Brain Ticklers

RESULTS FROM SUMMER 2010

Perfect	
*Beaudet, Paul R.	Father of member
Bukowski, Justin D.	OH A '90
Couillard, J. Gregory	IL A '89
*Hagadorn, Hubert W.	PA E '59
*Jenneman, Jeffrey H.	OK A '08
*Jones, John F.	WI A '59
*Kaliski, Burton S., Jr.	MA B '84
Kaliski, Stephen	Son of member
*Kimsey, David B.	AL A '71
*Mayer, Michael A.	IL A '89
Midgley, James E.	MI Γ '56
*Nabutovsky, Joseph	Father of member
Norris, Thomas G.	OK A '56
*Schmidt, V. Hugo	WA B '51
*Spong, Robert N.	UT A '58
*Strong, Michael D.	PA A '84
*Thaller, David B.	MA B '93
*White Jr., Warren N.	LA B '74
Other	
*Aron, Gert	IA B '58
Beal, Timothy A.	MI A '84
Brule, John D.	MI B '49
*Gaston, Charles A.	PA B '61
Handley, Vernon K.	GA A '86
Holloway, Benjamin R.	TX Z '02
Jacobs, Gregory P.	WI B '09
Jones, Donlan F.	CA Z '52
Lamb, James A.	NY Γ '66
Marks, Lawrence B.	NY I '81
Marks, Benjamin	Son of member
Marrone, James I.	IN A '61
Rasbold, J. Charles	OH A '83
Solt, Matthew	Son of member
Spring, Gary S.	MA Z '82
*Stribling, Jeffrey R.	CA A '92
Summerfield, Steven L.	MO Γ '85
Sutor, David C.	Son of member
*Voellinger, Edward J.	Non-member

* Denotes correct bonus solution

SUMMER REVIEW

The Summer set appears to have been somewhat easier than usual. The most difficult regular problem was No. 5 about the vertical belt sander. However, of the answers submitted, at least 80 percent were correct for all the problems.

FALL SOLUTIONS

Reader's entries for the Fall problems will be acknowledged in the Spring BENT. Meanwhile, here are the answers:

1 Cleo is not remarkably rich. There are four "remarkable" traits—artistic (ra), beautiful (rb), intelligent (ri), and rich (rr), with two women having each trait for a total of 8. Since each woman has a maximum of three traits, the distribution must be 3, 3, 2. Assume Ann (A) has two traits. One cannot be ri, because then A would also be rr and ra, three traits; A also cannot be rb and rr, for then she would also be ra, three traits. Therefore, A is ra, rb or ra, rr, but she cannot be ra, rb, for then both Barb (B) and Cleo (C) are ri, rr, and one is also rb, but that makes her also ra, four traits; and A cannot be ra, rr, for then both B and C are rb, ri, but this means both are also ra, which makes all three women ra. Therefore, A must have three traits. There are only four possibilities for this: (1) ra, rb, ri; (2) ra, rb, rr; (3) ra, ri, rr; and (4) rb, ri, rr. (1) and (4) can be ruled out, because for A, ri implies rr and rr implies ra, both of which would require A to have four traits; (3) can be eliminated since it requires both Barb (B) and Cleo (C) to be rb, which implies they are both ra, but this makes all three women ra. Therefore, A is ra, rb, rr, which makes both B and C ri. Now if C is rr, she is also ra, which leaves rb for B, but then B is also ra, so all three are ra. Thus, C must be ra, rb, ri, and B is ri, rr; so C is not rr.

2 An estimated 18.37 percent of the students cheated. Assume the "Answer Yes," "Answer No," and "Have You Cheated" cards are selected with equal probability by the 2,352 students, that is 784 for each card. Then 928 – 784 = 144 students had "Have You Cheated" cards and answered "Yes," indicating that they had cheated. And 1,424 – 784 = 640 students had "Have You Cheated" cards and answered "No," indicating that they had not cheated. The estimated fraction of cheaters is then 144/(144 + 640) = 0.1837.

