BRAIN TICKLERS

Results From Winter

Perfect Scores

Strong, Michael D.	PA	Α	'84
Voellinger, Edward	Non-member		

Other

Bannister, Kenneth A.	PA	Β	'82
*Berman, Alan D.	CA	Ε	'91
Berthold, Kristopher D.	TX	Β	'04
Bertrand, Richard M.	WI	Β	'73
*Bohdan, Timothy E.	IN	Γ	'85
Coleman, Benjamin J.	NH	Β	'20
*Cooper, Benjamin	Non-member		
Couillard, J. Gregory	IL	Α	'89
Dechman, Don A.	TX	Α	'57
Fine, Joseph M.	PA	Ζ	'75
Glaze, James B.	KS	Γ	'75
*Griggs Jr., James L.	OH	Α	'56
*Gulian, Franklin J.	DE	Α	'83
Gulian, Joseph D.	Son of member		
Hasek, William R.	PA	Γ	'49
*Janssen, James R.	CA	Γ	'81
*Johnson, Mark C.	IL	Α	'00
*Kimsey, David B.	AL	Α	'71
Lalinsky, Mark A.	MI	Γ	'77
Maino, Jon	PA	Θ	'20
Miller, David J.	MA	Ε	'71
*Norris, Thomas G.	OK	Α	'56
Oliver, Christopher R.	AL	Ε	'08
Parks, Christopher J.	NY	Γ	'82
Penlesky, Richard J.	WI	Β	'73
Richards, John R.	NJ	Β	'76
Riedesel, Jeremy M.	OH	Β	'96
Rowe, Steven A.	ME	Α	'81
Barr, David A.	Non-member		
*Schmidt, V. Hugo	WA	Β	'51
*Slegel, Timothy J.	PA	Α	'80
*Spong, Robert N.	UT	Α	'58
Spring, Gary S.	MA	Ζ	'82
Spring, Mitchell G.	Son of member		
Spring, Olivia	Non-member		
Summerfield, Steven L.	MO	Γ	'85
Withers, Derek R.	UT	Γ	'19
Zison, Stanley W.	CA	Θ	'83

*Denotes correct bonus solution

Winter Review

The hardest problem was number 5 (bowling balls) with only 31 percent correct answers.

The next hardest problem was number 3 (air bubble) with only 39 percent correct answers.

The Bonus (roller coaster) came in third with 53 percent correct answers.

Spring Answers

1: The solution to the cryptic addition is $\mathbf{688_9 + 68371_9 + 65_9 = 70245_9}$

The problem statement says that we're working in a base less than 10. There are nine different letters, so we must be working in base 9.

Column	4	3	2	1	0
Carries	C_4	C_3	C_2	C_1	
			A	L	L
	A	L	O	N	E
				A	T
	N	I	G	H	T

Having two Ts in column 0 tells us that L + E = 9 and C1 = 1. C_2 can be 0, 1, or 2. Having two different digits in column 3 tells us that C_3 = 1. Having two different digits in column 4 tells us that C_4 = 1 and N = A + 1.

The only way C_4 can be 1 is if L = 8 and I = 0. Knowing L = 8 tells us that E = 1.

We cannot reuse 0, 1, or 8 for A, H, or N so A must be 2, 3, 5, or 6. There are 12 possible A & O pairs only one of which does not reuse a previously assigned digit: A = 6 & O = 3 which implies that N = 7, H = 4, and G = 2. That leaves only one possible value for T: T = 5.

2: The exact probability of getting a sum of 50 is **0.0374894389**.

This question is related to the study of partitions. One could look to the large body of literature on the subject or use dynamic programming to find that the exact probability of getting a sum of 50 is 374,894,389 / 10,000,000,000.

3: The probability that the reported Celsius temperature is wrong is **2 / 15**.

In any given 9 degree Fahrenheit span, there are four intervals in which the reported Celsius temperature is wrong. Taking the span 32 deg F – 41 deg F as an example:

32.5 <= T < 32.9: Sign = 1 deg C (should be 0)

34.5 <= T < 34.7: Sign = 2 deg C (should be 1)

38.3 <= T < 38.5: Sign = 3 deg C (should be 4)

40.1 <= T < 40.5: Sign = 4 deg C (should be 5)

The probability of being in one of these spans is (0.4 + 0.2 + 0.2 + 0.4) / 9.0 = 2 / 15. Since the problem stated that the actual temperature was equally likely to be in a range of 72 degrees F (a multiple of 9 deg F), the answer is 2 / 15.

4: One interesting approach to this problem: Start with a 4x4 magic square made up of the numbers 1 through 16. For example:

1	3	16	14
8	15	2	9
13	6	11	4
12	10	5	7

Add 11 to each entry:

12	**14**	**27**	**25**
19	**26**	**13**	**20**
24	**17**	**22**	**15**
23	**21**	**16**	**18**

Reverse each entry:

21	41	72	52
91	62	31	2
42	71	22	51
32	12	61	81

Those reversed digits form a magic square.

