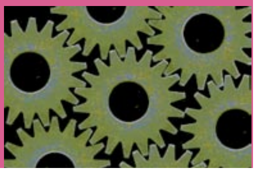


Brain Ticklers

RESULTS FROM SUMMER 2008

Perfect

*Baines Jr., Elliot A.	NY Δ '78
*Bohdan, Timothy E.	IN Γ '85
*Couillard, J. Gregory	IL A '89
*Gerken, Gary M.	Non-member
*Norris, Thomas G.	OK A '56
Rasbold, J. Charles	OH A '83
*Strong, Michael D.	PA A '84
*Weinstein, Stephen A.	NY Γ '96

Other

Alexander, Jay A.	IL Γ '86
Aron, Gert	IA B '58
Brule, John D.	MI B '49
Brzezinski, Mark A.	OH Ḃ '00
*Celestino, James R.	NJ B '00
*Fenstermacher, T. Edward	MD B '80
James, Catherine A.	Wife of member
Johnson, Roger W.	MN A '79
Jones, Donlan F.	CA Z '52
Kenny, Kevin B.	IL A '90
Kimsey, David B.	AL A '71
Lalinsky, Mark A.	M I Γ '77
*Newmiller, Jeffrey D.	CA Λ '86
Rentz, Peter E.	IN A '55
Rowe, David L.	Son of member
Santner, Jeffrey	PA K '09
Skowronski, Victor	NJ A '71
*Spong, Robert N.	UT A '58
Stribling, Jeffrey R.	CA A '92
Summerfield, Steven L.	MO Γ '85
Teichert, Gregory H.	UT B '10
Trimble, Alan R.	MD B '71
*Voellinger, Edward J.	Non-member

* Denotes correct bonus solution

THANK YOU, HOWARD!

Howard McIlvried has judged our Brain Tickler column since the April 1959 issue and has ably served as head judge since 1983. Howard, congratulations and thank you for your excellent work and 50 years of dedicated service!

SUMMER REVIEW

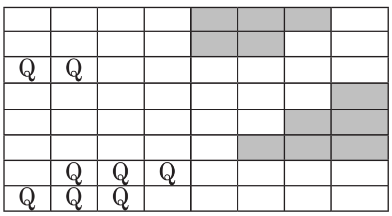
Problem 2 about three integers whose sums and differences are squares and problem 4 about integer solutions of a nonlinear equation were the most missed regular problems. About 2/3 of the Bonus solutions submitted were correct.

FALL SOLUTIONS

Reader's entries for the Fall problems will be acknowledged in the Spring BENT. Meanwhile, here are the answers:

1 The order of the recent marriages, from first to last, was Quentin/Emily, Tristram/Dinah, Ronald/Barbara, Peter/Celia, and Simon/Anne. The previous pairings were QA, TE, RD, PB, and SC. There are seven ways for P to wed earlier than A and later than Q and for B to wed earlier than C and later than T. Investigate each of these seven possibilities, and only Qw, Tx, yB, PC, and zA satisfies the other constraints. You will discover the previous pairings once all the constraints have been satisfied.

2 The maximum number of squares on a chessboard not attacked by eight queens is eleven. Place the queens according to the diagram below where the squares not attacked are shaded. Best way to solve this one is to try numerous possibilities.



3 The current month is August. When August 1 falls on Tuesday, then July 31 is on Monday, and September 7 is on Thursday. This satisfies all the constraints. This one is best solved by trial and error, which quickly leads you to try the only two adjacent months that each have 31 days.

4 You would expect to have to draw 4,829/630 = 7.665 … cards to get at least one card in each suit. For success at any point, you must first draw cards in three suits and then draw the missing suit on the next draw. You can be successful in as few as four draws, or it could take up to a maximum of 40 draws. You can calculate probabilities up the line as a change from the next lowest draw either by hand or using an Excel spreadsheet. 1-1-1 to 1-1-1-1; 2-1-1 to 2-1-1-1; 3-1-1 to 3-1-1-1; and 2-2-1 to 2-2-1-1 and so on. As a check, the probabilities must add exactly to one.

