Brain Ticklers

1. DONATE – MONEY – TO – THE + NEW = ENDOw + MENT
   translates to 129034-72495-3284+946=49126+7493. Moving the negative terms to one side and the positive terms to the other and then cancelling letters which occur on both sides in the same columns (for example, the W in NEW and ENDOw) and consolidating gives:

   E M D O T
   E N Y
   M O T H O
   D O 0 A 0 0

   Summing the individual columns produces the following five equations:

   T + Y + Ø = 10
   Ø + N + H + 1 = 20
   D + E + T + 2 + A = 10
   M + Ø + 1 = 10
   + E + M + 1 = Ø + 10

   First, we see that D = 1. Next, Ø = 9 – M, and E + M = Ø + 9, or E + M = 9 – M – 9, so E + 2M = 18, so possible values for (E, M, Ø) are (8, 5, 4), (4, 7, 2) and (2, 8, 1), but this last possibility must be rejected because D = 1, so Ø = 2 or 4. Now, N + H = 19 – Ø, and T + Y = 10 – Ø. If we assume Ø = 4, then T + Y = 10 – 4 = 6, so (T, Y) = (1, 5) or (2, 4) in some order, but D = 1 and Ø = 4, so Ø cannot equal 4 and must be 2.

   Then, we have: D = 1, Ø = 2, M = 9, Ø = 7, E = 18 – 2M – 4, T + Y = 10 – Ø = 8, so (T, Y) = (3, 5) or (5, 3); they cannot be (1, 7) or (2, 6) as 1 and 2 are already assigned; since N + H = 19 – Ø = 17, so (N, H) must be (8, 9) in some order, A – D + E + T + 10 = 1 + 4 + T + 10. Since A cannot be negative, T = 5, A = 0, and Y = 3. Finally, W must equal 6. This gives (A, D, E, H, M, N, Ø, T, W, Y) = (0, 1, 4, 8, 9, 7, 9, 8, 5, 2, 6, 3). Of the two possibilities, N = 9 gives the larger ENDOwMENT as 491267493 and produces the answer given above.

2. Ann and Bill’s ages are 19 and 36 (or vice versa). Let A = Ann’s age and B = Bill’s age. Then, (E1) 100A + B = x² and (E2) 100(A + 13) + B + 13 = y², or on expanding, 100A + B + 13 = 1313 = x². Subtracting (E1) from (E2) gives y² – x² = (y – x)(y + x) = 1313 = (13)(101). Therefore, y – x = 13 and y + x = 101.

   Solving gives x = 44 and y = 57, with 44² = 1936 and 57² = 3249. So, Alice is 19 (32 in 13 years) and Bill is 36 (49 in 13 years).

3. It takes at least 11 crossings to get the three couples safely across the river. The three right hand columns in the table list the individuals on the near bank, in the boat, and on the far bank for each crossing.

<table>
<thead>
<tr>
<th>Crossing</th>
<th>Near Bank</th>
<th>Boat</th>
<th>Far Bank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start</td>
<td>ABCXYZ</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>1</td>
<td>BCYZ</td>
<td>AX</td>
<td>---</td>
</tr>
<tr>
<td>2</td>
<td>BCYZ</td>
<td>A</td>
<td>X</td>
</tr>
<tr>
<td>3</td>
<td>ABC</td>
<td>YZ</td>
<td>X</td>
</tr>
<tr>
<td>4</td>
<td>ABC</td>
<td>–X</td>
<td>YZ</td>
</tr>
<tr>
<td>5</td>
<td>AX</td>
<td>BC</td>
<td>YZ</td>
</tr>
<tr>
<td>6</td>
<td>AX</td>
<td>–BY</td>
<td>CZ</td>
</tr>
<tr>
<td>7</td>
<td>XY</td>
<td>AB</td>
<td>CZ</td>
</tr>
<tr>
<td>8</td>
<td>XY</td>
<td>–Z</td>
<td>ABC</td>
</tr>
<tr>
<td>9</td>
<td>Z</td>
<td>XY</td>
<td>ABC</td>
</tr>
<tr>
<td>10</td>
<td>Z</td>
<td>–C</td>
<td>ABXY</td>
</tr>
<tr>
<td>11</td>
<td>---</td>
<td>CZ</td>
<td>ABXY</td>
</tr>
<tr>
<td>End</td>
<td>---</td>
<td>ABCX</td>
<td>YZ</td>
</tr>
</tbody>
</table>

   There is a 0.25 probability that your friend (call him Bob) took a blue marble. There are two possibilities: Bob took red and you took blue, or Bob and you both took blue. Initially, the bag has 3R and 2B, so there is a 0.6 probability that Bob chose red and a 0.4 probability that he chose blue. You might assume, therefore, that Bob’s probability of getting a B is 0.4 and that is the answer, but the fact that you drew a B decreases the probability that Bob got a B. If he chose red, that would leave 2R and 2B, and you would have a probability of 0.5 of picking B. Thus, the probability of Bob R, you B is 0.5(0.6) = 0.3. If Bob picked R, then 3R and 1B would be left, and your probability of drawing B would be only 0.25, so the probability of Bob R, you B is 0.25(0.4) = 0.1, and the relative probability is 0.1/(0.1+0.3) = 0.25.

