 000
f(4) = 10: 1111 -> 1101 -> 1100 -> 0100 ->
           0101 -> 0111 -> 0110 -> 0010 ->
           0011 -> 0001 -> 0000
```

Continuing in this fashion, it is also seen that f(5) = 21 and f(6) = 42.

For N even, $f(N) = 2*f(N-1)$. For N odd, $f(N) = 2*f(N-1)+1$. This can be combined into a single recurrence relation $f(N) = 2*f(N-1) + (-1)^{N-1}/2 + 1/2$. A particular solution to the above equation can be guessed of the form $f(N) = B*(-1)^N+C$, and plugging this into the recurrence relation gives $B*(-1)^N+C = 2B*(-1)^{N-1}+2C + (-1)^{N-1}/2 + 1/2$. Combining and equating like terms gives $B = -1/6$ and $C = -1/2$. A homogeneous solution to the recurrence relation $f(N) = 2*f(N-1)$ is clearly of the form $A*2^{N-1}$. The constant A can be found by taking the sum of the particular solution $-1/6\,(-1)^N-1/2$ and the homogeneous solution $A*2^{N-1}$, plugging in the initial condition $f(1) = 1$, giving $A = 4/3$. Thus $f(N) = 4/3*2^{N-1} - 1/6(-1)^N-1/2$. Note the last two terms equal either $-2/3$ or $-1/3$, depending on whether N is even or odd, respectively. These two terms can be combined into one term by taking advantage of modulo arithmetic; this is equivalent to $-1/3*2^{(N+1) \bmod 2}$, and leads directly to the simplified form stated above.

Computer Bonus The next six values of the specified function are:

f(4) = 246 = 2·3·41
f(5) = 1230 = 2·3·5·41
f(6) = 13830 = 2·3·5·461
f(7) = 60710 = 2·5·13·467
f(8) = 446010 = 2·3·5·14867
f(9) = 2992890 = 2·3·5·67·1489

NEW WINTER PROBLEMS

1 Looking for some PUZZLEs to fill up your WINTER? What is the solution to the following cryptarithm? PUZZLE + PUZZLE + PUZZLE + PUZZLE = WINTER. Standard rules apply. Each different letter stands for a different digit, and each different digit is always represented by the same letter; no leading zeros are allowed.

—**Jeffrey R. Stribling,** Ph.D.
CA A '92

2 Art, Bob, Cai, Dan, and Eli took a test and were ranked (with no ties) according to their performance. They were each informed of their own ranking, but no one was told details of the others' ranks. Art said he wasn't second, and Bob, who had recently claimed himself a psychic, announced confidently he was sure he was two places better than Dan. Cai overheard these remarks and came to the conclusion—for reasons that need not concern us—that one of them was right and the other wrong. After a pause for reflection Cai, who is a Tau Bate and hence highly intelligent, said that he could announce the correct ranking for all five people. But when he did so, he had got everyone in the wrong place except the bottom two. What is the correct order? (It is somewhat satisfying that Art and Bob are placed in the order of their truthfulness.)

—*101 Brain Puzzlers,* E.R. Emmet

3 Find a scalene triangle with integral sides and no right angle, such that at least one of its angles is an integral number of degrees. What are the lengths of the sides of such a triangle with the smallest perimeter?

—Adapted from *Technology Review*

4 The end of my sink faucet bends at a right angle to the horizontal so that the opening, which has an inside diameter of 2.0 cm, points vertically down. The water system has a pressure regulator that, with the faucet fully open, produces a steady downward stream with a circular cross section at a rate of 8.328 L/min. I noticed that the stream tapers down to a diameter of 0.9 cm at the point at which it reaches the drain. To the nearest millimeter, how far is the drain from the faucet opening? Use a value of 9.807 m/s^2 for the standard acceleration due to gravity.

—Adapted from *The Chicken from Minsk* by Yuri Chernyak and Robert Rose

5 Allen and Bill were nominated for president of the Golf Club. The club has 350 members, not all of whom voted. The tellers removed the votes from the ballot box and counted them one at a time. Allen not only won, but he was ahead of Bill throughout the entire count. If the probability of this occurring was exactly one in 100, how many votes did Allen and Bill each receive?

