RESULTS FROM WINTER

Perfect

*Couillard, J. Gregory	IL	Α	’89
*Gerken, Gary M.	CA	Η	’11
*Griggs Jr., James L.	OH	Α	’56
*Slegel, Timothy J.	PA	Α	’80
Spong, Robert N.	UT	Α	’58
*Strong, Michael D.	PA	Α	’84

Other

Barr, Robert A.	IL	Ε	’85
Barthel, Gerald R.	OH	Β	’67
Bohdan, Timothy E.	IN	Γ	’85
Dechman, Don A.	TX	Α	’57
*Gulian, Franklin J.	DE	Α	’83
Gulian, Joseph D.	Son of member		
Handley, Vernon K.	GA	Α	’86
Jordan, R. Jeffrey	OK	Γ	’00
Lalinsky, Mark A.	MI	Γ	’77
Mettler, Kelly M.	CA	Δ	’10
Riedesel, Jeremy M.	OH	Β	’96
Schmidt, V. Hugo	WA	Β	’51
Zison, Stanley W.	CA	Θ	’83

*Denotes correct bonus solution

THANK YOU HOWARD!

After more than 60 years of faithful service, our longtime Brain Ticklers’ head judge, **Howard G. McIlvried III,** *PA Γ ’53*, is calling it quits. His leadership and contributions will be missed by both readers and judges of this column. **Fred J. Tydeman,** *CA Δ ’73*, will be assuming the role of head judge. **Gary M. Gerken,** *CA H ’11*, a regular solver of Brain Ticklers with a strong record of solutions, has agreed to join the panel of judges. Gary will be writing the spring column going forward.

WINTER REVIEW

Tickler #2 (the logic problem involving couples with various vacation destinations) was the easiest problem for this issue, with all received submissions having the correct answer. Tickler #1 (the cryptarithm) proved both popular and straightforward, with all but one submission correct. The spherical geometry of Tickler #3 tripped up the most people, with ⅔ of submissions being correct. Equally difficult was the Bonus about the two spheres: the ⅔ correct submissions was tied for the lowest ratio, but nevertheless was higher than the columnist expected.

SPRING ANSWERS

1 The sum is **8991**, which can be achieved with a discard of 0 or 9. In the 9 discard case, the three primes are uniquely 23, 467, and 8501. We found 99 different sums generated from three addend primes with a single discarded digit. Of those 99 sums, only 8991 could be generated with two different discards. Six different addend sets could be created with a discarded 0, but only one set with the 9 discarded.

2 For 365 coins, no more than **7** weighings are required to determine if there is an eccentric penny, and the direction of the oddity.

It has been shown that the number of weighings needed for N coins is $\lceil \log_3(2N + 3) \rceil$. The judges found several specific algorithms for a year’s worth of pennies. A general solution for N coins is more challenging. Our generalized algorithm can be found at *www.tbp.org/pubs/brainTicklers.cfm*.

Note that the intent of the problem was to find the least upper bound on the number of weighings. A few readers pointed out that there are algorithms where the minimum number of weighings could be 2, although the maximum number of weighings may be much greater than 7. Given the ambiguous wording of the problem, the judges decided to accept both **2** and **7** as correct answers.

3 $\mathbf{19683 = (1 + 9 + 6 + 8 + 3)^3}$. ABCDE must be a five digit number with unique digits, so 12345 ≤ ABCDE ≤ 98765. The cube root sum is therefore ≥ 24 and ≤ 46. Note that the maximum sum of five digits is 35, further limiting the cube root to ≤ 35. Using trial and error, the 5 cubes that fit those constraints and have five unique digits are: 24^3 = 13824, 27^3 = 19683, 29^3 = 24389, 32^3 = 32768, 35^3 = 42875. Only the second cube meets the sum constraint, so ABCDE = 19683.

4 The cards are distributed as follows:

1	4
2	44
3	52
4	720
5	1096
6	3744
7	16440

Three cards can be drawn in C(52,3) = 22,100 ways. A royal flush can be chosen in exactly **4** ways (one for each suit). A straight flush can be drawn in 4(12) = 48 ways (one for each suit times the number of denominations which can begin a straight), then subtracting the royal flushes, or 48 – 4 = **44** ways. The three-of-a-kind can be drawn in (13)C(4,3) = 52 ways (for each denomination, choose 3 of 4 cards). A straight can be drawn in $(12)4^3$ = 768 ways (for each denomination that can begin a straight, choose one card from each of three consecutive denominations), less the 48 straight flushes for 768 – 48 = **720**. A flush can occur in (4)C(13,3) = 1144 ways (for each suit, choose 3 of 13 cards), less the 48 straight flushes or 1144 – 48 = **1096**. Pairs can be drawn in (13)C(4,2)C(48,1) = **3744** ways (for each denomination, choose two of four cards, and then choose one of the remaining 48 cards). Subtract the first six counts from 22,100 to get **16,440**.

