Brain Ticklers

RESULTS FROM WINTER 2011

Perfect

*Brule, John D.	MI	B '49
Couillard, J. Gregory	IL	A '89
Gay, Nicholas J.	KS	Γ '09
Griggs Jr., James L.	OH	A '56
Jones, John F.	WI	A '59
Krook-Magnuson, Chris	Non-member	
*Kumar, Vinny P.	PA	Δ '09
*Marx, Kenneth D.	OR	A '61
*Mayer, Michael A.	IL	A '89
Nabutovsky, Joseph	Father of member	
*Norris, Thomas G.	OK	A '56
Norris Jr., Thomas G.	PA	Γ '79
Overton, Leonard L.	IN	B '74
Newby, Dan	Nephew of member	
*Prince, Lawrence R.	CT	B '91
Rasbold, J. Charles	OH	A '83
Rosenthal, Jason	Son of member	
Rosenthal, Amy	NY	N '89
Schmidt, V. Hugo	WA	B '51
*Snelling, William E.	GA	A '79
*Spong, Robert N.	UT	A '58
*Stribling, Jeffrey R.	CA	A '92
*Strong, Michael D.	PA	A '84
White Jr., Warren N.	LA	B '74

Other

Alexander, Jay A.	IL	Γ '86
*Anonymous	Unknown	
Aron, Gert	IA	B '58
Bachmann, David E.	MO	B '72
Ballard, Jeffrey A.	FL	E '05
Bayne, Stephanie C.	LA	Γ '10
Beaudet, Paul R.	Father of member	
Becker, William H.	MD	A '65
*Berinato, Robert J.	GA	A '85
Bertrand, Richard M.	WI	B '73
Conway, David B.	TX	I '79
deVitry, David M.	PA	H '97
Dohner, John W.	CA	Γ '72
Forde, Jeffrey M.	CA	M'97
Handley, Vernon K.	GA	A '86
*Harms, Todd M.	TN	A '95
Henry, Jerry R.	MO	A '63
Jones, Donlan F.	CA	Z '52
Jones, Jesse D.	CA	Γ '95
Kane, Ronald J.	MI	Γ '74
*Kimsey, David B.	AL	A '71
Kneip, Paul M.	IA	A '89
Lew, Thomas M.	TX	Δ '84
Marks, Lawrence B.	NY	I '81
Hertz, Caryn M.	NY	I '81
Marks, Benjamin	Son of member	
Marks, Noah H.	PA	K '11
Marrone, James I.	IN	A '61
Mercer, Robert	Non-member	
Niemi, Michael G.	MD	Γ '07
Quintana, Juan S.	OH	Θ '62
Rentz, Peter E.	IN	A '55
Rhinehart, Todd M.	CA	Y '11
Saikali, Jeffrey	OH	B '96
Shah, Parth	Son of member	
Silver, Robert E.	NY	P '80
Spring, Gary S.	MA	Z '82
Stepanian, Shant P.	NJ	A '06
Stetson II, Scott B	Son of member	
Stevens, Robert E.	MO	B '85
Summerfield, Steven L.	MO	Γ '85
Sutor, David C.	Son of member	
Tessier, Thomas M.	MA	A '90
*Thaller, David B.	MA	B '93
Twete, Myles A.	OR	B '03
*Voellinger, Edward J.	Non-member	
Zison, Stanley W.	CA	Θ '87

* Denotes correct bonus solution

WINTER REVIEW

Problem 2 (the pirates) was the hardest, and problem 5 (the birds) was the next hardest regular problem.

SPRING SOLUTIONS

Reader entries for the Spring problems will be acknowledged in the Fall BENT. Meanwhile, here are the answers.

1 RNAGEL = 142857. This is the repeating digits of 1/7 = 0.142857142... and has the property that multiplication by 1, 2, 3, 4, 5, or 6 produces the same digits in a different order.