3 After one year, you will owe the banker $271,828,182.85 which is the constant, e, times one-hundred million. The expression for the annual payment is $(1 + i/n)^n$, where i is the annual interest rate and n is the number of times per year that interest is compounded. For continuous compounding, the payment is the limit of the above expression as n approaches infinity; when i is 1, this limit is defined as e, the base of natural logarithms. You can use a spreadsheet to calculate the payment with an increasing number of time intervals and quickly observe that the answer is approaching e times 10^8.

4 The distance of the center of the 1-cm-radius ball above the plane surface is approximately 4.6208 cm. Label the centers of the 4-cm, 3-cm, and 2-cm-radius balls a, b, and c, respectively. Then, as they lie on the plane surface and touch, their centers, from a top view, project a triangle ABC on the plane surface, where $AB = \sqrt{(7^2 - 1^2)} = 4\sqrt{3}$. $AC = \sqrt{(6^2 - 2^2)} = 4\sqrt{2}$, and $BC = \sqrt{(5^2 - 1^2)} = 2\sqrt{6}$. Let d be the center of the 1-cm-radius ball as it rests on the other three balls and D be its projection on the plane surface from a top view. Then, letting H equal the distance of d above the plane surface, $AD = \sqrt{\{5^2 - (H - 4)^2\}}$, $BD = \sqrt{\{4^2 - (H - 3)^2\}}$, and $CD = \sqrt{\{3^2 - (H - 2)^2\}}$. Use trial and error to find an H such that the areas of the three small triangles ABD, ACD, and BCD sum to equal the area of ABC. Use Herron's formula for calculating the area of a triangle when the lengths of the three sides are known.

5 The young girl should release from the swing with the swing 29° from its static position. She will land about 8.7 feet from the point directly below the swing's static position. For a pendulum, which the swing is, total energy is a constant and equals potential energy, $mg(h - h_o)$, plus kinetic energy, $mv^2/2$, at any point in its arc. Equating kinetic and potential energy gives $v^2/2 = gh_m - gh$, so $v = \sqrt{[2g(h_m - h)]} = \sqrt{[2gL(\cos\theta - \cos\theta_m)]}$, where θ is the angle the swing's chains make with the vertical and L is the length of the chains. Once the girl releases, the distance she travels from the release point is determined by the formulas for the trajectory of a projectile, which are $x_m = x_o + v_x t$ and $0 = y_o + v_y t - gt^2/2$. Now, $v_x = v\cos\theta$ and $v_y = v\sin\theta$. Solving for t (the time it takes the girl to hit the ground) gives $t = [v_y + \sqrt{(v_y^2 + 2gy_o)}]/g$. Substituting into the equation for x gives $x = x_o + (v\cos\theta/g)[(v\sin\theta + \sqrt{(v^2\sin^2\theta + 2gy_o)}]$, where $x_o = L\sin\theta$ and $y_o = L(1 - \cos\theta) + h_o$. Using trial and error (with L = 6 ft and h_o = 2 ft) on the swing's angle at the release point to discover that 29° gives the maximum attainable distance.

Bonus. The punter should aim at the six-yard line and expect to have the ball placed, on average, at the 14-yard line. The normal probability distribution is given by $p = [1/(\sigma\sqrt{(2\pi)})]\exp(-\theta^2/2\sigma^2)$, where θ is the deviation from the aim angle, θ_a and varies from -45° to +45°, and σ = 7.5 is the standard deviation. (Although the normal distribution goes from -∞ to +∞, we can truncate this range without loss of accuracy. After all, this is football!) Pick a yard-line aim point y_a; then the aim angle is given by $\theta_a = \tan^{-1}[(50-y_a)/(80/3)]$. The yard line for other angles is given by $y = 50-(80/3)\tan(\theta_a+\theta)$, except that when y is negative (indicating that the ball crossed the goal line), y = 20. The expected value of the yard line for spotting the ball is then the integral from -45° to +45° of $py\,d\theta$. It is easy to write a simple numerical integration computer program that varies y_a to find the minimum expected ball-placement spot. However, a spreadsheet also works well. Program the expressions for y and p into the spreadsheet (with y_a as a variable), and calculate y for unit increases in θ from -45 to 45. (This is sufficiently accurate for this problem.) Then, sum the expected values. Repeat with various values of y_a until the minimum listed above is found.

