Brain Ticklers

RESULTS FROM WINTER

Perfect			
*Bohdan, Timothy E.	IN	Γ	'85
*Couillard, J. Gregory	IL	A	'89
*Eanes, Robert Sterling	TX	Γ	'67
Gerken, Gary M.	CA	H	'11
Griggs Jr., James L.	OH	A	'56
*Gulian, Franklin J.	DE	A	'83
Gulian, Joseph D.	Son of member		
*Novak, Connor W.	Son of member		
*Richards, John R.	NJ	B	'76
*Slegel, Timothy J.	PA	A	'80
Strong, Michael D.	PA	A	'84
*Van Wyk, Rogell	IN	A	'59
Other			
Alexander, Jay A.	IL	Γ	'86
Aron, Gert	IA	B	'58
Chatcavage, Edward F.	PA	B	'80
Heske III, Theodore	PA	A	'86
Jordan, R. Jeffrey	OK	Γ	'00
Kimsey, David B.	AL	A	'71
Kramer, J. David R.	PA	Δ	'57
Lalinsky, Mark A.	MI	Γ	'77
Marks, Lawrence B.	NY	I	'81
Marks, Benjamin	Son of member		
Marrone, James I.	IN	A	'61
Mettler, Kelly M.	CA	Δ	'10
*Norris, Thomas G.	OK	A	'56
Rao, Sandesh S.	MI	B	'17
Riedesel, Jeremy M.	OH	B	'96
*Schmidt, V. Hugo	WA	B	'51
Schweitzer, Robert W.	NY	Z	'52
Sigillito, Vincent G.	MD	B	'58
Skorina, Frank K.	NY	M	'83
Vargas, Christina M.	FL	Δ	'07
*Voellinger, Edward J.	Non-member		

* Denotes correct bonus solution

WINTER REVIEW

The winter column saw opposite extremes on the difficulty spectrum. Problem #2, the logic problem involving toy makers, was decidedly the easiest. Not only did it have the most submissions of any problem, all answers received were correct. The double bonus about school mascots was somewhat orthogonal to the spirit of our traditional brain ticklers.

This was evident in the results with very few submissions and no correct answers. Poor Joe Miner! The bonus problem about blackjack odds was the second-most difficult—about 2/3 submitters with meticulous bookkeeping skills arrived at the correct solution.

SPRING ANSWERS

1 The five 9-digit numbers are: **656,356,768; 714,924,299; 688,747,536; 999,777,999; and 345,588,888**. The 5th root of a 9-digit number must lie between 40 and 63. On a spreadsheet, calculate the 5th powers over this range. Now, eliminate all numbers that contain a 0, two 1's, or three 2's. This leaves only 656,356,768, 714,924,299, and 992,436,543, and only the 1st and 2nd of these between them contain all the digits 1 through 9. The 4th root of a 9-digit number must lie between 100 and 178. Calculate the 4th powers over this range, and use the same elimination criteria. This leaves 688,747,536 as the only possibility for the 3rd number. Examination of the first three numbers shows that there are several possibilities for only two different digits; choices are: four 7's, five 8's; four 7's, five 9's; and three 7's, six 9's. Of the several possibilities, only 999,777,999 is divisible by 33 and 111. This leaves 345,588,888 for the 5th number.

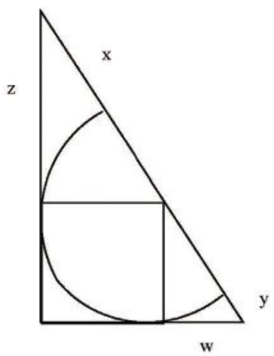
2 The probability of getting a bridge hand with no aces or face cards and exactly two 2's, three 3's, and four 4's is **99/246,511,475.** The number of ways to get two 2's is C(4, 2) = 6, where C(i, j) is the number of combinations of i objects taken j at a time. The number of ways of getting three 3's is C(4, 3) = 4, and the number of ways to get four 4's is C(4, 4) = 1. Finally, since they can only have face values of 5 through 10, the number of ways to get the other four cards in the hand is C(24, 4) = 10,626. The number of different bridge hands is C(52, 13) = 635,013,559,600. Therefore, the probability of getting a hand such as indicated above is P = 6(4)(1)(10,626)/635,013,559,600 =99/246,511,475=0.000000401604, or approximately once in every two-and-a-half-million hands.