A computer search shows that there are 7,040 magic squares made up of the digits 1 through 16 and 5,696 of those will lead to a reversed magic square using this procedure.

5: Part A: The expected number of boxes is **363 / 20 = 18.15**

Part B: The expected number of boxes is **188,107 / 9,240 ≅ 20.36**

This is an example of the Coupon Collector's Problem. One could approach this via closed form equations from the literature dedicated to this type of problem, via absorbing Markov chains or via simulation. The answers for our specific scenarios are above.

BONUS:

The flagpole's shadow rotates at approximately **69 deg/hr**.

In the diagram below:

Point O is the center of the Earth
Point N is the North Pole

- Point A is Albuquerque [Latitude = M (for Member) = 35 degrees]
- Point S is the solar subpoint [Latitude = T (for Tilt) = 23.5 degrees]

The thick blue arcs are great circles on the Earth's surface. The thin green lines are vectors from the Earth's center to surface points.

The dashed red arcs are angles between vectors.

Let's define four angles:

- *A* is the angle between planes *AON* and *SOA*
- *N* is the angle between planes *NOS* and *AON*
- *a* is the angle between vectors *OS* and *ON* = 90 – *T*
- *n* is the angle between vectors *OA* and *OS* (varies with time)

By the spherical law of sines:

$$\frac{\sin(A)}{\sin(a)} = \frac{\sin(N)}{\sin(n)}$$

When we get very close to solar noon:

- *A* gets close to 180 so sin(*A*) approaches 180-*A*
- *N* gets small so sin(*N*) approaches *N*
- *n* approaches *M* – *T*

Substituting, rearranging, and differentiating:

$$\frac{180-A}{\sin(90-T)} = \frac{N}{\sin(M-T)}$$

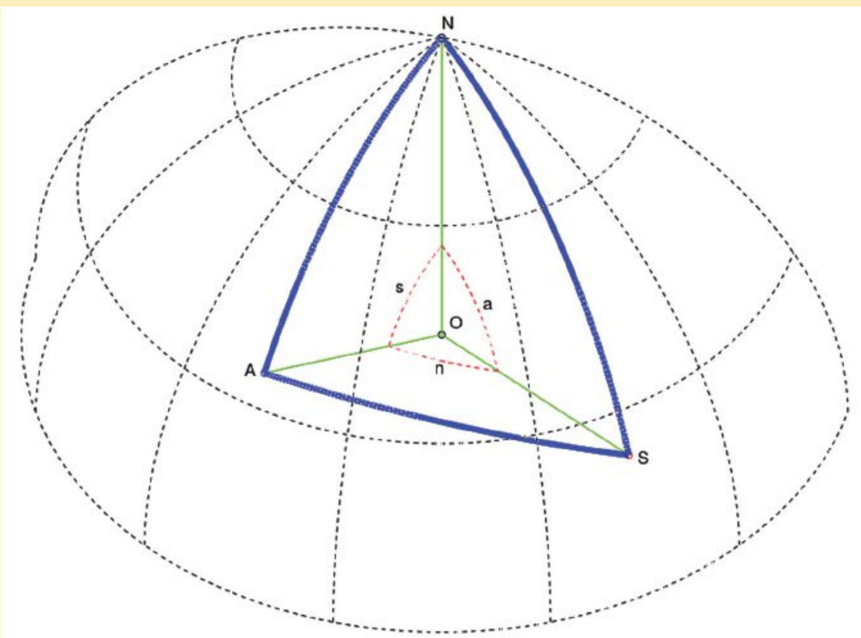
$$180-A = N * \frac{\sin(90-T)}{\sin(M-T)}$$

$$A = 180- N * \frac{\sin(90-T)}{\sin(M-T)}$$

$$\frac{dA}{dN} = -\frac{\sin(90-T)}{\sin(M-T)} = -\frac{\cos(T)}{\sin(M-T)}$$

The Earth rotates 15 deg/hr, so we finally have:

$$\frac{dA}{dt} = \frac{dN}{dt} * \frac{dA}{dN} = (-15\frac{deg}{hr}) * (-\frac{\cos(T)}{\sin(M-T)})$$



The flagpole shadow rotates on the north side of the flagpole as fast as the Sun rotates on the south side of the flagpole so our answer is 69 deg/hr.

DOUBLE BONUS:

136,694 out of the possible 15,820,024,220 deals are a gin.

Working out the combinatorics for many of the gin combinations is challenging. The overall problem has the additional challenge of tracking hands that qualify as gin in more than one way. Take, for example, a hand with the following cards: 2, 2, 2, 3, 3, 3, 3, 4, 5, 6. This qualifies as gin in two ways:

- 3 of a kind (2, 2, 2) + 3 of a kind (3, 3, 3) + 4 card straight (3, 4, 5, 6)
- 3 of a kind (2, 2, 2) + 4 of a kind (3, 3, 3, 3) + 3 card straight (4, 5, 6)

Adding up the ways of making each possible gin combination (141,982) and subtracting the number of hands that qualify as gin in more than one way (5,288) leaves us with 136,694 gins.