2 The spider catches the ant after 1,005 complete circuits. The length of the spider's circuit is πD cm, and that of the ant is $(\pi/2 + 1)D$ cm, where D is the diameter of the circle. The time required for the spider to complete one circuit is $\pi D/v_s$ min, and the time required for the ant to complete one circuit is $(\pi/2 + 1)D/v_a$ min. The critical issue is the position of the ant at the start of each spider circuit. The ant takes $(\pi D/2)/v_a$ min to traverse the upper semicircle, and the spider takes $(\pi D/2)/v_s$ min. Thus, if the ant is more than $(\pi D/2)(1/v_a - 1/v_s)$ min, or $x = (\pi D/2)(1 - v_a/v_s)$ cm, ahead of the spider at the start of a new spider circuit, the spider will not catch the ant before it escapes along the diameter.

The number of ant circuits equal to n_s spider circuits is $[2\pi/(\pi+2)]n_s(v_a/v_s)$. In general, this will equal a whole number plus a fraction of a circuit. For the spider to catch the ant, the fractional circuit must correspond to one diameter (since the ant starts at the left end of a diameter) plus an amount less than x. That is, we want the fractional part of $[2\pi/(\pi+2)]n_s(v_a/v_s) - 2/(\pi+2)$ to be less than $z = x/[(\pi/2 + 1)D]$. The approach is to try successive values of n_s until z falls in the critical range. (This is easily done on a spreadsheet.) For D = 100 cm, v_a = 700 cm/min, and v_s = 701 cm/min, x = 0.224079362 cm and z = 0.000871634. Thus, we want $1.220287673n_s - 0.388984529$ to be less than 0.000871634. These values with n_s = 1,005 give 1,226 ant cycles plus the equivalent of a diameter plus 0.00012636, which is less than the critical value. Thus, the spider catches the ant on the 1,006th circuit after 1,005 complete circuits. It looks like 660 circuits would work, but careful analysis shows that the ant barely escapes.

3 Doris stocked the shelf with catalogs. Let C, D, and E be the widths of a catalog, dictionary, and encyclopedia, respectively. There are three possibilities: (1) Al and Bob are correct; (2) Al and Connie are correct; or (3) Bob and Connie are correct. If (1), $2C + 3D + 3E = 4C + 3D + 2E$, then $E = 2C$. If (2), $2C + 3D + 3E = 4C + 4D + 3E$, then $D = -2C$. If (3), $4C + 3D + 2E = 4C + 4D + 3E$, then $E = -D$. Since book widths can't be negative, it must be Al and Bob who are correct. Doris can't have used encyclopedias, since then catalogs would also work; therefore, she used catalogs or dictionaries. Assume she used dictionaries; then $2C + 3D + 3(2C) = 8C + 3D = 15D$, so $D = 2C/3$, and $15D = 10C$, so both dictionaries and catalogs would work. Therefore, Doris must have used catalogs. (To see that only catalogs work, note that $2C + 3D + 3(2C) = 15C$, so $C = 3D/7$ and $15C = 45D/7$, and so dictionaries don't work.)

4 There are 144 different arrangements for the cards. Refer to the figure.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Place an arbitrary card in position 1 (16 possibilities). Assume it is the ace of spades. Place the other three aces in 7, 12, 14 or 8, 10, 15 (3 aces times 2 positions = 3!(2) = 12 possibilities). The remaining three spades must go in 8, 10, 15 or 7, 12, 14, depending on where the aces were located (3! = 6 possibilities); the rest of the cards are now forced. For each arrangement, there are 4 rotations and 2 reflections. Thus, the total number of different arrangements is 16(12)(6)/8 = 144.

5 Two thirds of the days are sunny, so the expected number of sunny days per year is 2(365)/3 = 243. Let P_r = probability of rain and P_s = probability of sun. Then, $P_r = 0.5P_s$ and $P_s = P_r + 0.5P_s$. Also, $P_s + P_r = 1$. Therefore, $P_s = 1 - P_s + 0.5P_s$. Solving gives $P_s = 2/3$, so the expected number of sunny days is 2(365)/3 = 243.3.

Bonus. For a person standing in the center of a flat face of the cylinder, g_c is 9.83 m/s^2. Let the height of the cylinder be H and its radius be R.

Consider a slice of this cylinder, parallel to its circular faces and of thickness dh, and let the distance of the slice from the top face be h. Now consider a concentric ring of radius r and width dr on this slice.

The volume of this ring is $dV = 2\pi r\,dr\,dh$, and its mass is $dM = 2\pi\rho r\,dr\,dh$, where ρ is density.