3 The ten item codes are the powers of 3 from 0 to 9: **1, 3, 9, 27, 81, 243, 729, 2187, 6561,** and **19683.** There are exactly 3^{10} possible orders, and this coding scheme produces a unique number for every possible order, from 0 to 3^{10}-1. To get the order number, the waitperson merely adds the codes for all the items ordered. (If two of an item are ordered, the code is added twice.) When an order is received in the kitchen, the chef converts the base 10 order into a base 3 number, a string consisting of 0's, 1's, and 2's, left padded with 0's to form a 10-*trit* (ternary digit) representation. Starting at the unit's trit and continuing through all 10 trits, the chef provides 0, 1, or 2 of the corresponding menu item. In general, if there are k items which can be ordered up to n times, the item codes are $(n+1)^{i-1}$ for i=1 to k.

4 A minimum of **seven pegs** is needed to transfer eighteen discs from the initial peg to another peg in less than 1 minute in the Super-Tower of Hanoi puzzle. A little thought will show that, if there are n empty pegs, n discs can be moved from the initial peg to another peg. Just distribute one disc onto each empty peg and then reassemble the discs in order on the peg with the largest disc; this takes 2n–1 moves. This leaves n–1 empty pegs, so the procedure can be repeated with one fewer disc, that is, n–1 discs can be moved in 2n–3 moves. Continuing in this way, d=n(n+1)/2 discs can be stored on n pegs. If n=5, then d=15, but we want to move 18 discs, so we need 6 pegs in addition to the original peg. With 6 pegs, 6 discs can be stored on P_1, 5 on P_2, 4 on P_3, and 2 on P_4. This takes 11+9+7+3=30 moves to store the discs. Then, it takes the same number of moves to rebuild the tower plus one move to transfer the largest disc, but this is a total of 61 moves (61 seconds) and we only have a minute. However, we can reduce the number of moves by 2 if, instead of moving 2 discs to P_4, we move 1 to P_4 and 1 to P_5. This only requires 4 moves instead of 6, a gain of two moves, for a total of 59 moves, just under a minute. With 7 pegs, just before the largest disc is moved, the discs will be distributed as shown in the table. The total number of moves required will be 11+9+7+1+1+1+1+1+7+9+11=59. The transfer could be completed with fewer than 7 pegs, but it would require more than 1 minute.

Peg	0	1	2	3	4	5	6
Discs	18	1-6	7-11	12-15	16	17	--

5 The maximum and minimum lengths of the hypotenuse are **442 cm** and **350 cm,** respectively. The following discussion refers to the diagram.



First, note that the radius of the semicircle is 120 cm. The large triangle contains two smaller triangles which are similar to the large triangle and to each other. Call the upper triangle A and the lower triangle B. Applying the Pythagorean Theorem to A gives $z^2 + 120^2 = (x + 120)^2$. Solving for x gives $x = \sqrt{(z^2 + 120^2)}$ -120. Similarly, for B, $y = \sqrt{(w^2 + 120^2)}$ -120. We need to find integral values for w and z such that $(w^2 + 120^2)$ and $(z^2 + 120^2)$ are perfect squares. Since A and B are similar triangles, we have $z/120 = 120/w$, or $w = 120^2/z$. That is, z is a factor of 120^2, but $120^2 = 2^6 3^2 5^2$, which has 7(3)(3) = 63 factors. Thus, there are only 63 possible values for z, and once we have z, we also have x, y, and w. It is a relatively simple matter to check all these possibilities on a spreadsheet, which yields the following results, in cm.

z	50	64	90	160	225	288
x	10	16	30	80	135	192
w	288	225	160	90	64	50
y	192	135	80	30	16	10
Hyp.	442	391	350	350	391	442

Thus, the maximum length of the hypotenuse is 442 cm, and the minimum length is 350 cm.

