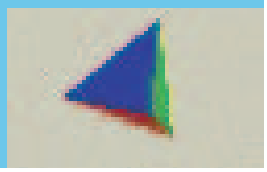

Brain Ticklers

RESULTS FROM WINTER

Perfect

*Bohdan, Timothy E.	IN	Γ	'85
*Gerken, Gary M.	CA	H	'11
*Schmidt, V. Hugo	WA	B	'51
Slegel, Timothy J.	PA	A	'80
* Strong, Michael D.	PA	A	'84
* Voellinger, Edward J.	Non-member		

Other

Alexander, Jay A.	IL	Γ	'86
Aron, Gert	IA	B	'58
Bertrand, Richard M.	WI	B	'73
*Couillard, J. Gregory	IL	A	'89
Egenriether, Brian J.	SC	Γ	'10
* Griggs Jr., James L.	OH	A	'56
Handley, Vernon K.	GA	A	'86
Jones, Donlan F.	CA	Z	'52
Jones, John F.	WI	A	'59
Lalinsky, Mark A.	MI	Γ	'77
Marks, Lawrence B.	NY	I	'81
Marks, Noah H.	PA	K	'11
Marks, Benjamin	Son of member		
* Prince, Lawrence R.	CT	B	'91
Quintana, Juan S.	OH	Θ	'62
Rentz, Peter E.	IN	A	'55
Richards, John R.	NJ	B	'76
* Spong, Robert N.	UT	A	'58
* Stribling, Jeffrey R.	CA	A	'92
Summerfield, Steven L.	MO	Γ	'85
Sutor, David C.	Son of member		
* Thaller, David B.	MA	B	'93
Vinoski, Stephen B.	TN	Δ	'85

* Denotes correct bonus solution

WINTER REVIEW

Problems No. 2 (soap bubbles) and No. 3 (three coins) were the most missed regular problems. For problem No. 4, we gave credit for the answer: The number of odd divisors including N but excluding 1.

Our solution to problem No. 3 of (1, 5, 22) is optimum if change is made from the largest coin first, then the middle denomination, and then the smallest (makes life easiest for the store clerk). However, as several readers pointed out, if one allows for the possibility of mixed combinations of the two largest coins to reduce the number of pennies needed, one gets (1, 12, 19) which has a smaller average. We accepted either answer.

SPRING SOLUTIONS

Readers' entries for the Spring problems will be acknowledged in the Fall BENT. Meanwhile, here are the answers:

1 Ten lights are lit. A light changes status for each divisor of its number. If the number of divisors is odd, the light will be lit, if even, it will be off. The equation for the number of divisors of $N = p_1^a p_2^b \ldots p_n^m$ is $(1+a)(1+b)\ldots(1+m)$. This will be odd only if $a, b, \ldots$, m are even, that is, if N is a square. Therefore, only lights 1, 4, 9, 16, 25, 36, 49, 64, 81 and 100 will be lit.

2 The correct order is **DAEFCB.** There are C(6, 2) = 15 different two-letter combinations, and thus 15 questions, where $C(n, m)$ is the combinations of n objects taken m at a time. Consider the questions in the order: A<B, A<C, A<D, A<E, A<F, B<C, B<D, B<E, B<F, C<D, C<E, C<F, D<E, D<F, E<F, where < means "comes before," reading from left to right. Then, the seven students gave the following answers:

Name	Guess	Answers
Greg	BCDAEF	NNNYY-YYYY-YYY-YY-Y
Hal	DAEFBC	YYNYY-YNNN-NNN-YY-Y
Ivan	ABEFDC	YYYYY-YYYY-NNN-NN-Y
Jill	BCFDEA	NNNNN-YYYY-YYY-YN-N
Kate	AEBDFC	YYYYY-YYNY-NNN-NY-Y
Lila	CFEABD	YNYNN-NYNN-YYY-NN-N
Mel	DCAEFB	YNNYY-NNNN-NYY-YY-Y

Since each student got a different non-zero even number correct, their scores must have been 2, 4, 6, 8, 10, 12, and 14. Examining the table carefully, we see that Hal and Jill agree on only three questions, so one of them must have 2 correct and the other 14 correct, which means that one of their guesses is within one transposition of being correct. Assume it is Hal's, and try switching B and C to get DAEFCB which gives YYNYY-NNNN-NNN-YY-Y. Comparing this to the results in the table, we get for the number of matches: G-6, H-14, I-8, J-2, K-10, L-4, M-12. If you try other combinations, you will find that none give the specified breakdown of correct answers.

3 The probability that a twelve-wafer roll will have at least one wafer of each of the eight flavors is **0.09331**. The number of different 12-wafer rolls is 8^{12} = 68,719,476,736; in general, the number of rolls with n flavors missing is $C(8, 8\text{-}n)(8\text{-}n)^{12}$, so we have:

No. missing flavors	Possibilities
1	110,730,297,608
2	60,949,905,408
3	13,671,875,000
4	1,174,405,120
5	29,760,696
6	114,688
7	8

The probability that at least 1 flavor is missing is:
$P = (1/8^{12})\sum_{n=1}^{7}(-1)^{n+1}C(8, 8\text{-}n)(8\text{-}n)^{12}$
=62,307,508,096 / 68,719,476,736 = 0.90669, and the probability that all 8 flavors are present is $1\text{-}P$ = 0.09331.

