SUMMER REVIEW

The most difficult Summer problem was #5, asking for the smallest integer having at least 1,000 factors. Less than one-third of the entries were correct.

FALL ANSWERS

Here are the solutions to the Fall Brain Ticklers. Fall entries will be acknowledged in the next issue.

1. There are 2,040 pips on the dominoes that make up a 136 domino, double-fifteen set. Eight dominoes (0-15, 1-14, … 7-8) each have fifteen pips. The remaining dominoes can be paired to have thirty pips per pair, such as 2-7 and 8-13. Thus, 136(15) = 2,040. Alternately, 136 dominoes contain 272 numbers. Each number appears 272/16 = 17 times, and the number of pips is 17(0 + 1 + 2 + … + 15) = 2,040.

2. The solution to PEN × INK = LETTER is 615 × 354 = 217710. This cryptic multiplication can be solved without computer assistance, but with a lot of effort. Analyzing the 44 possible combinations of N, K, and R yields 26 possible combinations of N, K, R, and E. Each has around 20 possible combinations of P and I that yield a six-digit product. Check each with a hand-held calculator for a unique L and T and a repeat E.

3. There is a 0.3 probability that the Q of spades will drop if the K is played. Once the declarer sees the dummy, there are $C(26, 13)$ = 10,400,600 possible hands for East. The number of these hands with i spades is given by $C(5, i)C(21, 13 - i)$ where $C(n, i)$ is the combinations of n things taken i at a time. For $i = 0$ to 5, we get 203,490; 1,469,650; 3,527,160; 3,527,160; 1,469,650; and 203,490. Of these, for cases where East has 1 or 4 spades, the Q will be a singleton 1/5 of the time. And for cases where East has 2 or 3 spades, the Q will be a doubleton 4/10 of the time. After the first round of spades, we know that spades are not distributed 5-0 or 4-1 with the Q the singleton. Therefore, the total possible hands for East are reduced to 10,400,600 – 2(203,490) – (0.2)(2)(1,469,650) = 9,405,760. Of these hands, the hands that have a doubleton Q are (0.4)(2)(3,527,160) = 2,821,728. And the probability of the Q dropping on the second round of spades is 2,821,728/9,405,760 = 0.3.

4. This problem had a typographical error, and a unique solution was not possible. We apologize. This problem will be returned to our files for later use and will not be counted in the grading for "Perfect" Fall entries.

5. Thirty-two knights can be placed on a standard 8 × 8 chess board so that no knight threatens another knight. Place the knights on all the white or all the black squares. A knight can threaten only squares of the opposite color.

Bonus. The estimated period of the lake's unimodal oscillation (also called the sieche) is about 61 minutes. With L = length, h = depth, g = gravitational constant, and n = number of nodes, Merian's formula for the period of each oscillation is $2L/n\sqrt{(gh)}$ = 2(70,000)/(1) $\sqrt{(9.806)(150)}$ = 3,650 seconds or about 61 minutes. Merian's formula can be found by a library search or an Internet search. Howard McIlvried has independently derived Merian's formula. Send a request to *dondechman@aol.com* if you would like a copy.

COMPUTER BONUS. The next solution to the equation $x^7 + y^3 = z^2$ is $x = 17$, $y = 76{,}271$, and $z = 21{,}063{,}928$. This requires quite a bit of computer run time! Run time can be reduced by realizing that either x, y, or z must be even and the other two odd. This is the only other solution to this equation known to the judges. Be sure to let us know if you find another!

NEW WINTER PROBLEMS

1. A circle is divided into 13 sectors. Two players alternate by placing a coin either on a vacant sector or on two adjacent vacant sectors. The player who places the last coin wins. Which player has the advantage, and how should he/she proceed?

—Sam Loyd, circa 1880

2. A committee of four is to be chosen from a club which has a membership of 10 men and 12 women. In how many different ways can the committee be selected if it is to include at least two women, keeping in mind that Mr. and Mrs. Bickerson refuse to serve at the same time?

—*Introductory Combinatorics* by Richard A. Brualdi

3. Mary cashed a check, and the teller made a mistake and paid her the amount written in cents in dollars and the amount written in dollars in cents. Later, after spending $3.50, Mary realized that she still had twice the amount the check called for. What was the amount of the check?

—*Technology Review*

4. Five schoolgirls use their only nickel to weigh themselves on a scale in pairs with one staying on as the other gets off and another gets on. The 10 readings are 183, 186, 187, 190, 191, 192, 193, 194, 196, and 200 pounds. What are the weights of each of the five schoolgirls?

—*Discover*

5. A postal-service company will ship a package of up to 70 pounds with a length plus girth of up to 108 inches. A customer wishes to wrap and mail a uniform density right circular cone with the maximum volume. What is this cone's height, and what is its maximum allowable density, assuming the wrapping paper has negligible weight? Assume the girth is the circumference at the base of the cone and the length is the height of the cone.

—**Robert A. Grimm,** *IN* A *'50*

BONUS. The island of Bongo is a rectangle whose area is an integral number of square kilometers. When it was first colonized, the inhabitants decided to divide it into five counties with each county in the shape of a right triangle. Abongo, Ebongo, and Ibongo each had the same area. Obongo was larger, and Ubongo was the largest. Abongo, Ebongo, and Ibongo later merged into a single county that was still triangular. Bongo's shorter side is 45 kilometers. What is Bongo's area?

—Roger Schofield in *New Scientist*

COMPUTER BONUS. Using each integer exactly once, separate the integers 2 through 25 inclusive into two groups, such that the difference between the product of the integers in the first group and the product of the integers in the second group is as small as possible. What is this difference?

—**John S. Stanczak,** *MI* Δ *'70*

The judges are:
H.G. McIlvried III, *PA* Γ *'53*,
R.W. Rowland, *MD* B *'51*,
F.J. Tydeman, *CA* Δ *'73*,
and the columnist for this issue,
—**Don A. Dechman,** *TX* A *'57*

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