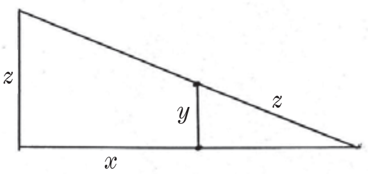
n	*probability*
4	0.1054982
5	0.1582473
6	0.1627366
7	0.1430960
8	0.1158512
9	0.0892652

And so on.

Then, multiply each probability by its n to get 7.665 as the expected number of cards drawn. But, if you are expert at using probability formulae you can quickly reach the same value. E(S&H&D&C) = E(S)+E(H)+E(D)+E(C)-E(S or H)-E(S or D)-E(S or C)-E(H or D)…+E(S or H or D)+E(S or H or C)…-E(S or H or D or C). For a deck of ab = n cards, with a suits and b in each suit, the expected value of the number of draws for i of a to occur is $E_i = (n + 1)/(ib + 1)$. For a single suit (S,H,D,C), i = 1; for two suits (S or H, S or D, …), i = 2; for three suits (S or H or D, S or H or C, …), i = 3; and for all four suits (S or H or D or C), i = 4. This simplifies to E =53 (4/14–6/27 +4/40–1/53) = 4,829/630. For more study, get a probability book with a chapter on playing-card probabilities. *The Theory of Gambling and Statistical Logic* by Richard A. Epstein deals with a number of similar problems.

5 The equation of the beetle's path is $u = (1 - v^{0.5})\,(1 - v)^{0.5}$, where u = x/L, and v = y/L. The accompanying diagram shows that, at any position of the ladder, the remaining height of the ladder on the wall above the floor, z, is equal to the remaining distance of the bug to the end of the ladder, also z.

Using the relationships of similar triangles, $y/z = z/\mathrm{L}$ and $x/(\mathrm{L} - z) = (\mathrm{L}^2 - z^2)^{0.5}/\mathrm{L}$. From the first equation, $z/\mathrm{L} = (y/\mathrm{L})^{0.5} = v^{0.5}$. Using this to eliminate z from the second equation yields $x/\mathrm{L} = u = (1 - z/\mathrm{L})(1 - (z/\mathrm{L})^2)^{0.5}$ which is equal to the answer above.



Bonus. The period of oscillation of the system is $T = 2\pi[(L/\mathrm{g})(M/(m + M))]^{0.5}$, where M is the mass of the larger bead, m the mass of the smaller bead, L the length of the wire, and g the acceleration due to gravity. Let θ be the angle the pendulum makes with the vertical, x = the horizontal distance of m from its rest position, y = the horizontal distance of M from its rest position, and F = tension in pendulum wire = $m\mathrm{g}\cos\theta$. The restoring force acting on m is $-m\mathrm{g}\sin\theta$, and the horizontal force acting on M is $F\sin\theta$. Because $F = ma$, we can write two equations: $mx'' = -m\mathrm{g}\sin\theta$ and $My'' = m\mathrm{g}\cos\theta\sin\theta = m\mathrm{g}\sin 2\theta/2$, where the apostrophes represent differentiation with respect to time t. Since for small angles $\sin\theta = \theta$ and since $\theta = \sim(x - y)/L$, we have $x'' = -(\mathrm{g}/L)(x - y)$ and $y'' = (m\mathrm{g}/ML)(x - y)$; or, upon rearranging, $x'' + \mathrm{g}x/L = \mathrm{g}y/L$ and $y'' + m\mathrm{g}y/(ML) = m\mathrm{g}x/(ML)$. Solving the first of these equations for y gives $y = (L/\mathrm{g})x'' + x$. Differentiating twice with respect to t gives $y'' = (L/\mathrm{g})x'''' + x''$. Substituting the values for y and y'' into the second equation and rearranging gives $x'''' + [\mathrm{g}(m + M)/ML]x'' = 0$. This is a linear fourth degree differential equation, which is easily solved by standard methods (see any book on differential equations). The general solution is $x = c_1\sin at + c_2\cos at + c_3 t + c_4$, where $a = [\mathrm{g}(m + M)/(ML)]^{0.5}$. Substituting the four conditions $(0, x_m)$, $(\pi/2, 0)$, $(\pi, -x_m)$, and $(3\pi/2, 0)$ for (at, x), where x_m is the maximum value of x, gives $c_1 = c_3 = c_4 = 0$ and $c_2 = x_m$. Therefore, $x = x_m\cos at$. Substituting this equation into the equation for y gives $y = -(m/M)x_m\cos at$. Now, $aT = 2\pi$. Therefore, the period $T = 2\pi/a = 2\pi[ML/\mathrm{g}(m + M)]^{0.5}$. Thus, m and M move back and forth at the same frequency, exactly 180º out of phase.

