Brain Ticklers

SUMMER REVIEW

The summer problems were apparently less challenging than the typical Brain Ticklers set, with more than half the entries achieving perfect scores. Due to the wording error in Problem 1 (buried treasure on the island), it was not counted towards scoring perfect entries. Nonetheless, many responders observed that the location of the gallows was independent of the location of the treasure, and a majority correctly located the treasure anyway, which was the intent of the problem. Kudos to our readers. Problem 3 (assigned seats in a theatre) stumped the most readers, with the somewhat surprising result that the probability is independent of the number of seats.

FALL SOLUTIONS

Readers' entries for the Fall Ticklers will be acknowledged in the Spring '16 *Bent*. Meanwhile, here are the answers:

1 The arrangement of husbands, wives, hometowns, and childhood ambitions is shown in the table.

HUSBAND	A	B	C	D	E
WIFE	B	C	A	E	D
HOMETOWN	C	E	D	A	B
AMBITION	E	D	B	C	A

By the conditions of the problem, all Eve's statements are true; therefore, Basil was not born in Andover; and the man born in Chippenham wanted to become an Engineer. Alice always lies, so the would-be Dentist is not married to Barbara. Arthur was not born in Andover (can't have same letters); Basil was not born in Andover (see above); Clarence was not born in Andover (Clarence can't be Clarissa's subject); Earl was not born in Andover (Earl cannot be Eve's subject); so Dudley was born in Andover. Earl did not want to be an Engineer (same letters); Dudley did not want to be an Engineer (born in Andover, not in Chippenham); Clarence did not want to be an Engineer (Eve's second statement and not born in Chippenham); Basil did not want to be an Engineer (Basil cannot be Barbara's subject); therefore, Arthur wanted to be an Engineer. The man who wanted to be an Engineer was born in Chippenham (Eve's second statement). Since the would-be Engineer is Arthur, Barbara's first statement is true; so Arthur's wife is not Dorothy and must be Barbara. If the would-be Barber is Earl, then Dorothy's statement is false (the would-be Barber would be her husband, but that is false), so the initial premise is false, and the would-be Barber is Clarence (would-be Barber cannot be Dudley, since Dudley cannot be Dorothy's subject); and the would-be Barber cannot be Basil due to same letter). Since Dorothy's statement is true, Dorothy is married to Earl. Earl's hometown has to be Bristol (process of elimination). Basil's ambition has to be Dentist (only letter D is left). Earl's hoped for job has to be Architect (only letter A is left). Dudley's ambition has to be Chemist (only letter C is left). Dudley's wife has to be Eve (only letter that now fits). Clarence's hometown has to be Delhi (only letter that now fits). Basil's hometown has to be Ealing (only letter that now fits). Clarence's wife has to be Alice (only letter that now fits), and, finally, Basil's wife has to be Cathy (only spot left).

2 The Swan is at the **top left corner.** There are 15 steps in Peter's path: 5 A's, 5 B's, 2 C's, and 3 D's. Now, each north cancels a south, and each east cancels a west. The end of Peter's path is A(DABC)B. The DABC in parentheses merely goes around a square and cancels out, but since Peter visits The Bull only once, the final A and B cannot cancel. So C cancels A, and D cancels B, leaving 3 A's and 2 B's. Only the top left corner is far enough from The Bull for Peter's path to stay on the map.

**3** **350** is the largest integer that can be formed in nine different ways, and no more, by adding together a positive integral multiple of 5 and a positive integral multiple of 7. Let p and q be two primes. Then, the integer $N = 10pq$ is the largest integer that can be formed in exactly nine ways by adding together a positive integral multiple of p and a positive integral multiple of q. N is given in

9 ways by: $N = (nq)p + [(10-n)p]q$, with $n = 1$ to 9. If $p = 5$ and $q = 7$, $N = 1(7)(5) + 9(5)(7) = 2(7)(5) + 8(5)(7) = \ldots = 9(7)(5) + 1(5)(7) = 350$.

4 The 18 x 19 near-square can be filled with **seven** squarelets, as shown,
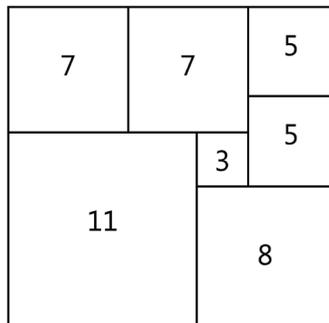
while the 22 x 23 near-square requires **eight** squarelets.
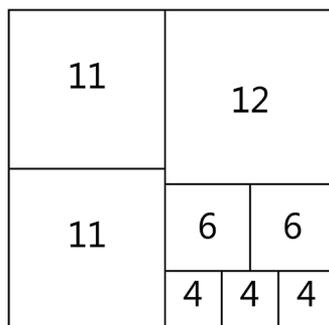
The solution is found by trial and error.

