Brain Ticklers

RESULTS FROM WINTER 2012

Perfect

Eisenhauer, William D.	OR	B	'94
Jones, John F.	WI	A	'59
Kimsey, David B.	AL	A	'71
*Mangis, J. Kevin	VA	A	'86
Prince, Lawrence R.	CT	B	'91
*Rasbold, J. Charles	OH	A	'83
Schmidt, V. Hugo	WA	B	'51
Silver, Robert E.	NY	P	'80
Slegel, Timothy J.	PA	A	'80
Smith, Ronald E.	PA	A	'86
Stein, Gary M.	FL	Δ	'04
Vegeais, James A.	IL	A	'86

Other

Alexander, Jay A.	IL	Γ	'86
Aron, Gert	IA	B	'58
Bird, David W.	VT	A	'74
Bohdan, Timothy E.	IN	Γ	'85
Brule, John D.	MI	B	'49
Couillard, J. Gregory	IL	A	'89
deVitry, David M.	PA	H	'97
Giannini, Mark C.	CA	AΓ	'13
Handley, Vernon K.	GA	A	'86
Havas, Donald W.	NY	N	'67
Jones, Donlan F.	CA	Z	'52
Kern, Peter L.	NY	Λ	'62
Lalinsky, Mark A.	MI	Γ	'77
McDonough, Thomas J.	MO	B	'11
Melancon, Thomas B.	WA	B	'79
Rentz, Peter E.	IN	A	'55
Shaffer, Daniel A.	OH	N	'11
Shevenock, Stephanie A.	MI	H	'13
Sigillito, Vincent G.	MD	B	'58
Spong, Robert N.	UT	A	'58
Stetson II, Scott B.	CA	T	'12
Stribling, Jeffrey R.	CA	A	'92
Strong, Michael D.	PA	A	'84
Summerfield, Steven L.	MO	Γ	'85
Svetlik, J. Frank	MI	A	'67
Voellinger, Edward J.	Non-member		
Wiesner, Jeffrey J.	WI	A	'77
Wiesner, David J.	Son of member		

* Denotes correct bonus solution

WINTER REVIEW

Winter No. 3, about determining the six-digit lock combination, was by far the most popular Tickler with many entries, most giving the correct unique answer. The Bonus problem, about finding all the unique circular paths starting and ending in the center of a 5x5 checkerboard, was extra difficult with only two correct answers.

Fall Recap. Because of a computer glitch, we missed acknowledging two entries for the Fall 2011 column: **Eamonn T. Harter,** *ID Γ '06*, (perfect) and **Lawrence R. Prince,** *CT B '91*.

Also, **Gert Aron,** *IA B '58*, has submitted a clever solution to Fall Tickler No. 1 (about the cost of turkeys). Since there are 72 = 8(9) turkeys, the cost ($x67.9y) must be divisible by both 8 and 9. Divisibility by 8 means the last 3 digits are divisible by 8. For 79y to be divisible by 8, y must equal 2. Divisibility by 9 means the sum of the digits is divisible by 9; for x+6+7+9+2 = x+24 to be divisible by 9, x must equal 3, so the cost is $367.92.

SPRING SOLUTIONS

Readers' entries for the Spring problems will be acknowledged in the Fall BENT. Meanwhile, here are the answers:

1 O/NE + T/WO + S/IX = NI/NE decodes as 6/24 + 7/56 + 9/18 = 21/24. The approach is to try various values for NE; picking NE fixes I since NINE is divisible by 9 and also fixes T since TEN is divisible by 7. As an example, try NE = 12; then I = 5, T = 7, and NI/NE = 15/12. A little trial will show that the left hand side can barely exceed 1, so NE = 12 will not work. Try NE = 24; then I = 1, T = 7, and NI/NE = 21/24. Since O and X must be even, O must be 6 or 8, and X must be 0, 6, or 8. A little trial will quickly arrive at the above solution.

2 N integers can be arranged into a strictly increasing sequence followed by a strictly decreasing sequence in $2^{N\text{-}2} - 1$ ways, when reversals are not considered different arrangements and there are at least two members in each sequence. Since N is a member of both sequences, the remaining $N - 1$ integers must be divided into two groups with X integers in one group and $N - X - 1$ in the other. This can be done in T = [C(N-1, 1) + C(N-1, 2) + C(N-1, 3) + … + C(N-1, N-2)]/2 ways, where C(i, j) is the number of combinations of i objects taken j at a time. The factor of 2 arises because half the permutations are reversals of the other half. Once the split is made, there is only one permutation that leads to an ascending followed by a descending sequence. Now, $(1 + 1)^Z$ = C(Z, 0) + C(Z, 1) + C(Z, 2) + … + C(Z, Z) = 2^Z. Since each group must have at least one number, T does not include C(Z, 0) and C(Z, Z). Since Z=N-1, $T = (2^{N\text{-}1} - 2)/2 = 2^{N\text{-}2} - 1$.