Computer Bonus. The next two palindromic squares that have an even number of digits are 637,832,238,736 = 798,644² which has 12 digits and 4,099,923,883,299,904 = 64,030,648² with 16 digits! This is quite a distance from the first such number 698,986, whose square root is 836. An eight-digit palindrome can be expressed as *abcddcba* where *a* can equal 1, 4, 5, 6, or 9 and *b*, *c*, and *d* can equal 0 through 9. Write a computer program to check each possible combination to see when a perfect square occurs. Do this for 6, 8, 10, 12, 14, and, finally, 16-digit palindromes.

NEW WINTER PROBLEMS

1 Find the smallest positive integer that has a remainder of 1 when divided by 2, a remainder of 2 when divided by 3, a remainder of 3 when divided by 4, a remainder of 4 when divided by 5, a remainder of 5 when divided by 6, and a remainder of 6 when divided by 7.

—**Christopher R. Oliver,** *AL E '08*

2 Five pirates find a chest containing 100 gold pieces and decide to divide them according to the following scheme. In order of seniority, each pirate proposes a distribution of the gold pieces. All of the pirates then vote, and if half or more of the pirates accept the proposal, the gold pieces are divided as proposed. If not, then the proposer must walk the plank and drown, and they start the process over again with the next most senior pirate making a proposal. Assuming the pirates are very intelligent, greedy, and interested in surviving, how does the senior pirate propose to split the 100 gold pieces?

—*How to Ace the Brain Tickler Interview* by John Kador

3 My wife and I recently attended a party at which there were four other married couples, and various people shook hands. No one shook hands with himself or herself or with his or her spouse, and no one shook hands with the same person more than once. After the handshakes were over, I asked each person, including my wife, how many hands he or she had shaken. To my surprise, each had shaken a different number of hands. How many hands did my wife shake?

—*Knotted Doughnuts and Other Mathematical Entertainments* by Martin Gardner

4 Professor Asterisk treated his friend, Professor Scalene, to a ride on the local bus. He asked the driver for two transfer tickets, which had consecutive five-digit serial numbers. He mentally added the ten digits and informed Professor Scalene that the sum was 62. Professor Scalene asked if the sum of the digits of the serial number on either of the tickets were in the range of 29 to 39, inclusive. After Professor Asterisk answered, Professor Scalene remarked, "I now know what the two serial numbers are." What were the serial numbers?

—*Hard-to-Solve Brainteasers* by Jaime and Les Poniachik

5 A sparrow, flying horizontally in a straight line, is 20 m directly below an eagle and 40 m directly above a hawk. Both the eagle and the hawk always fly directly at the sparrow, and they reach it simultaneously. If the sparrow is flying at a speed of 15 m/s and the hawk is flying at 30 m/s, how fast is the eagle flying?

—*Chases and Escapes* by Paul J. Nabin

Bonus. A regular dodecahedron has 12 faces in the shape of regular pentagons. If some, including none or all, of the faces are painted red and the rest are painted blue, how many dodecahedrons that are distinguishable from each other are possible?

—*Classic Puzzles* by Gyles Brandreth

Computer Bonus. A, B, and C are different nine-digit integers that each use the digits 1 through 9 once and only once. What is the value of C that satisfies the equation, A x B = C², and where A and B are also perfect squares?

—*Madachy's Mathematical Recreations* by Joseph S. Madachy

Send your answers to any or all of the Winter Brain Ticklers to Jim Froula, **Tau Beta Pi, P.O. Box 2697, Knoxville, TN 37901-2697** or email to *BrainTicklers@tbp.org* only as plain text. The cutoff date for entries to the Winter column is the appearance of the Spring BENT in late March. The method of solution is not necessary.

We welcome any interesting problems that might be suitable for the column. The Computer Bonus is not graded. Jim will forward your entries to the judges who are **Dr. H.G. McIlvried III,** *PA Γ '53;* **F. J. Tydeman,** *CA Δ '73;* **J. L. Bradshaw,** *PA A '82;* and the columnist for this issue,

—**D.A. Dechman,** *TX A '57.*

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