Brain Ticklers

RESULTS FROM WINTER

Perfect

Name	State	Chapter	Year
*Couillard, J. Gregory	IL	A	'89
*Ebersold, Dakota	Son of member		
*Gee, Albert	CA	A	'79
Gee, Aaron J.	CA	Ψ	'16
Gee, Avery	Member's daughter		
*Gerken, Gary M.	CA	H	'11
*Griggs Jr., James L.	OH	A	'56
*Norris, Thomas G.	OK	A	'56
Norris Jr., Thomas G.	PA	Γ	'79
Prince, Lawrence R.	CT	B	'91
*Richards, John R.	NJ	B	'76

Other

Name	State	Chapter	Year
Aron, Gert	IA	B	'58
Bertrand, Richard M.	WI	B	'73
Burbey, Ingrid K.	WI	A	'82
Dechman, Don A.	TX	A	'57
DeSelms, Bradley C.	MO	A	'82
Ferruzza, David	NJ	Γ	'58
*Gulian, Franklin J.	DE	A	'83
Gulian, William F.	Member's son		
Handley, Vernon K.	GA	A	'86
*Johnson, Mark C.	IL	A	'00
Jones, Donlan F.	CA	Z	'52
Kay, Ewan S.	NY	I	'14
*Kimsey, David B.	AL	A	'71
Klaver, Naftali	Member's son		
Lalinsky, Mark A.	MI	Γ	'77
Quan, Richard	CA	X	'01
Rentz, Peter E.	IN	A	'55
*Schmidt, V. Hugo	WA	B	'51
Schweitzer, Robert W.	NY	Z	'52
Sigillito, Vincent G.	MD	B	'58
Skorina, Frank K.	NY	M	'83
Slegel, Timothy J.	PA	A	'80
*Spong, Robert N.	UT	A	'58
*Strong, Michael D.	PA	A	'84
Summerfield, Steven L.	MO	Γ	'85
Svetlik, J. Frank	MI	A	'67
*Voellinger, Edward J.	Non-member		

* Denotes correct bonus solution

WINTER REVIEW

The intrepid Brain Ticklers submitters took problem 1 (cryptic addition) to heart, using the WINTER to ensure this PUZZLE was done properly—100% of the submissions with an answer to this problem were correct. Interestingly, the hardest was problem 3 (determining the sides of a specific triangle). Here, only 60% of the submissions for this problem were correct, with most wrong answers describing otherwise valid solutions that did not meet the minimum perimeter requirement. The winter columnist thought the Bonus (finding triangular numbers with certain constraints) might prove challenging, but over 85% of the submissions for the Bonus were correct.

SPRING ANSWERS

Readers' entries for the Spring Ticklers will be acknowledged in the Fall *BENT*. Meanwhile, here are the answers:

1 It takes John nine transfers to get 2 liters of water in both the 4-liter and 5-liter pails. There are four different nine-transfer schemes, one for each possible initial pour source/target pail combination. One such transfer sequence is shown in the table. The values in the table represent the amounts of water in each vessel after each transfer.

10a	10b	5	4
10	10	0	0
5	10	5	0
5	10	1	4
9	10	1	0
9	10	0	1
4	10	5	1
4	10	2	4
8	10	2	0
8	6	2	4
10	6	2	2

2 ONE = 435; THREE = 17955; SIX = 820; and **TEN = 153.** Start by listing the 14th (105) to 44th (990) and 141st (10011) to 446th (99681) triangular numbers. (A spreadsheet is a good way to do this.) Note that the last two digits of ONE and TEN are the reverse of each other, so look for pairs of three-digit triangular numbers that have this property. There are eight such possible ONE-TEN pairs: 153-435; 253-435; 435-253; 435-153; 630-703; 630-903; 703-630; and 903-630. From the listing, we see that E must be 0, 3, or 5, and T must be 1, 2, 4, 6, 7, or 9. Next, go through the list of five-digit triangular numbers looking for values that end in 00, 33, or 55, don't have any other repeated digits, and have an initial digit of 1, 2, 4, 6, 7, or 9. The only such numbers are 15400, 17955, 40755, and 79800. We can eliminate 15400 and 40755 as THREE because no potential value of TEN starts with 10 or 45. We are left with two ONE-TEN-THREE possibilities: 435-153-17955 and 630-703-79800. SIX must consist of three digits that are not used in ONE, THREE, or TEN, so pick a pair of values for ONE and TEN that is compatible with one of the values for THREE and look for three-digit triangular numbers which don't have any digits in common. Of all the possibilities, only SIX = 820 works with ONE = 435, THREE = 17955, and TEN = 153.