WINTER REVIEW

Winter #4 about the maximum number of cards in a 5-Spot deck was as difficult as the Bonus and kept seven entries out of the “Perfect” section.

SPRING SOLUTIONS

Readers’ entries for the Spring Ticklers will be acknowledged in the Fall Bent. Meanwhile, here are the answers.
For a 100 story building in the shape of a square antiprism, the ratio, \( R \), of total floor space to the square base is 113.8. The general equation for an \( N \) story building is \( R = N + k(N^2 - 1)/(6N) \), where \( k = 2(\sqrt{2} - 1) \). The cross section of the OWTC at any level is an octagon with alternating longer and shorter sides; only at the exact middle is the cross section a regular octagon. Let \( L \) be the length of a side of the square base at each end of the antiprism. Define \( x \) as the length of each of four sides of an octagonal cross section that starts from \( L \) at the base and narrows to 0 at the top. Let \( y \) be the length of the alternate four sides of the octagon that vary from 0 at the base to \( L \) at the top. Let \( a \) and \( b \) be the lengths of the perpendiculars from the center of the octagon to sides of lengths \( x \) and \( y \) respectively. Now, \( x = (1 - f) L \), where \( f \) is the fraction of the building’s height at which the cross section is located, and \( y = fL \). Now, \( a \) goes from \( L/2 \) at the bottom to \( L\sqrt{2} \) at the top, so \( a = L/2 + (L\sqrt{2} - L/2) \), which is \( (L/2)[1 + (\sqrt{2} - 1)f] \), and \( b = L(2) \) \[ 1 + (\sqrt{2} - 1)(1 - f) \]. The area of \( A_1 \) is \( aL/2 = (L/2)[1 + (\sqrt{2} - 1)f] \). \( A_2 \) is \( bL/2 = L(2)[1 + (\sqrt{2} - 1)(1 - f)] \), the area of \( A_3 \) is \( bL/2 = (L/2)[1 + (\sqrt{2} - 1)(1 - f)] \), and the area of \( A_4 \) is \( bL/2 = (L/2)[1 + (\sqrt{2} - 1)(1 - f)] \). The area of \( A_5 \) is \( 4(A_1 + A_2) = L^2[1 + 2(\sqrt{2} - 1)f - 2(\sqrt{2} - 1)y^2] = L^2[1 + kf - ky]\). Based on this equation and varying \( f \) from 0 to 0.99 in increments of 0.01, a spreadsheet can quickly calculate the ratio of the floor space of a 100 story building to the square base as 113.8. To get a general equation, we must sum the floor space from 0 to \( N - 1 \), where \( N \) is the number of stories. To start, replace \( f \) with \( n/N \) and, since we want a ratio to the base, we can let \( L = 1 \). Therefore, we have \( S = \sum_{n=1}^{N} \left[ 1 + k(n/N - n^2/N^2) \right] \), which can be evaluated using the formulas for the summation of integers \( [n(n+1)/2] \) and squares \( [n(n+1)(2n+1)/6] \) to give \( R = N + k(N-1)N/(2N) - k(N-1) \). The following array, found by inserting the semiprimes into the squares \[ \begin{array}{|c|c|c|c|c|} \hline n & 33 & 38 & 85 & 94 \\ \hline 09 & 35 & 85 & 38 & 94 \\ 49 & 74 & 10 & 34 & \\ 15 & 25 & 58 & 69 & \\ 94 & 33 & 14 & 26 & \\ \hline \end{array} \]

A novel approach is to find a 4×4 array of 16 semiprimes with the property that the differences between entries in the 1st and 2nd columns, between the 1st and 3rd columns, and between the 2nd and 4th columns are constants (call them \( \Delta_1 \), \( \Delta_2 \), and \( \Delta_3 \), respectively), with a similar situation for the rows (call the row constants \( \delta_1 \), \( \delta_2 \), and \( \delta_3 \), respectively). The following array, found by inspection of a listing of all semiprimes less than 100, has this property, with \( \Delta_1 = 5 \), \( \Delta_2 = 16 \), \( \Delta_3 = 25 \), \( \delta_1 = 1 \), \( \delta_2 = 24 \), and \( \delta_3 = 60 \).