5 The cryptic addition decodes to:

TWELVE	592032
TWELVE	592032
NINE	8482
NINE	8482
NINE	8482
NINE	8482
FIVE	7432
FIVE	7432
SEVENTY	1232856

Start by numbering the columns 1 to 7 from right to left. Cx will refer to column x, and cx will refer to the carry from Cx. We see immediately that E ≠ 0, for then Y would also be 0. Now, C5 can have a maximum carry of 2, but 2*T+2 cannot be 20 (since E ≠ 0), so S is 1. E ≠ 0 or 1, so try the next value and let E=2, which makes Y=6 with c1=1. Now, SE=12=2*T+c5. If c5=2, T=(12-

2)/2=5. Consider C2; we have [4(V+N)+1]≡5 (mod 10), so V+N=1 or 11, but S=1, so V+N=11. From C5, we get 2*W+c4=V+20. Now, V≥3 (S=1, E=2, T=5, and if V=0, then N=11). Try V=4, then N=7, and ∑C4=2*E+4*N+2*F+c3 ≤2*2+4*7+2*8+4=52, then 5+2*W=5+2*9=23, which makes V=3, but we assumed V=4. So, V=3 and N=8. Therefore, W=9, and from summing C4, we get F=(2*2+4*8+c3)/2=7. That leaves 0 and 4; I must be 4 (to get c2=2), so L=0.

Bonus There are **(N-1)(N-2)/2** ways that N people can be seated at a round table so that no person sits next to the same pair of neighbors (unordered) twice. The solution depends on the number of unique pairs of neighbors a given person has, which is given by the number of combinations of N-1 objects taken 2 at a time: C(N-1, 2) = $(N-1)(N-2)/2$. For $N = 7$, 15 seating arrangements are possible. Finding a complete set is not an easy task for large values of N. One solution (of 35 basic solutions) for $N = 7$ is:

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|2345671|    |3642571|
|2435761|    |4276351|
|2546731|    |4365271|
|2643751|    |4572361|
|2735641|    |5237461|
|3247561|    |5426371|
|3472651|    |6235471|
|3526741|
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Double Bonus Even though **running** (versus walking) in a steady vertical rainfall gets one wet faster, one still gets less wet by running. The target area (of a hypothetical spherical person) that can be struck by the falling rain is a circle (the "circle of wetness") with a radius of R and area of πR^2. Define N as the number of raindrops per unit volume of air; then the number of raindrops that strike the stationary sphere per unit time is: $D = Nu(\pi R^2)$, where u is the speed of a raindrop. Now, consider the sphere moving with speed v. The raindrop density N remains unchanged, but the velocity of the falling rain relative to the sphere

changes to $u' = \sqrt{(u^2 + v^2)}$. Using u' in the D equation, one can see that more raindrops strike the sphere per unit time, so the sphere gets wet quicker the faster it moves: but, moving faster cuts the time spent in the rain. Since u' is not linear in v, D is not linear in v, so doubling the speed results in less than twice the amount of rain falling on the sphere, and since the time in the rain is cut in half, one will stay dryer (be less wet) by running.

NEW WINTER PROBLEMS

1 Given one red cube and a supply of green cubes of the same size, what is the maximum number of green cubes that can simultaneously touch the red cube on at least part of a face under the condition that the surface of the red cube is completely covered by the touching green cubes? Touching means contact over a finite area; contact at only a corner or an edge is not considered touching.

—*Wheels, Life and Other Mathematical Amusements* by Martin Gardner

2 Joan has drawn a right triangle on a 76.2 mm x 127 mm (standard 3"x5") index card and cut it out. She then cut the triangle into two right triangles and further cut each of those triangles into two right triangles. She discovered that all 12 sides of the four triangles were integral numbers of millimeters long. What was the size of the original triangle?

—An Enigma by Bob Walker in *New Scientist*

3 For dates from January through September, a digital 24-hour watch displays the date (month and day) and time as M:DD:hh:mm:ss. Occasionally, such as at 23:46:57 on August 19, each of the digits 1 through 9 is used exactly once. How many times does this occur each year?