3 The dimensions in cm of the paving blocks being manufactured are: **(48, 7, 1), (27, 8, 1), (20, 9, 1), (13, 12, 1), (18, 4, 2), (8, 6, 2), (12, 3, 3), and (7, 4, 3).** The given requirements of the blocks' properties leads to the relationship $4(L + W + T) = 2LWT/3$, or $6(L + W + T) = LWT$, where L, W, and T are the length, width, and thickness, respectively, of the blocks. Solving for L gives $L = 6(W + T)/(WT - 6)$. If it is assumed that $T = 1$, integer values are generated for W = 7, 8, 9, and 12, giving corresponding values for L = 48, 27, 20, and 13, respectively. Similar analysis for $T = 2$ gives integer values for $W = 4$, $L = 18$, and $W = 6$, $L = 8$; $T = 3$ gives integer values for $W = 3$, $L = 12$, and $W = 4$, $L = 7$. Because W and T are interchangeable in the equation for L, using higher values for T does not generate any new solutions, so the sizes of paving blocks are as shown above.

4 The values for A, B, C, and D are 1, 3, 4, and 8. Four different digits can form 24 different four-digit integers, and each of the four digits will appear as the final digit six times in the 24 permutations. We are told that of the 24 integers, 4 are primes and 8 are semi-primes (the product of two not necessarily different primes), so 12 of the integers must be odd; thus, exactly two of the digits must come from the set (1, 3, 7, 9). None of the digits can be 5, since this would not allow enough primes and semi-primes. Therefore, (A, B) = (1, 3), (1, 7), (1, 9), (3, 9), or (7, 9). We do not have to consider (3, 7) because a prime square is not possible for an integer ending in 3 or 7. Similarly, we are told that 12 integers are divisible by 2; therefore, (C, D) must be even digits and are one of: (2, 4), (2, 6), (2, 8), (4, 6), (4, 8), or (6, 8). This gives 5×6 = 30 combinations of (A, B) and (C, D) to systematically try (a spreadsheet is very helpful). A powerful first test is to look for a perfect square among the 12 odd integers for any of the (A, B)(C, D) combinations. If none of these integers is the square of a prime, this (A, B)(C, D) combination is not a solution. If one of the integers is the square of a prime, check the rest of the requirements to see if they are met (a factor table is helpful in checking primes and semi-primes). Of the 30 possible combinations, only 1, 3, 4, 8 meets all the stated requirements.

5 The most likely final total is **13.** Consider a die toss which produces a total of 14 or more, and let *T* equal the total prior to the final toss. Now, for any toss that gives a total of 14 or more, there is an equally likely toss which gives a total of 13 (a toss with a pip value of 13 – *T*). Thus, for every toss that gives a total of 14 or higher, there is a toss which will give a final total of 13. For example, if you have a 10, you could throw a 4 to reach 14, but you are just as likely to throw a 3 and reach 13. So the probability of getting a 13 is at least as high as any other total, but there is one other case to consider. Suppose you have a 7, then the only toss that will end the game is a 6 to reach 13, and there is no equally likely toss that produces a higher total than 13. This means that 13 appears more often than any other value and is, therefore, the most likely final total.