—*New Scientist:* Colin Singleton

Bonus Consider a triangular number T which is simultaneously the sum of two cubes and the difference of two cubes (e.g., $T = A^3 + B^3 = C^3 - D^3 = n(n+1)/2$) such that $D = B + 1$. The first such number is $91 = 3^3 + 4^3 = 6^3 - 5^3$, and the second is $48427561 = 360^3 + 121^3 = 369^3 - 122^3$. What is the general formula for the i^{th} such number?

—*Elementary Number Theory,* David M. Burton

Computer Bonus The prime-counting function, usually denoted as $\pi(x)$, returns the number of primes less than or equal to a positive integer x. Similarly, the semi-prime-counting function, usually denoted as $\pi_2(x)$, returns the corresponding number of semi-primes (integers with exactly two prime factors, not necessarily distinct). For example, $\pi(20)=8$ and $\pi_2(20)=6$. What is the minimum value of x such that $\pi_2(x)/\pi(x) > \pi$ (the mathematical constant 3.1415…..)?

—Jeffrey R. Stribling, Ph.D., *CA A '92*

Postal mail your answers to any or all of the Brain Ticklers to Curt Gomulinski, Tau Beta Pi, P. O. Box 2697, Knoxville, TN 37901-2697 or email to *BrainTicklers@tbp.org* as plain text only. The cutoff date for entries to the Winter column is the appearance of the Spring *Bent* in early April (the digital distribution is several days earlier). The method of solution is not necessary. We welcome any interesting problems that might be suitable for the column. The Computer Bonus is not graded. Curt will forward your entries to the judges who are **H. G. McIlvried III,** *PA Γ '53;* **F. J. Tydeman,** *CA Δ '73;* **J. C. Rasbold,** *OH A '83;* and the columnist for this issue,

J. R. Stribling, *CA A '92.*

LETTERS TO THE EDITOR

(Continued from page 9)

and were divided into groups of three. Each group was given sealed instructions and informed that we were to open and follow them and to provide a report by the following morning. Our team's instructions started with observing the Red Cedar River. Fortunately, I had a car so we pushed it until it started and took off on our assignment.

With complicated instructions involving the quantity of water flowing and the heat loss through the windows of the bars in East Lansing, etc. etc., we settled in an apartment and provided the requested information by morning.

A few days later we were invited to attend a dinner with our wives or girlfriends, during which we were given back our unopened reports and informed that we were now officially members of ΤΒΠ!

Fun Times!

John D. L'Hote, MI A '49

Cartoon Protest

• I unintentionally opened the Spring 2016 *Bent* to the last page first. I was upset when I read the PC Weenies cartoon belittling ethics as "boring" and "fiction." This is an affront to the many engineers that I've worked with who work hard at conducting their business ethically and encouraging others to do the same. It is a serious disservice to the profession to encourage members who may consider ethics as abstract or idealistic and therefore discredit the profession as a whole.

John R. Smith, P.E., TX A '71

[Artist response: My intent in the referenced comic was not to belittle ethics, but rather draw attention to the fact that ethical standards have decreased across many companies.]

"Pseudo-Profession" Fear

• I believe you, your team, and Dr. Gray are doing great work. Based on President Blackford's Council's Corner *(Spring 2016 Bent)*, I guess I am not the only person that has gained an alarming amount of appreciation of Tau Beta Pi and *The Bent* as my career—and age—has progressed.

As J.P. wrote, "The first time I heard about Tau Beta Pi … I received my letter to join." I didn't even know why I should join but I did—thankfully. Now, after multiple decades, I've become convinced that Tau Beta Pi may be the number one catalyst to stimulate and transform engineering from a "pseudo-profession" to a fully honored, respected, and trusted profession.

I could write pages of testimony why I begrudgingly use the term "pseudo-profession" but that will come at a later time as I witness an increasing rate of poor performances blamed on "engineering" where degreed and/or Professional Engineers were NOT involved—the Flint, MI, water crisis, the General Motors ignition switch scandal, etc.

We really need to ELEVATE the profession into the fabric of our "public"—I guess through government … but hopefully without politics. Is that possible?

Randy E. Smith, PA B '84