Bonus. The gravitational acceleration acting on a person standing at the center of the base would be **24.8 m/s²** and acting on a person at the apex, it would be **2.75 m/s².** We first calculate the center of mass of the cone. Consider a cone with its apex at the origin of a Cartesian coordinate system and its axis lying on the z axis. The x and y components obviously lie on the axis of the cone, so all that is needed is the z component. The equation for the center of mass is $z_{cm} = (1/M)\int z\,dm$, where M is the total mass of the cone, and dm is the mass of a thin slice of the cone perpendicular to the cone's main axis. Now, by similar triangles, $r/z = R/H$, where R is the radius of the cone's base, H is the distance from the apex to the middle of the base, and r and z are the radius and distance from the origin of a thin slice of the cone. Now, $dm = \sigma\pi r^2 dz = \sigma\pi(R/H)^2 z^2 dz$, where σ is the density of the cone. $M = \int dm = \sigma\pi(R/H)^2 \int_0^H z^2 dz = \sigma\pi(R/H)^2[z^3/3]_0^H = \sigma\pi(R/H)^2(H^3/3) = \sigma\pi R^2 H/3$. Now, $z_{cm} = (1/M)\int dm = (\sigma\pi(R/H)^2/M) \int_0^H z^3 dz = (\sigma\pi/M)(R/H)^2(H^4/4) = (3\sigma\pi/\sigma\pi R^2 H)(R/H)^2(H^4/4) = 3H/4$. Thus, the apex is 3/4 the height of the cone from the center of mass. Now, $F = mg_o = GMm/R^2$, so $g_o = GM/R^2$, where $G = 6.673\times10^{-11}$ m³/kg-s². The volume of the earth $V_e = (4\pi/3)R_e^{\,3} = V_c = (\pi/3)R_c^{\,2}H_c = (2\pi/3)R_c^{\,3}$, so $R_c^{\,3} = 2R_e^{\,3}$ or $R_c = 1.26R_e = 1.26(6{,}370{,}000) =$ 8,026,200 m. $V_c = 2\pi(8{,}026{,}200)^3/3 = 1.0827\times10^{21}$ m³ and $M_c = 5{,}518V_c = 5.9743\times10^{24}$ kg. A person on the apex is 3/4 of the height of the conical planet from the center of mass. Therefore, $g_o = (6.673\times10^{-11}$ m³/kg-s²$)(5.9754\times10^{24}$ kg$)/(0.75\text{x}2\times8.0262\times10^6$ m$)^2 = 2.75$ m/s². For someone at the middle of the base, $g_o = (6.673\times10^{11}$ m³/kg-s²$)(5.9754\times10^{24}$ kg$)/(0.25\times2\times8.0262\times10^6$ m$)^2 = 24.75$ m/s².

Double Bonus HOWIE + FRED + CHUCK + JEFF = JUDGE translates to **380ab + 61b5 + 73472 + cb66 = c459b** in base 13. Or, 103400 + 13499 + 207287 + 28307 = 352493 in base 10.

NEW SUMMER PROBLEMS

1 A man and his daughter make a 64 km trip with their saddle horse which travels 16 km per hour but can carry only one person at a time. The man walks at the rate of 6 km per hour and the daughter at the rate of 8 km per hour. The trip is made in the following manner. The two start out at the same place and time with one walking and the other riding. After the rider has gone a certain distance, the rider dismounts, ties the horse, and immediately starts walking ahead. When the walker comes up to the horse, the walker mounts and rides forward another certain distance, whereupon they dismount, and the above process is repeated. The two reach the halfway point at the same time, at which point they take a half-hour break to feed the horse. They then proceed in the same manner as in the first half of the journey. At what time does the pair arrive at their destination if they start at 6 a.m.?

—*Ingenious Mathematical Problems and Methods* by L. A. Graham

2 If a decimal integer ends in a particular digit, repeated enough times, the hexadecimal representation of the number will end with either 1 or 3 repeated hex digits. For example, a decimal number ending in a string of 1's will produce a hexadecimal number ending in the repeated digits "1C7," and a decimal number ending in a string of 6's will produce a hexadecimal number ending with the repeated digit "A" (A stands for the value 10). How long is the smallest decimal integer consisting entirely of 6's which has a hexadecimal representation that ends with a string of 25 consecutive A's?

—Franklin J. Gulian, *DE A '83*

3 Twice the larger of two numbers is three more than five times the smaller and the sum of four times the larger and three times the smaller is 71. What are the numbers?

—*Mean Girls*, 2004

4 Mary stood beside a large pile of turnips (fewer than 20,150), which she was to distribute evenly among the people in the group in front of her. Since it was unlikely that the number of turnips would divide evenly among the people, she was given permission to add or subtract turnips to produce an even division to the nearest integral value. A quick division of the number of turnips by the number of people gave an answer between 99 and 100, but closer to 99. However, realizing that everyone would prefer 100 turnips to 99, she decided to do the calculations a different way. First, she assumed everyone would get 99 turnips, so she divided the number of turnips by 99. Then, she assumed everyone would get 100 turnips, so she divided the number of turnips by 100 and found that this gave an answer closer to the actual number of people than the division by 99, so she added some turnips to the pile and gave everyone 100 turnips. How many people were there and how many turnips were in the original pile?