—*Puzzles 101: A PuzzleMaster's Challenge* by Nob Yoshigahara

4 Find three three-digit integers that meet the following conditions.

The nine digits making up the three integers are all different. At least one of these integers is divisible by two, at least one is divisible by three, at least one by four, at least one by five, at least one by six, at least one by seven, at least one by eight, at least one by nine; and all three integers are divisible by eleven. What are the three integers?

—An Enigma by Susan Denham
in *New Scientist*

5 Place the integers 1 through 12 on a regular hexagon such that six of these integers are at the vertices and the other six are at the midpoints of the sides, and each of these midpoint integers is the sum of the two adjacent vertex integers. How many different ways can this be done? Now, repeat for the integers 1 through 16 on a regular octagon. How many different ways can this be done for the octagon? Rotations and reflections are not considered to be different arrangements.

—An Enigma by Bob Walker
in *New Scientist*

Bonus A Bingo card consists of a 5 by 5 grid with numbers in each cell, except for the center square, which is marked "Free." The five columns are labeled B, I, N, G, and O and contain, respectively, numbers in the ranges 1 through 15, 16 through 30, 31 through 45, 46 through 60, and 61 through 75. The caller has 75 balls, numbered 1 through 75, and randomly draws balls and calls numbers until someone yells "Bingo!" Bingo is achieved when a player fills in a column, row, or major diagonal. Consider the case of a single player with a single card. With seven or fewer numbers having been called, what is the exact ratio of the probability of getting a Bingo using the "Free" cell to the probability of getting a Bingo not using the "Free" cell?

—**D.A. Dechman,** *TX A '57* and
F.J. Tydeman, *CA Δ '73*

Computer Bonus Using the ten digits, 0 through 9, once and only once, create four integers A, B, C, and D such that $A \times B = C \times D = N$, where N is a minimum. Then, find four other integers where N is a maximum. Leading zeroes are not allowed. An apparent solution is $86 \times 259 = 74 \times 301 = 22274$, but this is not correct because 22274 is neither the minimum nor the maximum value of N.
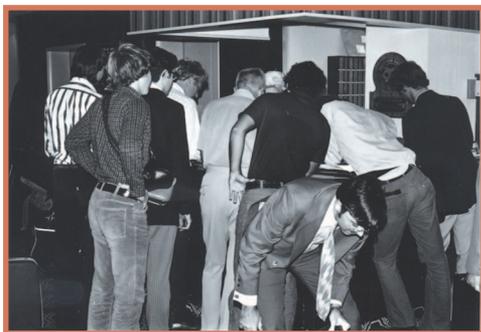
—*The Canterbury Puzzles*
by H. E. Dudeney

Send your answers to any or all of the Winter Brain Ticklers to Curt Gomulinski, Tau Beta Pi, P. O. Box 2697, Knoxville, TN 37901-2697 or email to *BrainTicklers@tbp.org* as plain text only. The cutoff date for entries to the Winter column is the appearance of the Spring *Bent* in early April (the electronic version is a few days earlier). The method of solution is not necessary. We welcome any interesting problems that might be suitable for the column. The Computer Bonus is not graded. Curt will forward your entries to the judges who are **H.G. McIlvried, III,** *PA Γ '53;* **F. J. Tydeman,** *CA Δ '73;* **J.C. Rasbold,** *OH A '83;* and the columnist for this issue,

D.A. Dechman, *TX A '57*t

SEND US YOUR witty captions for this photo from *The Bent* archives, and if it is judged one of the best, you will win a TBΠ t-shirt.

The picture below was made during the 1974 Convention that

was held in Flint, MI, and hosted by the Michigan Zeta Chapter at the General Motors Institute, which is now Kettering University. Here delegates wait to register for the Convention.

Email entries to pat@tbp.org, or mail them to HQ by Monday, February 1, 2016.

The photo for the fall contest, right, published in the winter 1977 issue, was taken during the 1976 Convention that was held on the campus of Texas A&M University. The camera caught delegate Beverlee G. Steinberg, *CA Θ '77,* checking to see if the punch really was all gone.

A total of 25 captions were submitted by 11 readers. The winning caption is:

"Hey Genie! I know you're in there! I'm not leaving until I get my third wish!" submitted by Drake R. Kijowski, *IN A '76.* Because Drake was the winner of the summer contest, he suggested that we award a shirt to another winner instead.

The next two entries were almost tied for second place and both members will receive a t-shirt:

"OK, no problem here— the nuclear inspections in Iran are complete." was sent by Joel P. Prevost, *LA A '69,* and Joseph A. Ricci, *DC B '75,* offered this gem: "Oh! So that's what the inside of the Stanley Cup looks like!"

Thanks again to all of our participants past, present, and future, for sharing your brand of engineering humor!