Brain Ticklers

RESULTS FROM SUMMER 2007

Perfect

	Couillard, J. Gregory	IL	A	'89
	Griggs, James L., Jr.	OH	A	'56
*	Kimsey, David B.	AL	A	'71
	Rasbold, J. Charles "Chuck"	OH	A	'83
*	Spong, Robert N.	UT	A	'58
	Stribling, Jeffrey R.	CA	A	'92
*	Trimble, Alan R.	MD	B	'71
*	Voellinger, Edward J.	Non-member		

Other

	Bessert, Holly S.	WI	B	'04
*	Kokkinopoulos, Ted	Non-member		
	Lew, Thomas M.	TX	Δ	'84
	Marrone, James D.	IN	A	'87
*	Melton, Walter C.	TX	A	'56
	Quintana, Juan S.	OH	Θ	'62
	Rentz, Peter E.	IN	A	'55
	Scholz, Gregory R.	PA	B	'00
	Strong, Michael D.	PA	A	'84
*	Venema, Todd M.	OH	H	'08

* Denotes correct bonus solution

SUMMER REVIEW

We regret that there was a formatting error in Summer No. 3. We did not deduct for missing this problem when assessing perfect scores.

We also regret that an incomplete version of the Summer Double Bonus answer was used in the Fall column. The correct minimum value approaches -2, which occurs when B=C in the limit, $(A,B,C) = (0,\pi/2,\pi/2)$.

FALL SOLUTIONS

1 Tommy committed the crime. John-George and Fabian-Chubby are both offsetting pairs, where one is a truth-teller and one is a liar for each pair. So, either Elvis is guilty, Ringo is guilty, or neither is guilty, and only one of the five remaining statements is true. Assuming Paul's statement is true, where Tommy is guilty, results in the other four statements being lies. Each of the other four statements, if assumed to be true, result in at least one additional statement being true as well.

2 My number was 256. There are eleven even, three-digit perfect squares. Of these, only three can result in a perfect square on the third next runner under the given constraints. Namely, 196-791-297-289, 256-527-258-529, and 484-845-486-784. But, neither 289 nor 784 can yield a similar four-person chain. But, 529-305-630-361 and 361-623-264-256 get back to my original number. It saves a lot of work if you write a short computer program to find these sequences. Email *dondechman@verizon.net* for a QBasic computer program.

3 The height of the infinite stack is $(3 + 5\sqrt{2})/6 \approx 1.6785$. The height of the centerline of the first cylinder is ½. The additional height of the centerline of the second cylinder is $(\sqrt{2})/3$. The height of each subsequent cylinder's centerline increases sequentially by $(\sqrt{2})/4$, $(\sqrt{2})/8$, $(\sqrt{2})/16$, and so on. This simplifies to ½ + $(\sqrt{2})/3$ + $(\sqrt{2})/2$, which further simplifies to the above answer.

4 The expected value of the distance of the nearest point from the base is $h/7$. This problem is an easier version of the Spring 2006 Bonus problem. For simplicity, let the triangle height = 1 and the triangle base = b. Then, the area of the triangle is $b/2$. For a thin horizontal slice, with a thickness of dx, at a distance x from the triangle's apex, the probability of a random point being in that slice is $bx\mathrm{d}x/(b/2)$, which simplifies to $2x\mathrm{d}x$. For that point to be the closest point to the base, the other two points must each be in the area of the triangle above that slice, which has a probability of $[(bx^2/2)/(b/2)]^2 = x^4$. Plus, the closest point can occur with the first, second, or third point or three different ways. Thus, the expected distance of the nearest point to the base is the sum of the product of the probability of each slice times the distance of that slice from the base, which is the integral from 0 to 1 of $2x(3x^4)(1 - x)\mathrm{d}x$. Integration yields the expected distance equals $6(x^6/6 - x^7/7)$ where x = 1, which simplifies to 1/7. For any h, this proportions up to $h/7$.

5 The widgets that cannot fit in a box with integral dimensions have a length L such that $L^2 = (8x - 1)4^{n-1}$ where x and n are positive integers. The problem boils down to finding the integer values of L^2 that cannot be expressed as the sum of three squares. Number theory has shown that such values are given by the answer above. The problem can also be solved by writing a short computer program to generate the "no fit" widgets for L^2 up through 500. Then note that the answers can be sorted and grouped 7, 15, 23, 31, 39 ... ; 28, 60, 92, 124, 156 … ; 112, 240, 368, 496 ... ; 448 … ; and so on. Then, the answer stated above can be easily deduced. Email *dondechman@verizon.net* for a QBasic computer program.