3 The solution with the smallest positive value of $(x + y + z)$ is x = 21, y = -168, z = 154, which sum to 7. The secret to solving this equation is realizing that 987,654,321 – 8(123,456,789) = 9. Let 987,654,321= A and 123,456,789 = B. Then, $nA - 8nB = 9n$, where $x = n$ and $y = -8n$. Let $x + y + z = m$. Then, $9n + z = m^3$ and $n - 8n + z = z - 7n = m$. Eliminating z gives $16n = m^3 - m = m(m - 1)(m + 1)$, which is solvable in integers if 16 divides $m(m - 1)(m + 1)$. There are an infinite number of solutions, the one with the smallest positive m being x = 21, y = -168, z = 154, and m = 7.

4 The length of the field is 1248 meters, and the strip is 24 meters wide. Let L = length of a field and S = width of the unplowed strip. Then, the area of a field $A_F = 100L$ and the area of the strip $A_S = 2(L + 100)S - 4S^2 = A_F/2 = 50L$, or $2S^2 - (L + 100)S + 25L = 0$. Solving for L gets $L = 2S(50\text{-}S)/(25\text{-}S)$ which has four integer solutions.

S	L
15	105
20	240
23	621
24	1248

The only L whose digits form an increasing sequence is 1248.

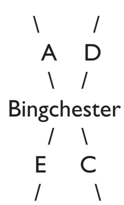
5 The gambler's probability of throwing a 7 with one standard die and one loaded die is 1/6, the same as if he had two standard dice. Let p(x) be the probability of throwing an x with the loaded die and P(x) be the probability of throwing an x with the standard die. Then, the probability of throwing a 7 is p(1)P(6) + p(2)P(5) + p(3)P(4) + p(4)P(3) + p(5)P(2) + p(6) P(1), but P(i) = 1/6 for i = 1 to 6 and $\sum$ p(i) = 1, so the probability of throwing a 7 is 1/6. Thus, no matter how his die is loaded, the gambler's probability of throwing a 7 is still 1/6.

Bonus The order of the integers 1 through 32, arranged in a circle so that the sum of each adjacent pair is a perfect square, is 1-8-28-21-4-32-17-19-30-6-3-13-12-24-25-11-5-31-18-7-29-20-16-9-27-22-14-2-23-26-10-15-1 (or its reverse). This problem is much easier to solve once you realize that certain combinations must occur; this considerably cuts down the size of a decision tree. Because each integer must contribute to two sums and the only possible sums for 25 through 32 are 36 and 49, the following combinations must occur: (4-32-17), (5-31-18), (6-30-19), (7-29-20), (8-28-21), (9-27-22), (10-26-23), and (11-25-24). Also, we must have (9-16-20), since the only sums that 16 can be part of are 25 and 36, but 20 can be used only once, so we can extend (7-29-20) to (7-29-20-16-9), which we can further extend by adding (9-27-22) to give (7-29-20-16-9-27-22). Now, the only other possibility for 18 is 7, since 36 would require using 18 again, so we can extend further to (5-31-18-7-29-20-16-9-27-22). Now, for 19 we need 6 or 17, but 6 would form a loop; therefore, we have (6-30-19-17-32-4). Finally, 8 cannot be paired with 17 or 28 (already used), so must be paired with 1 extending (1-8-28-21). At this point a manageable decision tree completes the solution.

Computer Bonus. Repeatedly applying the algorithm, if N is even, divide by 2 and if N is odd, multiply by 3 and add 1, to a positive integer N always eventually results in 1. For integers up to 10,000, the number 6171 requires the most steps, reaching a maximum value of 975,400 at step 78 before ending at 1 at step 261.

NEW SUMMER PROBLEMS

1 Bingchester is an important junction where train lines cross thus:



In addition, roads parallel the train tracks. A, C, D, and E represent stations on the lines whose distances from Bingchester by road are: A, 4 miles; C, 7 miles; D, 10 miles; and E, 7 miles. The areas between A, C, D, and E are covered by impenetrable woods so it is impossible for anyone to get from one of these places to another without passing through Bingchester either by road or by train.

