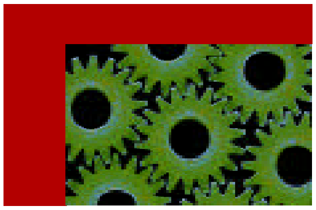

BRAIN TICKLERS

SUMMER REVIEW

The most difficult of the Summer Ticklers were No. 2, about the batting average, and No. 5, about estimating the height of a bridge, with only about half the entries having correct solutions to these problems. The Bonus, about the brightness of Venus, also proved to be difficult.

FALL ANSWERS

Here are the solutions for the Fall Brain Ticklers. Fall entries will be acknowledged in the next issue.

1. All of Jones' statements are true, and all of Smith's and Robinson's statements are false. Assuming Smith made all true statements leads to a contradiction, since S2 conflicts with S3 plus R1. Assuming Robinson made all true statements also leads to a contradiction, since this makes J2, J3, and J4 false, which is in conflict with R4. No such conflict occurs with assuming that all of Jones' statements are true, and it is then fairly easy to determine that all of Smith's and Robinson's statements are false.

2. The lengths of Scalenia's three frontiers are 26, 80, and 90 kilometers. Let A, B, and C be the shortest, intermediate, and longest frontiers, respectively. Let
$A + B + C = w^2$, $A + B - C = x^2$,
$A + C - B = y^2$, $B + C - A = z^2$.
Then, adding the last three equations gives, $A + B + C = x^2 + y^2 + z^2 = w^2$.
Also,
$A = (x^2 + y^2)/2$, $B = (x^2 + z^2)/2$, and
$C = (y^2 + z^2)/2$. Try a few small values for w and discover that, for $w = 7$,
$7^2 = 2^2 + 3^2 + 6^2$. But that would result in $A = 6.5$. So, try $14^2 = 4^2 + 6^2 + 12^2$
which yields $A = 26$, $B = 80$, and $C = 90$.
A simple computer program could also be written to check all As from 1 to 500, all Bs from $A + 1$ to 501, and all Cs from $B + 1$ to $A + B - 1$ and will quickly identify the solution with the shortest perimeter that satisfies the requirements.

3. The greatest change in the clock's relative brightness is a 25% decrease as the clock changes from
11:11:10 to 11:11:11 and a 25% increase as the clock changes from 11:11:11 to 11:11:12. Although there are other time changes with a larger change in the number of elements lit, such as going from 11:59:59 to 12:00:00, the change in relative brightness is less.

4. The only solution is 7678/33333 = .2303423034… which can be identifed by first expressing the repeating decimal as the fraction ONAND/99999. Then EVER x 9/N = ONAND. Try N = 3 first, since it is a factor of nine. Then the units digit from 3 x E plus the carry from 3 x R must equal 3. A little analysis shows that the only potential E,R combinations are 4,6; 7,8; and 7,9. Of these, only E = 7, R = 8 leads to a solution. Using the same logic, N = 1 does not produce a solution. More work is required when N does not divide 9, but no other solutions are found. The problem can also be solved with a simple computer program that tries all combinations of E and N from 1 to 9 and of V and R from 0 to 9 with E, V, R, and N all different digits and determines when the ONANDONAND… requirement is met.

5. There is a 53/120 = 0.4417 probability of being able to form a three-digit prime number. There are 10!/(7!)(3!) = 120 combinations of three balls drawn from an urn containing 10 numbered balls. Analysis of the primes between 102 and 987 identifies 53 different combinations that form primes.

Bonus. The curve is a half-cycloid that is tangent at the 50 km depth limitation, then a horizontal line, and then a mirror-image half-cycloid back up to the original elevation. The time for the trip is 663.75 seconds. In the 17th century, the Bernoulli brothers, Jacob and John, proved that the fastest way for a bead to slide from Point A down to Point B is along a cycloid. Kinetic plus potential energy is $0.5mv^2 - mgy = 0$ since the particle starts at rest and total energy remains constant. The speed is $v = ds/dt = \sqrt{(2gy)}$, and, thus, $dt = ds/\sqrt{(2gy)}$. But $ds = (dx^2 + dy^2)^{0.5}$ and $ds/d\theta = ((dx/d\theta)^2 + (dy/d\theta)^2)^{0.5}$ and $x = 25{,}000(\theta - \sin\theta)$ and $y = 25{,}000(1 - \cos\theta)$. Substituting and integrating with respect to θ from zero to π yields $t = \pi(25{,}000/9.8) = 158.674$ seconds to fall along the cycloid, which is also the time to rise. The time spent going horizontal is $(500{,}000 - 50{,}000\pi)/(\sqrt{2(9.8)(50{,}000)}) = 346.402$ seconds. So the total time is $2(158.674) + 346.402 = 663.75$ seconds.