New Summer Problems

1: Wrongs Make a Right

Contrary to conventional wisdom, two wrongs can make a right, at least in the sense that the cryptic addition WRONG + WRONG = RIGHT has a solution. In fact, it is also possible to have three WRONGs equal a RIGHT. Our question is, what is the minimum value of *w* (*w* > 1) such that *w* WRONGs do not equal a RIGHT? The usual rules of cryptics apply and one can assume all the values are in base 10. Of course, the same letter can have different values in different solutions.

—**Howard J. McIlvried,** *PA Γ '53*

2: Tennis Rankings

The Nets and the Deuces play each other in a tennis tournament.

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from page 13.

Summer Problems *Continued*

Each team has three players ranked 1 through 3, where the player ranked 1 is the strongest player of the three, and rank 3 is the weakest. In the tournament, each player from one team is paired against one player from the opponent in a head-to-head match. If a player's ranking is A and his opponent's ranking is B, then the first player's probability of winning the match is B ÷ (A + B). The Nets win the coin toss, which gives them the privilege of determining the pairings. What is the Nets best pairing strategy, and using that strategy, what is their probability of winning the tournament by winning a majority of the matches?

—**Don A. Dechman,** *TX A '57*

3: From the Movies

In the eponymous movie, Will Hunting, responding to the challenge of Professor Lambeau, drew eight, simplified (that is, no vertex has exactly two neighbors), unique trees with exactly 10 vertices. How many *N* such trees are there with exactly 11 vertices? Of those *N* trees, what is the maximum number of non-leaf vertices found on a single tree?

—*adapted from:*
Good Will Hunting, 1997

4: Gizmo Orders

Through an unfortunate logistics error, the SuperGizmo Company has received a large number of two sizes of boxes—one size holds 20 Gizmos and the other holds 43 Gizmos. They decide to only accept orders that they can fill with an exact combination of full boxes. Thus, they could fill orders for 558 (15x20+6x43), 559 (13x43), 560 (28x20), or 561 (13x20+7x43), but not 562. What is the largest order that cannot be filled? What is the general solution for any two containers of capacity *x* and *y* Gizmos, assuming *x* and *y* are relatively prime and are both greater than one?

—*Madachy's Mathematical Recreations*
by Joseph S. Madachy

5: Toroid Planet

Sometime in the future, five Tau Bate astronauts are exploring a recently discovered solar system and land on one of its small planets. To their amazement, they discover the planet is in the shape of a perfect torus. Furthermore, they detect a series of channels, each of which is a perfect circle formed by the intersection of a plane with the planet's surface. Suspecting that the channels were dug by intelligent beings, the five start out looking for artifacts.

Hannah follows one channel and finds its length to be 30 kms in circumference.

Jack follows a longer channel which does not cross Hannah's.

Dana's channel is 50 kms and crosses Jack's.

Ron's channel is 60 kms and also crosses Jack's.

Finally, Sarah follows a channel which is the maximum possible circumference.

What is the length of Sarah's channel?

—*All-Start Mathlete Puzzles*
by **Richard I. Hess, Ph.D.,** *CA B '62*

BONUS: Fortunata considered twenty linear equations based on the English spellings of the numbers 1 through 20 (and yes, O is a variable, not to be confused with the digit 0):

O + N + E = 1
T + W + O = 2
...
T + W + E + N + T + Y = 20

There was no simultaneous solution to all twenty equations, but she did find a solution to nineteen of the equations. Her solution had exactly four of the specified variables equal to zero. What common English word could she spell by arranging the four letters that name those variables?

—**Daniel L. Stock,** *OH A '80*

COMPUTER BONUS:

On a remote island lives a tribe with a strange custom. On their New Year's Day, each member of the tribe must deliver to the chief one unit of grain for each year they are old. The grain is weighed using a set of seven sacred stones, each of which is an integral number of grain units. A traditional part of the ritual is that no more than three of the stones can be used for each weighing, although they need not all be on the same pan of the balance scale used. Another part of the tradition is that if any tribe member lives to an age such that their contribution of grain cannot be weighed using at most three stones, the levy will be suspended forever. Last year an elder died just a few months shy of this critical age, much to the relief of the chief. Assuming the seven stones are chosen to make the critical age the maximum possible, what was the elder's age when they died, and what are the weights of the seven stones?

—*Sunday Times*

Email your answers (plain text only) to any or all of the Summer Brain Ticklers to BrainTicklers@tbp.org or by postal mail to **Dylan Lane, Tau Beta Pi, P.O. Box 2697, Knoxville, TN 37901-2697**.
The method of solution is not necessary. The Computer Bonus is not graded. Where possible, exact answers are preferable to approximations. The cutoff date for entries to the Summer column is the appearance of the Fall *Bent* which typically arrives in mid-September (the digital distribution is several days earlier). We welcome any interesting problems that might be suitable for the column. Dylan will forward your entries to the judges who are **F.J. Tydeman,** *CA Δ '73;* **J.R. Stribling,** *CA A '92;* **G.M. Gerken,** *CA H '11;* and the columnist for this issue,

— **J.C. Rasbold,** *OH A '83*