An extract from the timetable reads thus:

A dep: 9:15	C dep: 9:10
B arr: 9:23	B arr: 9:27
B dep: 9:25	B dep: 9:30
C arr: 9:44	A arr: 9:38
D dep: 8:55	E dep: 8:58
B arr: 9:26	B arr: 9:22
B dep: 9:29	B dep: 9:25
E arr: 9:53	D arr: 9:56

Trains run every 15 minutes, precisely on time, so that the time of a previous or following train can be determined by subtracting or adding 15 minutes.

The most famous attraction in Bingchester is the Moaning Lisa, hung in the train station. Unfortunately, someone has defaced the picture by adding a beard and mustache. The picture was seen unadorned by several people at 9:25:30, and the damage was discovered at 9:28:30.

Five men are suspected of the desecration, and one is certainly guilty. The five suspects make the following true statements:

Paul: I saw Ron in A at 9:14 and I was in E at 9:52.
Quentin: I was in E at 9:01, and in C at 9:58.
Ron: I was in D at 10:09. I left my bicycle at A.
Sam: I was in C at 8:56, and in D at 10:03.
Ted: I was in A at 9:40, and in E at 8:59.

They all have bicycles which they can peddle at a steady speed of 15 m.p.h. They can take their bicycles on the train, but no one rides anybody else's bicycle. Two of their bicycles were found at Bingchester after the defacing.

Whose bicycles were they? What were the exact movements of all five suspects? Who defaced the Moaning Lisa?

—*Brain Puzzler's Delight*
by E. R. Emmet

2 Solve the following cryptic multiplication, where each different letter represents a different digit: ABCDEF = BCDEFA x M.

—*The Crucible*

3 On an analog watch, in less than half a second after the precise time that the second hand passes one of the twelve hourly marks, the minute hand passes over the hourly hand. If this occurs before noon, at what time do the minute hand and hour hand coincide?

—Source Unknown

4 What is the minimum number of people, born on random days in 1981, that need to be in a room to have at least 50% probability that either there are (at least) two pairs of persons who have the same birthday or there are three (or more) people who share a common birthday or both?

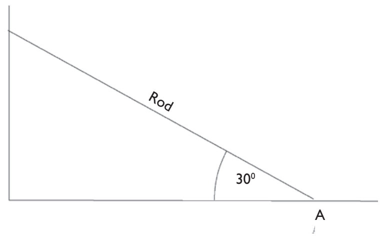
—**D.A. Dechman,** *TX A '57*

5 What is the minimum number of knights that can be placed on a standard 8x8 chess board, so that every square (including those occupied by knights) is threatened by a knight and what is such a configuration? A square is threatened if a knight can move to that square on its next move. A knight moves two squares in one direction and one square in another direction (perpendicular to first direction) to end up on a square of opposite color. The move can occur even if intervening squares are occupied. Present your answer as an 8x8 grid with 'o' representing an unoccupied square and 'N' representing a square occupied by a knight. Hint: the minimum is less than 16.

—*Amusements in Mathematics*
by H. E. Dudney

Bonus. A rigid, uniform rod one meter long is held leaning against a frictionless, vertical wall at a 30 degree angle to the horizontal by a peg

(Continued on page 45.)



at point A (where the stick touches the ground) on a frictionless, horizontal surface. If the peg is suddenly removed, what is the horizontal speed of the center point of the rod as it passes point A?

—Allan Gottlieb's Puzzle Corner in *Technology Review*

Computer Bonus. In how many ways (order matters) can 14 married couples be seated in chairs numbered consecutively 1 to 28 about a round table in such a manner that there is always one man between two women and none of them is ever next to his own wife?

—*100 Great Problems of Elementary Mathematics* by Heinrich Dorrie

Postal mail your answers to any or all of the Brain Ticklers to **Curt Gomulinski, Tau Beta Pi, P. O. Box 2697, Knoxville, TN 37901-2697,** or email to *BrainTicklers@tbp.org* plain text (no HTML, no attachments). The cutoff date for entries to the Summer column is the appearance of the Fall Bent during early October. The method of solution is not necessary, unless you think it will be of interest to the judges. We also welcome any interesting new problems that may be suitable for use in the column. The Computer Bonus is not graded. Curt will forward your entries to the judges, who are: **H.G. McIlvried III,** *PA Γ '53*; **D.A. Dechman**, *TX A '57*; **J.L. Bradshaw**, *PA A '82*; and the columnist for this issue,

—F.J. Tydeman, *CA Δ '73*