Double Bonus. The problem was, given a triangle ABC, find a point P such that triangles ABP, ACP, and BCP have equal perimeters.

The solution we like best was submitted by Jeffrey R. Stribling, *CA* A *'92*, and is based on a construction discovered by Apollonius of Perga, a Greek geometer who lived about 200 B.C.

Label the sides opposite the vertices A, B, and C as a, b, and c. Construct three circles tangent to each other with centers at A, B, and C and radii of $s - a$, $s - b$, and $s - c$, respectively, where $s = (a + b + c)/2$. Then, the center P of the large circle that exactly circumscribes these three circles, that is, is tangent to all three circles, is the desired point.

Apollonius' method of constructing this circle starts by dropping a perpendicular from A to side a. Call its intersection with the small A circle D.

Then, draw a straight line from the point of tangency of the small B and C circles through D, and extend this line until it intersects the small A circle outside the triangle at a point E, which is the point of tangency with the circumscribing circle.

Construct a straight line from point E through point A. The desired point P is then the intersection of this line with lines similarly drawn starting from B and C.

It is now easy to prove that the perimeters of ABP, ACP, and BCP are all equal to the diameter of the circle with center P.

Try this construction as a homework exercise! By the way, there is no solution if the largest of the interior angles of triangle ABC is too obtuse.

NEW WINTER PROBLEMS

1. Some space travelers landing on a distant planet find a thriving civilization similar to our own. However, those on the planet have eight fingers on each hand, and thus they use a hexadecimal system. By coincidence, they use coins with values of 1, 5, 10, 25, and 50, just as we do, but these numbers are hexadecimal. Furthermore, they love to gamble as evidenced by their toll bridge, which instead of being a fixed toll, flashes the toll as a random number from 1 through 100, inclusive, and hexadecimal, of course. To use the exact change lane, how many coins of each denomination must a traveler carry in order to have the least total value of coins and still be able to exactly pay any toll?

—**William W. Verkuilen,** *WI* B *'92*

2. What minimal number of knights is required so every square on a standard 8 x 8 chessboard is either occupied by a knight or threatened by a knight? A square is threatened if a knight can move to that square. Recall that a knight moves two squares in any direction and then one square perpendicular to that direction to land on a square of opposite color. The move can occur even if the intervening squares are occupied.

— Daryl Cooper

3. An integer N consisting of five all-different nonzero digits has the curious property that it is equal to the sum of all the different three-digit integers formed by the three-digit permutations of its five digits. Find N.

—**Thomas R. Diaz-Davilla,** *PR* A *'96*

4. In the tiny kingdom of Podunk, there are exactly 1,000 inhabitants, and they have their own vehicles with three-digit license-plate numbers from 000 through 999. Plate 000 belongs to comedian Zero Mousetell. One day, someone noticed that the king's flag had been flown upside down, a considerable insult. A surveillance camera caught the departing car and showed that the first two digits of its license plate were 00 but the third digit was blocked by a pole. Zero was the prime suspect. Most citizens were out of town attending a fair, but those at the fair did not see Zero there. On a random basis, how many citizens, not counting Zero, would have had to remain in town for Zero to have just less than a 50% chance of being the culprit?

— Kurt F. Hafner Jr.

5. To produce this odd-looking product table, I wrote four digits in ascending order from left to right across the top and repeated them down the left side and then filled in the product matrix in the usual way. Then I transliterated the numerals, replacing each digit with a different one consistently throughout. Finally, I rubbed out all the entries in the table except 1 x 1 = 0 and 7 x 9 = 54. What are the four digits I started with?

x	1	3	7	9
1	0			
3				
7				54
9			54	

— Martin Hollis

Bonus. Dissect a square into the least number of right triangles with legs in the ratio of 2 to 1 such that the area of each triangle is different. What is this least number?

— Karl Scherer

Computer Bonus. What is the largest 11-digit prime that contains each of the digits 0 through 9 at least once.

—**Richard I. Hess,** *CA* B *'62*

The judges are: H.G.McIlvried III, *PA* Γ *'53;* R.W. Rowland, *MD* B *'51;* F.J.Tydeman, *CA* Δ *'73;* and the columnist for this issue,

—**Don A. Dechman,** *TX* A *'57.*