We can generate variations of the magic square by permuting the letters and numbers of the Greco-Roman square. Since there are 4 letters and 4 numbers and 2 squares, there are \( 2(4!)^2 = 1152 \) possible magic squares, which, upon eliminating rotations and reflections, gives 1152 as 144 fundamentally different magic squares with a magic sum of 167.

**Computer Bonus** The smallest social number of degree 28 is 14,316. This Tickler requires a program to get the prime factorization of a number \( N = p_1p_2p_3...p_n \). Then, the sum of the divisors is given by the following product, which has one factor for each different prime:

\[
\prod_{k=1}^{n} [(p_k^{m_k}-1)/(p_k-1)] - N.
\]

The exponent \( m \) is that associated with prime \( p_i \) in the prime factorization.

Try successive numbers until you find one that returns to its original sum on the 28th cycle; with 14,316, the next sum is 19,116, and the sums gradually increase to 629,072, then slowly decrease back to 14,316.

**NEW SUMMER PROBLEMS**

1. There was a young and adventurous man who found among his grandfather’s papers a map on a piece of parchment that revealed the location of a hidden treasure. The instructions read:
"Sail to ___ latitude and ___ longitude where thou wilt find a deserted island. [When this Tickler first appeared in a 1955 *Bent*, the island’s location was given as 16°N 16°W, but that is obviously incorrect, and the true coordinates, which were known to the young man at the time he found the map, have since been lost.] There lieth a large meadow, not pent, on the north shore of the island where standeth a lonely oak and a lonely pine. There thou wilt see also an old gallows on which we once were wont to hang traitors. Start thou from the gallows and walk to the oak counting thy steps. At the oak thou must turn right by a right angle and take the same number of steps. Put here a spike in the ground. Now must thou return to the gallows and walk to the pine counting thy steps. At the pine thou must turn left by a right angle and see that thou takest the same number of steps, and put another spike in the ground. Dig halfway between the spikes; the buried treasure is there."

The instructions were quite clear and explicit, so our young man chartered a ship and set sail. He found the island, the field, the oak and the pine, but to his great sorrow the gallows were gone. Too long a time had passed since the document had been written; rain and sun and wind had passed since the document had been written; rain and sun and wind had disintegrated the wood and returned it to the soil, leaving no trace even of the place where it once had stood. The young man, not having been a Tau Bate, gave up in despair. Perhaps YOU can find the gallows’ former location for him.

—One, Two, Three...Infinity
by George Gamow

Consider five points arranged as the vertices of a regular pentagon and labeled A through E. Identical resistors are used to connect each of the following pairs of points: A-B, A-C, A-D, B-C, B-D, B-E, C-D, C-E, and D-E. Note that the pair A-E is not connected. The resistance between points A and E is 1 ohm. When three of the nine resistors are removed, the resistance between points A and E remains 1 ohm.

Which resistors are removed, and what is the value of one of the resistors?

—Hubert W. Hagadorn, PA E ’59

A new theater, with exactly 200 seats, has just opened. Every night 200 people, each with an assigned seat, line up. However, the first person in line is an acoustic engineer, who is interested in assessing the sound quality throughout the theater. The engineer picks a seat at random, which may or may not be his assigned seat. As the rest of the people enter in order, if their assigned seat is vacant, they sit in it. If not, they pick any unoccupied seat. What is the exact probability that the last person in line sits in their assigned seat?

—The Everything Brain Strain
Book by Jake Olefsky

The DryFroot Company makes one pound (16 oz) boxes of Froot-Mix. Each box is made to the same recipe and has a positive integral number of ounces of apples, bananas, cranberries, and dates. The company also makes one pound boxes of MixedFroot, and although the recipe is different, each box still contains an integral number of ounces of the same four fruits. Finally, DryFroot produces giant boxes of TootyFrooties by mixing four pounds of Froot-Mix with three pounds of MixedFroot. In each of the three products, the weights of the four fruits are different. The respective boxes list the ingredients in descending order by weight as follows: Froot-Mix: dates, cranberries, bananas, apples; MixedFroot: apples, bananas, cranberries, dates; TootyFrooties: bananas, cranberries, dates, apples. How many ounces of each fruit are there in a box of each product?

—An Enigma by Keith Austin in
New Scientist

Solve the following cryptic equation:

\[
TBP = \sqrt{\text{PUZZLE} + 1}
\]

All the usual rules apply. There are no leading zeros. Each letter represents a different digit, and the same letter always represents the same digit.

—Adapted from Allan Gottlieb’s
Puzzle Corner in Technology Review

**Bonus** A dozen POWs share a cell. What is important to them is that they escape as soon as possible.

They have found a way to remove the bars on the window of their cell so they can climb through and, once on the ground, plan to head due north, climb over the east-west fence near the window, and disappear. But they can only hope to escape unseen if they do it in the dark, one man at a time, and with each man having from the time he starts, a clear two minutes with all four guards at least 200 meters away from his turn directly below their window.