Bonus. The golf ball can be launched at either 35.0° or 38.2° with hang times of 6.82 seconds or 7.72 seconds, respectfully. Consider an x-y coordinate system. Newton's laws of motion applied to the golf ball give $m\mathrm{d}u/\mathrm{d}t = -F\cos\Theta$ and $m\mathrm{d}v/\mathrm{d}t = -F\sin\Theta - mg$ where Θ is the angle between the tangent of the ball's trajectory and the x axis, u is the component of the ball's velocity in the x direction, v is the component of the ball's velocity in the y direction, m is the mass of the golf ball, and F is the drag force. Let V equal the ball's velocity. Then, $\cos\Theta = u/V$, $\sin\Theta = v/V$, and $V = (u^2 + v^2)^{½}$. Making these substitutions, Newton's equations become $m\mathrm{d}u/\mathrm{d}t = -Fu/(u^2 + v^2)^{½}$ and $m\mathrm{d}v/\mathrm{d}t = -Fv/(u^2 + v^2)^{½} - mg$. But, $F = \mathrm{B}V^2 = \mathrm{B}(u^2 + v^2)$ where $\mathrm{B} = \rho_a \mathrm{C}_\mathrm{D}\mathrm{A}/2$. Making these substitutions, we have $\mathrm{d}u = -(\mathrm{B}/m)u(u^2 + \mathrm{v}^2)^{½}\mathrm{d}t$, $\mathrm{d}v = (-(\mathrm{B}/m)v(u^2 + v^2)^{½} - g)\mathrm{d}t$, $\mathrm{d}x = u\mathrm{d}t$, and $\mathrm{d}y = v\mathrm{d}t$. These last four equations are not solvable explicitly, so they must be solved by stepwise numerical integration. The answer above was obtained using time increments of 0.0001 seconds and trial and error for the initial Θ to achieve y = -40 feet at x = 715 feet.

Computer Bonus. There are 376 Smith numbers less than 10,000. The first ten are 4, 22, 27, 58, 85, 94, 121, 166, 202, and 265. The last three are 9,942, 9,975, and 9,985. Again, you can email Don Dechman for a QBasic computer program.

NEW WINTER PROBLEMS

1 Ann, Beth, and Clara have been challenged to find a solution to the following cryptic:

FOUR is a perfect square;
FIVE is a Fibonacci number; &
SIX is a semi-prime (product of two primes).

Each girl found a different solution, and the values of FOUR, FIVE, and SIX in Clara's solution were not used in Ann's or Beth's answer. What was Clara's solution? The usual rules apply. No leading zeroes, and each letter represents a different digit.

—Richard England in *New Scientist*

2 Consider a regular pentagram (five-pointed star) with a circumscribing circle of unit radius. Fold the five points of the star so that their tips meet in a point to form a pyramid with a regular pentagon as its base. What is the volume of this pyramid?

—*Slicing Pizzas, Racing Turtles, and Further Adventures in Applied Mathematics* by Robert B. Banks

3 Find the positive root of the following equation: $(x + 9)^{1/3} - (x - 9)^{1/3} = 3$. Express your answer exactly in terms of surds.

—*Mathematics Teacher*

4 In a garden is a circular pond with a horizontal bottom and vertical sides. The owner of the garden decides to erect a statue in the middle of the pond. In preparation, three identical cubical (solid and impermeable) concrete blocks are placed side by side in the pond to form a base for the statue. When the first block is placed in the pond, the water level rises three inches. When the second block is placed, the water level rises another four inches. And, when the third block is added, the water level again rises four inches. What is the length of the edges of the blocks?

—*Almost Impossible Brain Bafflers* by Tim Sole and Rod Marshall

5 Using a standard set of 28 dominos, Carl laid out in a row, in typical domino style with ends matching, e.g., 6-3, 3-5, 5-5, 5-1. He started with 0-0 and then added dominos, building on either end of the string, so that the total number of pips in the row equaled 1, then 4, then 9, working through each perfect square and continuing as far as possible. His row could not have been shorter at any stage. When he reached the highest square possible, he was left with five unused dominos, four of which were doubles. What were those five

dominos? List the complete order of the domino row of your solution.

—Adrian Somerfield in *New Scientist*

Bonus In the game of Thirty Five, each player is dealt a hand of nine cards from a standard deck of 52 cards. To win a pot, one must be able to select cards in a single suit to obtain a score of 35, where face cards count 10, aces count 1, and all other cards have their designated value. What is the probability of getting a hand that can win the pot?

—**Howard G. McIlvried III,** *PA Γ '53*

Double Bonus In 2006, the age of a famous composer would be, if he were still alive, equal to 10 times 10 less than his age when he died. His age at death equals 2006 minus the reverse of the year he died. Who is this composer?

—**Dr. Aziz S. Inan,** *CA E '78*

Send your answers to any or all of the Winter Brain Ticklers to **Jim Froula, Tau Beta Pi, P. O. Box 2697, Knoxville, TN 37901-2697**, or email to *BrainTicklers@tbp.org* only as plain text. The cutoff date for entries to the Winter column is the appearance of the Spring BENT in late March. The method of solution is not necessary. We welcome any interesting problems that might be suitable for the column. The Double Bonus is not graded. Jim will forward your entries to the judges who are **H. G. McIlvried III,** *PA Γ '53;* **F. J. Tydeman,** *CA Δ '73;* **J. L. Bradshaw,** *PA A '82;* and the columnist for this issue,

D. A. Dechman, TX *A* '57.

Write Your Own Alumnus Note!

Your fellow Tau Bates are interested in news about you. Send items about civic activities, honors won, weddings, births, promotions, changes in address, etc. to Tau Beta Pi, Box 2697, Knoxville, TN 37901-2697 or to *alumnote@tbp.org*. Material for publication must be received for the **Spring** issue by February 1; **Summer** issue by May 1; **Fall** issue by August 1; and **Winter** issue by November 1. Include name, address, chapter, class year, and email address or phone no. Thank you!

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