4 There are **35,890** different paths from A to S. Consider cell S, and let T_i be the total number of paths from A to i. Now, there are two ways to reach S: through Q and through R, so the total number of paths from A to S includes $T_Q + T_R$, but this is not all the paths from A to S, because it does not include any paths with an R-Q step. However, a little thought will show that each path from A to P leads to a path to S by appending -R-Q-S. Thus, the total number of paths from A to S is $T_S = T_R + T_Q + T_P$, or in general, $T_{i+1} = T_i + T_{i-1} + T_{i-2}$. Starting with $T_A = 1$, $T_B = 1$, and $T_C = 2$, we get $T_D = 4$, $T_E = 7$, $T_F = 13$, …, $T_S = 35{,}890$. Alternatively, you can solve for systems of 1, 2, 3, 4, 5, etc. cells by inspection, and the above pattern will quickly reveal itself.

5 The three digits are **1, 7,** and **8.** There are C(9, 3) = 84 possible choices of three different non-zero digits, not too many to list. Any three-digit integer divisible by the product of two small primes or a small square is not a semiprime. Using certain well-known relationships, we can quickly eliminate by inspection most possibilities. If the last two digits of a number are divisible by 4, the number is also; thus, any choice that contains two digits that form a number divisible by 4 can be eliminated [45 cases]. If the sum of the

digits is divisible by 9, so is the number [5 cases]. Any choice that includes 5 and 2 or 7 has a permutation divisible by 25 [6 cases]. Any choice that has a digit sum divisible by 3 and an even digit will have a permutation divisible by 6 [6 cases], and a digit sum divisible by 3 plus a 5 has a permutation divisible by 15 [1 case]. Finally, using a factor table, one can quickly eliminate 19 more cases because one permutation is prime. This leaves only two possibilities—(1, 5, 8) and (1, 7, 8). The first of these is eliminated because 518 =2×7×37, so the answer is (1, 7, 8). The six semiprimes are: 178 = 2×89; 187 = 11×17; 718 = 2×359; 781 = 11×71; 817 = 19×93; and 871 = 13×67.

Bonus Al's age is **49**, and he guessed Beth's age as 50; Carl's age is **47**, and he guessed Beth's age as 48; Dawn's age is **78**; and Beth's age is **68**. The logic for this solution is as follows. A asked B, "Is your age a multiple of 17?" B must have said no. A then asked, "Is your age a multiple of 3?" Again, B must have said no. A then asked, "Is your age a prime number?" (It would make no sense for A to ask this question if B had answered yes to either or both questions 1 and 2.) B's answer could be either yes or no. A then asked, "Are you older than I am?" Again the answer could be yes or no. Finally, A asked, "Have you had your 51st birthday?" At this point, A thinks he can deduce B's age. What combination of answers to the five questions would make this possible? Consider NNNYN; then if A were 49, and B was older than A but less than 51, that would make B 50 which would be A's guess, but A is wrong. C would use the above logic to deduce A's age as 49, which is stated to be correct. C realizes that B does not always tell the truth and postulates that she alternately tells the truth and lies. Thus, her answers could be (1) FTFTF or (2) TFTFT. Consider case (1). Then, instead of the answers to the five questions being NNNYN, they should have been YNYYY, but Q1 and Q3 can't both be true, so case (2) must hold. Then, the answers should have been NYNNN. This means that B's age is not a multiple of 17, is a multiple of 3, and is not prime, and that B is younger than A and younger than 51 but older than C whose age is a prime number. The only prime number less than 51 that would let C deduce B's age is 47. If C's age is 47, then he would guess B's age as 48, but this is also wrong. Finally, we come to D. From C we deduce that two of B's answers are false, but which two? Not 2 and 4 as C assumes. Since D's age is a multiple of 13, it must be 26, 39, 52, 65, 78, or 91. Assume that the false answers are 1 and 5. Then, the correct answers are YNNYY. So B's age is a multiple of 17, not a multiple of 3, older than 51, and younger than D. B's age is then 68 or 85, and D's age is 78 or 91 or 104. But if D's age is 91 or 104, he wouldn't know if B's age was 68 or 85. Since he knows B's age, D must be 78 and B is 68.

Computer Bonus You can stay afloat indefinitely by the loop: 101→1701/3→567→7567/7→1081→10817/29→373→3737/37→101. This is only one of several possible loops. You originally have four choices: 7101, 1701, 1071, and 1017. 1701 gives the above loop: 7107/3→2367, but 2367 gives only primes for all four possibilities, so you sink. Using condensed notation(where the boldface **7**'s are the inserts), 1017 leads to: 101**7**/3→**7**339/41→179**7**/3→599**7**/3→**17**999/41→43**7**9/29→**17**51/17→**17**03/13→131**7**/3→43**7**9, which loops back to 439; 1071 also forms a loop.