The guards each walk an east-west route, two inside the fence and two outside. Robinson starts 300 meters west of the prisoners’ window, marches to a point exactly outside their window, and turns around and marches back. Smith starts outside their window, marches 400 meters to the east, turns around, and marches back.

Outside the fence, Phillips starts 500 meters west of their window, marches 550 meters to his turn around point, 50 meters east of their window. Quinn starts at Phillips’ turn around point, marches 500 meters east and turns around.

The guards march back and forth on their beats at a uniform speed of 100 meters per minute. (Assume no time is spent in turning.)

Because it is winter, the POWs reckon that it is dark enough for their purposes at 6 p.m., until 6 a.m. The guards of one shift begin their routes at the above indicated locations at precisely 6 p.m.

How many prisoners can successfully escape? At what time(s) should they start their attempts? Express your answer in minutes.

—Brain Puzzler’s Delight
by E.R. Emmet

**Computer Bonus** A k-digit automorphic, or circular, number is a number whose square ends in the same k digits as the number itself.
That is, N is a k-digit automorphic number if \( N^2 \equiv N \pmod{10^k} \). For example, 76 is a 2-digit automorphic number because 76^2 = 5776. K-digit automorphic numbers are known to come in pairs. What is the leftmost (that is, most significant) digit of each of the two 1,001-digit, base-10, automorphic numbers?

—J.C. Rasbold, OH A ’83

Send your answers to any or all of the Summer Brain Ticklers to Curt Gomulinski, Tau Beta Pi, P. O. Box 2697, Knoxville, TN 37901-2697 or email to BrainTicklers@tbp.org.

plain text only. The cutoff date for entries to the Summer column is the appearance of the Fall Bent in early October. It is not necessary to include the method of solution. The Computer Bonus is not graded. Entries come in pairs. What is the leftmost digit of each of the two 1,001-digit, base-10, automorphic numbers?

Three Tau Bates have been named as Gates Cambridge Scholars among the 94 students chosen from around the world chosen for 2015. The scholarships are full-cost awards to pursue a full-time postgraduate degree at the University of Cambridge.

Allison M. Kindig, IA B ’15, is an industrial engineering senior at the University of Iowa. She will pursue a M.Phil. in engineering for sustainable development at Cambridge. She is a 2014 Scholar, 2015 Fellow, and vice president of Iowa Beta. Kindig plans to work on helping communities achieve energy and food security.

Shruti Sharma, MA B ’15, who is majoring in materials science and engineering at MIT, will pursue a Ph.D. in engineering at Cambridge. She conducts research on the use of three dimensional printing for prosthetic prototyping and production. Sharma is the elected president of the MIT student body and founder of the Girls Leadership and Mentorship (GLAM) program.

Chiedozie Ibekwe, MS B ’11, plans to pursue a M.Phil. in public policy and “will utilize my manufacturing and supply chain management expertise to advise African policymakers on crafting and executing effective industrial policies to boost manufacturing and diversify African economies.” Ibekwe has been employed at GE, currently as a lead buyer at GE Oil & Gas, since graduating with a degree in electrical engineering.

GATES CAMBRIDGE SCHOLARS

TELL US THE TALE...WIN A T-SHIRT

Send us your clever caption(s) for this photo from The Bent archives, and if it is judged one of the best, you will win a TBII t-shirt.

The picture above, published in the Spring 1974 issue, was included in an article about Northrop University (at the time, Northrop Institute of Technology) where the California Pi Chapter was located. As part of an undergraduate senior research project investigating the effects of industrial noise on brain wave activity, the student above records the brain waves of a fellow student.

Email entries to pat@tbp.org, or mail them to HQ by August 10, 2015.

The photo for the Spring Contest, to the right, showed guests staying to dance following the Michigan Regional Banquet held on April 5, 1974. It was attended by members, faculty, and guests of Michigan Alpha, Michigan Gamma, Michigan Zeta, and the Flint Alumnus Chapter.

From a field of 21 entries, this caption, submitted by David W. Kortebein, IL A ’85, was #1 with the judges: “Yes dear, I do think that my new sport coat will still be in style 40 years from now.” Second place goes to Michael B. Dubey, NY E ’77, for his caption: “Really? I didn’t know that engineers could dance!” Congratulations to both winners who will receive a TBII t-shirt.

An entry submitted by Richard “Lance” Null, AZ B ’90, is deserving of honorable mention: “Maintain 5 psi pressure on female. Right foot, left foot. Rotate right 90 degrees…”

Thanks to all of the participants for your witty commentaries. Keep those entries coming!!