Double Bonus. It would require an infinite number of gallons of nanotech paint to coat the inner surface of the horn of plenty. And it would be very difficult to do so since the horn is infinitely long! Although the volume of this horn is finite, π cubic units, the surface is infinite! Dust off your Calculus 101 book, and you'll agree.

NEW WINTER PROBLEMS

1 What is the smallest right triangle that will fit completely inside another right triangle, such that all six sides of the two triangles have integral values, and the larger triangle has one side that is shorter than all three sides of the smaller triangle?

—*Almost Impossible Brain Bafflers* by Tim Sole and Rod Marshall

2 Find the smallest positive integer such that, if you place a 4 in front of it, you get a number that is exactly four times as large as you get if you place a 4 at the end of the number.

—*The Numerology of Dr. Matrix* by Martin Gardner

3 Al has two 12-hour clocks that, when fully wound, will run for nearly eight days. Both clocks were keeping different times, with each being wrong by a different exact number of minutes per day, although less than one hour each. Al took his clocks to the local clock mender, who works only from 9:30 a.m. to 5 p.m. Monday through Friday. He immediately wound both clocks fully and set them to the correct time, a whole number of minutes after the hour, and put them on the shelf for observation.

The following Monday, as he went to take down the clocks to start working on them, they started to strike eight o'clock simultaneously. This occurred some hours and minutes past the correct time. What day and exact time did the clock mender set them originally?

—*Puzzles, Mathematical Diversions, and Brainteasers* by Erwin Brecher

4 Two dominoes are chosen at random from a 28-domino, double-six set. What is the probability that an end-to-end chain of all the dominoes can be formed with the two chosen dominoes at the ends, with one in the first position, and the other in the last position? The usual rule applies that the numbers on the ends of adjoining dominoes must match.

—*Classical Mathemagic* by R. Blum, A. Hart-Davis, B. Longe, and D. Niederman

5 A number of identical planes, each with a tank that will hold just enough fuel to travel exactly halfway around the world, are based on a small island on the equator. If the planes can refuel only from the island or from another plane, what is the smallest number of planes that are required for one plane to complete an equatorial trip around the world, with each plane involved returning safely to the island? Assume that a plane can refuel and transfer fuel instantaneously and that all planes travel at the same speed and use fuel at the same uniform rate. Ignore rotation of the earth and winds.

—*Almost Impossible Brain Bafflers* by Tim Sole and Rod Marshall

Bonus. In the game of Yahtzee, players try to achieve specific combinations of five dice. A player first tosses all five dice and then retosses those that don't have the desired values and then finally repeats this process for a third toss. Suppose you are trying to maximize the number of 6s. After your tosses, your score is the sum of all the 6s thrown. What is the expected value of your score? Your maximum score is 30 since you stop at any time that you have tossed five 6s.

—*Howard G. McIlvried III, PA Γ '53*

Computer Bonus. Fermat, in about 1650, asked for all solutions in positive integers for $x^2 + 2 = y^3$. It turns out that $x = 5$, $y = 3$ is the only solution. Now consider $x^2 - 15 = y^3$. One solution in positive integers is $x = 4$, $y = 1$. Find all other solutions.

—*Don A. Dechman, TX A '57*