—Keith Austin in *New Scientist*

5 My uncle's ritual for dressing each morning except Sunday includes a trip to the sock drawer, where he (1) picks out three socks at random, then (2) wears any matching pair and returns the odd sock to the drawer or (3) if he has no matching pair, returns the three socks to the drawer and repeats steps (1) and (3) until he completes step (2). The drawer starts with 16 socks each Monday morning (eight blue, six black, two brown) and ends up with four socks each Saturday evening. On which day of the week is he least likely to get a pair from the first three socks chosen? On average, which day of the week requires the greatest number of times that my uncle grabs three socks from the drawer?

—Richard Hess, *CA B '62*, in *The Mathemagician and Pied Puzzler*

Bonus Suppose you are given four non-standard but otherwise fair, dice: one blue, one green, one red, and one white. For simplicity, assume that the four dice have the numbers 1 through 24 on their faces. After a long sequence of rolling pairs of these dice, you conclude the following: When rolled simultaneously, two thirds of the time the blue die shows a higher value than the green die. When rolled simultaneously, two thirds of the time the green die shows a higher value than the red die. When rolled simultaneously, two thirds of the time the red die shows a higher value than the white die. Now, if the blue and white dies are rolled simultaneously, what is the least probability P that the blue die will show a higher value than the white die on a given roll? For such P, give an example of the distribution of the numbers 1 through 24 on the four die.

—Puzzle Corner by Allan Gottlieb in *Technology Review*

Double Bonus The game of Sprouts, invented by J.H. Conway and M.S. Patterson, is a pencil and paper game played as follows. Any number of points are placed on the paper, and the players take turns connecting two of the points. Each move consists of two parts: (a) drawing a line between two points or looping back to the starting point and (b) placing a new point on the line anywhere except at an endpoint. There are three rules: (1) a line cannot cross itself or any other line; (2) a line cannot pass through an existing point but must start and end on existing points (can be the same point); and (3) no point can have more than three lines that start or end on it. The player who cannot make a move loses. A little thought will show that the game must always end. In how many different configurations can a two-point game end? Isomorphic transformations, such as order of play, rotations, and reflections are not different games.

—*The World Book of Math Power*

Postal mail your answers to any or all of the Summer Brain Ticklers to **Tau Beta Pi, P.O. Box 2697, Knoxville, TN 37901-2697** or email to *BrainTicklers@tbp.org* as plain text only. The method of solution is not necessary, and the Double Bonus is not graded. Where possible, the judges consider exact answers to be preferable to approximations. The cutoff date for entries to the Summer column is the appearance of the Fall *Bent* in mid-September (the electronic version is distributed a few days earlier). We welcome any interesting problems that might be suitable for the column. HQ will forward your entries to the judges who are **H.G. McIlvried III,** *PA Γ '53;* **F.J. Tydeman,** *CA Δ '73;* **J.R. Stribling,** *CA Δ '92*, and the columnist for this issue,

J. C. Rasbold, *OH A '83*

General Revision to the Constitution and Bylaws Ratified

The 2017 Convention approved a general revision to the Constitution and Bylaws of Tau Beta Pi and sent it to the chapters for ratification. In accord with the Association's amending procedure, with 289 chapters (246 collegiate and 43 alumni) eligible to vote, 217 or more affirmative chapter votes are required to ratify an amendment, and 73 or more negative votes would defeat it.

Headquarters received 190 valid ballots by the voting deadline of April 1, 2018 (plus 6 invalid for lack of a chapter quorum). An additional 9 ballots (plus 2 invalid) were received after the deadline prior to the Executive Council meeting. The Council acted on April 28 and voted on behalf of those chapters submitting an invalid or not ballot. The proposed general revision was therefore ratified and is in effect.

Amendment	Outcome
1. Adopt the general revision of the Constitution and Bylaws. *(All Constitution Articles and Bylaws were amended)*	1. Unresolved by chapter vote; 188 affirmative, 11 negative. Ratified by Council vote for invalid and missing chapters.