5 $2^{4{,}700{,}063{,}497} \equiv$ **3** (mod 4,700,063,497). Use the right-to-left binary method for modular exponentiation. The length of the exponent determines the complexity of the algorithm. The exponent 4,700,063,497 is a 33 bit value in length with 12 one bits. Therefore, there are 33 steps to the calculation, with 12 updates to the result. The calculations are possible (but tedious) by hand; the judges found an extended precision calculator helpful. Curiously, the number 4,700,063,497 is known to be the smallest integer n greater than one such that $2^n \equiv 3 \pmod{n}$.

Bonus. The longest chain with less than 1,000 links is **989** with breaks between links 164 and 165, 594 and 595, 824 and 825. The distance between the posts of the equilateral triangle is 163,185. The shortest chain length that can be wrapped in multiple ways is **125** links, with breaks between links 2 and 3, 72 and 73, 102 and 103, or alternatively with breaks between 57 and 58, 92 and 93, 117 and 118. In this case, the distance between the posts of the triangle is 2,625.

Computer Bonus. 2016 + 2080 + 2145 = 79². The sum of three consecutive triangular numbers is given by $(3n^2 + 9n + 8)/2$. Solve this equation for integral n, k: $2k^2 = 3n^2 + 9n + 8$. The first three solutions are $n = 5$, $k = 8$, $8^2 = 64 = 15 + 21 + 28$; $n = 14$, $k = 19$, $19^2 = 361 = 105 + 120 + 136$; and $n = 63$, $k = 79$, $79^2 = 6241 = 2016 + 2080 + 2145$.

NEW SUMMER PROBLEMS

1 cos(arcsin(10.0)) has what value?

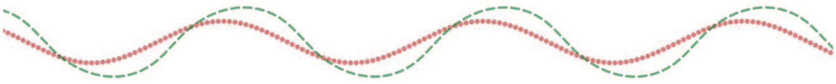
—**Fred J. Tydeman**, *CA Δ '73*

2 Samantha's square courtyard (total area less than 100 m²) has an area equal to an integral number of square meters. She decides to install an octagonal fish pond in the courtyard. To mark the sides of the pond, she draws lines from each corner of the square to the midpoints of the two sides not touching said corner. She finds that the perimeter of the pond, thus delimited, is an integral number of meters. What is the area of the courtyard, and what is the perimeter of the pond?

—An Enigma by Andrew Gibbons in *New Scientist*

3 A student rides their bicycle across the mud, leaving tracks as in the diagram. One of the colored tracks represents the path of the rear tire, the other the front tire. Is the direction the student traveling from left to right, or right to left? Does the red or green color represent the rear-tire track?

—adapted from *The Adventure of the Priory School* by Arthur Conan Doyle



4 What is the exact probability that the first decimal digit of 2^N is a 1, as N becomes large?

—*Challenging Mathematical Problems with Elementary Solutions* by A.M. & I.M. Yaglom

5 As part of a Mission Impossible team, you have a vital switch to throw in exactly 31 minutes. Unfortunately, your watch has just stopped. All you have are two lengths of fuses, which burn irregularly, and a supply of matches. One fuse takes 50 minutes to burn completely when lit from either end, and the other which burns in 24 minutes. How do you use these two fuses to time exactly 31 minutes? (A fuse can not be folded to make a shorter time, but it can be burned from both ends to get half the time.)

—**Richard I. Hess**, *CA B '62*

Bonus. Divide a rectangular sheet of paper into 8 identical squares. On the front side of the paper, number the 8 squares as shown:

1	8	7	4
2	3	6	5

On the back of square N write the Nth letter of the alphabet.

Fold the sheet along the lines (like a paper road map) to form a square packet such that the "1" square is face up on top. Describe the folding sequence (that is, what folds onto what) needed such that the squares are in serial order 1 through 8 from top to bottom. Other than the top square, the squares need not be number side up.

Repeat the task, this time with the front side squares numbered as below:

1	8	2	7
4	5	3	6

—*My Best Mathematical and Logic Puzzles* by Martin Gardner

Double Bonus. A circular 30 cm diameter pizza is cut into 8 identical slices. The slices are arranged on a flat circular plate that is marginally smaller in diameter than the original 30 cm, such that the slices do not overlap each other nor do they overlap the edge of the plate. What is the maximum number of slices S that can fit on the smaller plate? What is the diameter D_S of the smallest plate that can hold S slices? What is the diameter D_{S-1} of the smallest plate that can hold S-1 slices?

— **Timothy J. Slegel,** *PA A '80*

Send your answers to any or all of the Summer Brain Ticklers to *BrainTicklers@tbp.org* as plain text only or by postal mail to **Dylan Lane, Tau Beta Pi, P.O. Box 2697, Knoxville, TN 37901-2697**. The method of solution is not necessary, and the Computer Bonus/Double Bonus is not graded. Where possible, exact answers are preferable to approximations. The cutoff date for entries to the Summer column is the appearance of the Fall *Bent* in mid-September (the digital version is distributed a few days earlier). We welcome any interesting problems that might be suitable for the column. Dylan will forward your entries to the judges who are **F.J. Tydeman,** *CA Δ '73;* **J.R. Stribling,** *CA A '92;* **G.M. Gerken,** *CA H '11;* and the columnist for this issue,

— **J.C. Rasbold,** *OH A '83*