NEW SUMMER PROBLEMS

1 What conclusions (about shingles, happiness, Klingon, Ph.D. candidate, mom, and mixed dominance) can be drawn from these eight premises?

1. Everyone who works in the Reliable Data Dump has shingles.
2. All hippies are unhappy.
3. No one whose mother is, or has been a shaman suffers simultaneously from shingles and mixed dominance.
4. Christopf L. Biggleswade works in the Reliable Data Dump.
5. All hippies are fluent in Klingon.
6. Christopf L. Biggleswade is a hippy.
7. Everyone fluent in Klingon is a candidate for the Ph.D. degree, or else his mother is, or has been, a shaman.
8. No one who is unhappy can be a candidate for the Ph.D.

—Professor H. Webb

2 You have an eight-pint jug full of wine and want to give exactly half to your friend. However, all you have is a three-pint jug and a five-pint jug, both empty. How do you arrive at four pints in each of the five-pint and eight-pint jugs by just pouring wine from one jug to another (in the minimum number of pours)? Show your answer as the state of each jug after each pour.

—Master Book of Mathematical Recreations by Fred Schuh

3 A, B, C and D all played each other once at soccer. Some of the figures in the table of results (in which they are not necessarily arranged in the order of points scored) are given below.

	Played	Won	Lost	Drawn	Goals For	Goals Against
A	3					5
B	3	3			4	
C	3					
D	3			1	0	3

With the additional information that the match between A and C was a draw (3-3) and that A scored more than 5 goals altogether, please fill in the table, and provide the scores of each game.

—Brain Puzzler's Delight by E.R. Emmet

4 Find 10 primes in arithmetic progression, that is find an expression of the form $a + nb$, which generates primes for values of n from 0 through 9 inclusive. We want the expression where $a + 9b$ is the smallest.

—Penguin Dictionary of Curious and Interesting Numbers by David Wells

5 Solve the following cryptic addition problem with the largest TIGER.

```
  BEAVER
+ TIGER
--------
  RABBIT
```

Usual rules: Base ten, no leading zeros, different letters are different digits, same letter is same digit throughout.

—150 Puzzles in Crypt-Arithmetic by Maxey Brooke

BONUS There are three pegs located on the circumference of a circle: peg A is at 12 o'clock; peg B is at 4 o'clock; and peg C is at 8 o'clock. Initially, there are N disks (all different sizes) on peg A, stacked in size from largest at the

bottom to smallest at the top. Find the minimum number of moves to transfer all of the disks to peg *B*, one at a time, never having a larger disk on top of a smaller one. Only clockwise moves (to adjacent peg) are permitted. Also, what is the minimum number of moves to transfer all the disks to peg *C* (from peg *A*) with the same rules? We want direct (not recursive) formulas. As a check, fill in the following:

N 0 1 2 3 4 5 6 7 8 9
B 0 1 5
C 0 2

where, *N* is the number of disks on the *A* peg, *B* is the number of moves to move those *N* disks to the *B* peg, and *C* is the number of moves to move those *N* disks to the *C* peg (from the *A* peg).

—Allan Gottlieb's Puzzle Corner in *Technology Review*

COMPUTER BONUS A bingo card consists of a 5 x 5 grid with numbers in each cell, except the center, which is marked FREE. The five columns are labeled B, I, N, G, and O. Column B consists of five different numbers in the range 1 through 15, column I of five different numbers from 16 through 30, N of four numbers between 31 and 45, G of five numbers between 46 and 60, and O of five numbers between 61 and 75. The caller has 75 balls, numbered 1 to 75, and randomly calls numbers (without replacement) until someone calls "Bingo." Bingo is scored by getting five numbers (or four numbers and the FREE cell) in a row, vertically, horizontally, or two main diagonals. What is the expected number of calls to get Bingo on an arbitrarily selected Bingo card?

—**H.G. McIlvried III** *PA Γ '53*

Postal mail your answers to any or all of the Brain Ticklers to **Curt Gomulinski, Tau Beta Pi, P.O. Box 2697, Knoxville, TN 37901-2697,** or email to *BrainTicklers@tbp.org* as plain text (no HTML, no attachments). The cutoff date for entries to the Summer column is the appearance of the Fall BENT during early October. The method of solution is not necessary, unless you think it will be of interest to the judges. We also welcome any interesting new problems that may be suitable for use in the column. The Computer Bonus is not graded. Curt will forward your entries to the judges, who are: **H.G. McIlvried III** *PA Γ '53;* **D.A. Dechman,** *TX A '57*; **J.C. Rasbold**, *OH A '83;* and the columnist for this issue,

F.J. Tydeman, *CA Δ '73*

ROLL OUT THE BARREL ... WIN A T-SHIRT

Send us your witty caption for this photo from our archives, and if it is judged one of the best, you will win a TBP t-shirt. It shows Robert C. "Red" Matthews, second left, who went on to be Secretary-Treasurer, and other Illinois Alpha Class of 1902 members taking part in pre-initiation activities (my, how times have changed!) on the University of Illinois at Urbana-Champaign campus in 1901. Email entries to *tbp@tbp.org* or mail them to Headquarters by August 21.