Numbers Matched	Formula	Ways	$Payout	$Payout * Ways
5W+PB	C(5,5)C(64,0)C(25,0)C(1,1)	1	Jackpot	Jackpot
5W	C(5,5)C(64,0)C(25,1)C(1,0)	25	1000000	25,000,000
4W+PB	C(5,4)C(64,1)C(25,0)C(1,1)	320	50000	16,000,000
4W	C(5,4)C(64,1)C(25,1)C(1,0)	8,000	100	800,000
3W+PB	C(5,3)C(64,2)C(25,0)C(1,1)	20,160	100	2,016,000
3W	C(5,3)C(64,2)C(25,1)C(1,0)	504,000	7	3,528,000
2W+PB	C(5,2)C(64,3)C(25,0)C(1,1)	416,640	7	2,916,480
1W+PB	C(5,1)C(64,4)C(25,0)C(1,1)	3,176,880	4	12,707,520
PB	C(5,0)C(64,5)C(25,0)C(1,1)	7,624,512	4	30,498,048
All draws	C(69,5)C(26,1)	292,201,338		

Bonus The jackpot would have to be **$648,918,318** for the game to be fair for a $3 ticket. The table above shows the calculations. The multiplication factor is 2 for the $1,000,000 payout and has an expected value of (24×2+13×3+3×4+2×5)/42 = 109/42 for the other cases. In the formulas, C(i, j) is the number of combinations of i objects taken j at a time. W refers to the white balls and PB is the power ball. The expected revenue for all possible draws is $3×292,201,338 = $876,604,014. For all possible draws, the expected winnings, excluding the jackpot, is 2×$25,000,000 + (109/42)×(sum of the other seven possible payouts) = $50,000,000 + (109/42)×$68,466,048 = $227,685,696. Therefore, for a fair game, the jackpot must be $876,604,014 - $227,685,696 = $648,918,318.

Double Bonus You were asked to arrange the 28 dominoes in a standard double-6 set in the perimeter of a square such that the sums of the pip values on each side of the square are the same. One solution with a sum of 44 on a side is:

0-0 | 0-1 | 1-1 | 1-3 | 3-5 | 5-6 | 6-6 | 6
6-0 | 0-2 | 2-1 | 1-4 | 4-4 | 4-6 | 6-2 | 2
2-2 | 2-3 | 3-4 | 4-2 | 2-5 | 5-5 | 5-0 | 0
0-3 | 3-3 | 3-6 | 6-1 | 1-5 | 5-4 | 4-0 | 0

Excluding rotations, we found 30,229,400 unique solutions. Of these, 12,039,392 solutions have a sum of 44 on a side, 6,150,616 have a sum of 45, and another 12,039,392 sum to 46. There are 1,989,807,482,880 possible square formations using all 28 dominoes, so on average only one in 66,000 formations is a solution to the given problem.

NEW SUMMER PROBLEMS

1 At lunch one day, math Prof. Lohwater mentioned to economics Prof. Baird that he had to go pick up a birthday present for his daughter, Tatiana. Baird knew that Lohwater had three children of different ages, but did not recall which was which and asked her age. Lohwater, being in a puzzling mood, wrote on a paper napkin a polynomial f(x) with integer coefficients and said, "Tatiana's age is the only positive integer which is a zero of this polynomial." Baird was not much impressed and not in the mood for algebra, but decided to give it the old college try. More or less at random, Baird picked 7, but was told f(7) = 77. Then he tried a larger integer n, which gave f(n) = 85. Lohwater exclaimed that his daughter was older than n. How old is Tatiana?

—*The Surprise Attack in Mathematical Problems* by L.A. Graham

2 Inside a hemispherical building is a concentric hemispherical atrium. Each floor of the building is an annulus which extends from the outside of the atrium to the inside of the outside wall. On each floor, a regular geometric figure has been painted. On the top floor, the outside wall circumscribes an equilateral triangle which circumscribes the atrium. Similarly, on the floor immediately below the top floor, a painted square inscribes the outer wall and circumscribes the atrium. One floor down, the figure is a regular pentagon, and the figure is a regular hexagon on the level below that. The top floor is 5 m higher above ground level than the floor below it, and the painted square is 60 m on a side. What is the area of the floor with the painted hexagon, what is its outside perimeter and its height above ground?

—**Cecil T. Ho,** *NY Beta '68*

3 Give me an integer that, when divided by the product of its digits yields a quotient of 3, and if you add 18 to the integer, its digits are reversed.

—*Little Man Tate*, 1991

4 A data entry service has assigned a different code number in the inclusive range of 1 through 10 to each of the days of the week: SUN, MON, TUE, WED, THU, FRI, and SAT. It turns out that the code numbers for a pair of days have a common factor greater than 1 if—and only if—the abbreviations for the days have at least one letter in common. In addition, if one substitutes code numbers for days, MON + TUE < WED + THU < FRI + SAT. In the order SUN through SAT, what are the code numbers of the days of the week?

—Susan Denham in *New Scientist*

5 Five Tau Bates, who understand not a word of Chinese, enter a restaurant in China, where none of the staff speak any English. Undaunted, they sit down and look at the menu, which has nine items listed, each identified by a Chinese symbol, but which, for the sake of simplicity, we will call A, B, C, D, E, F, G, H, and I. They each order one item from the menu by pointing to it. When the server brings the five dishes, he sets them in the middle of the table so that the diners can share. The same thing happens for the next two nights. However, by the time they return on the fourth day, they have deduced what culinary item each letter represents, and each is able to order his favorite meal with total confidence. What dishes did they order each night? Present your answer as three strings, each with five letters.

—*Hard-to-Solve Brainteasers* by Jaime and Lea Poniachik

Bonus The ancient Egyptians had a unique way of depicting rational numbers when the numerator was greater than 1; they expressed such fractions as the sum of a series of unit fractions, that is fractions with a numerator of 1. Furthermore, in a so-called Egyptian fraction, none of these unit fractions could be repeated. For example, 5/6 could be written as 1/2 + 1/3, and 7/12 as 1/2 + 1/12 or 1/3 + 1/4 or 1/4 + 1/6 + 1/8 + 1/24, but not as 1/4 + 1/4 + 1/12.

In ancient Egypt, five women went into a bakery to buy bread, but only three identical loaves were left. The baker said not to worry as she and her two apprentices would each take a loaf and divide it into pieces which were unit fractions of a whole loaf. Each loaf would be cut into the same number of pieces, but the pieces from each loaf would form a different Egyptian fraction. That is, no two loaves were cut into exactly the same set of pieces, although the Egyptian fractions for the three loaves may have had some sizes in common. The baker was more skilled with a knife than the apprentices, as she was capable of cutting off a slice as small as 1/100 of a loaf, but the smallest piece the apprentices were competent to slice off was 1/50 of a loaf. The pieces would be distributed to the five customers in such a way that each would get the same number of pieces, the volume of the pieces for each customer would total 3/5 of a loaf, and each buyer's pieces would form a different Egyptian fraction. What size pieces did each woman get?

We want the answer with the fewest pieces. Present your answer as five different series of unit fractions. For simplicity, the answers need only list the denominators of the unit fractions.

—*Ancient Puzzles-The Greatest Puzzles Ever Solved,* by Tim Dedopulos

Computer Bonus Many states have license plates with a format consisting of three letters followed by four digits. Assume each such license plate represents an integer in the lowest possible base greater than 10, with a = 10, b = 11, c = 12, . . ., z = 35. For example, the license plate **TBP1885** would represent an integer in base 30, the lowest base where T is a valid digit, and its base 10 equivalent would be $21{,}428{,}584{,}445_{10}$. Exactly four of these license plates, when expressed in their base 10 equivalent, can be expressed as x^y, where x and y are positive integers ($y > 3$). One of these plates is **BBG1802** which is in base 17; its base 10 equivalent is $282{,}475{,}249_{10} = 49^5 = 7^{10}$. In license plate format, what are the other three integers?

—Dakota Ebersold, son of member

Postal mail your answers to any or all of the Summer Brain Ticklers to Curt Gomulinski, Tau Beta Pi, P. O. Box 2697, Knoxville, TN 37901-2697 or email to *BrainTicklers@tbp.org*, as plain text only. The cutoff date for entries to the summer column is the appearance of the Fall *BENT* in mid-September (the electronic version is distributed a few days earlier). The method of solution is not necessary, and the Computer Bonus is not graded. We welcome any interesting problems that might be suitable for the column. Your entries will be forwarded to the judges who are **H.G. McIlvried III,** *PA Γ '53*; **F.J. Tydeman,** *CA Δ '73*; **J.R. Stribling,** *CA Δ '92*, and the columnist for this issue,

—J.C. Rasbold, *OH A '83*