The gravitational force of the ring on a mass m located at the center of the top face of the cylinder is $dF = 2\pi\rho G m \cos\theta\, dM/s^2$, where s is the distance from a point on the concentric ring to the center of the top face and θ is the angle between the centerline of the cylinder and a line from the concentric ring to the center of the top face ($\cos\theta$ is necessary because only the vertical component of the force contributes to g).

Now, $s^2 = r^2 + h^2$, and $\cos\theta = h/s = h/\sqrt{(r^2 + h^2)}$. Therefore, $F = 2\pi\rho G m \int_0^H \int_0^R h\,dr\,dh/(r^2 + h^2)^{3/2} = 2\pi\rho G m \int_0^H [-1/(r^2 + h^2)^{1/2}]\,|_0^R h\,dh = 2\pi\rho G m \int_0^H [1/h - 1/(R^2 + h^2)^{1/2}]h\,dh = 2\pi\rho G m \int_0^H [1 - h/(R^2 + h^2)^{1/2}]dh = 2\pi\rho G m[h - (R^2 + h^2)^{1/2}]\,|_0^H = 2\pi\rho G m[H + R - (R^2 + H^2)^{1/2}]$. Now, $M = \pi R^2 H\rho$, or $\pi\rho = M/R^2H$. Therefore, $F = 2GmM[H + R - (R^2 + H^2)^{1/2}]/R^2H$, but $F = mg_c$, so $g_c = 2GM[H + R - (R^2 + H^2)^{1/2}]/R^2H$, but $H = 2R$, so $g_c = (3 - \sqrt{5})GM/R^2$. Now, the volume of the earth is $V_e = (4/3)\pi R^3 = (4/3)\pi(6.37\text{x}10^6)^3$ m^3 = $1.0827\text{x}10^{21}$ m^3, and its mass is $M_e = 5{,}518V_e = 5.974\text{x}10^{24}$ kg. $V_c = \pi R^2 H = 2\pi R^3 = (4/3)\pi R_e^3$, so $R = (2/3)^{1/3}R_e$, or $R = 0.8736\,R_e = 5.565\text{x}10^6$ m. Now, G = $6.673\text{x}10^{-11}$ m^3/s^2 kg, so $g_c = (3 - \sqrt{5})(6.673\text{x}10^{-11})(5.974\text{x}10^{24})/(5.565\text{x}10^6)^2$ = 9.83 m/s^2, surprisingly close to g for a spherical earth.

Computer Bonus. 967,718,839 is the smallest prime P for which the ratio of the number of double primes to the number of primes (both ≤ P) is less than $1/e$. There are 49,288,471 primes ≤ P, of which 18,132,215 are double primes, giving a ratio of 0.3678794378; $1/e$ = 0.367879441.

NEW SUMMER PROBLEMS

1 Al, Bob, Carl, Don, and Ed took a mathematics test with five questions. For a correct answer, 10 points were given; if the answer were wrong, either 7, 2, or 0 points were given, depending on the method used and the way the work was presented. (No points other than 10, 7, 2, or 0 were awarded, and it is perfectly possible for two students to get the same wrong answer, but be given different points.) At least one student got each question right. Some information about the answers of different students is shown in the table.

	1	2	3	4	5
Al	5	?	11	?	4.5
Bob	3.5	17	5	?	3.4
Carl	4	43	5	?	2.8
Don	2.5	17	11	6	3.8
Ed	7	17	7	7	5.2

The total points for Al, Bob, Carl, Don, and Ed were 34, 19, 31, ?, and 9, respectively. The total points for each question, #1 – #5, were 14, 29, 22, 42, and ?. You may also find it helpful to know that Ed got more points for question 5 than Bob. Determine the number of points awarded each student for each question.

—*Brain Puzzler's Delight*
by E. R. Emmet

2 2010 was the 75th anniversary of the creation of the WPA, which provided millions of jobs during the Great Depression. Solve the following two cryptic addition problems simultaneously in base 11 (use a lower case *a* to represent the digit 10).

USA + FDR = WPA
USA + WPA = PARK

Different letters represent different digits, the same letter always represents the same digit, and there are no leading zeros.

—**Howard G. McIlvried III,** *PA Γ'53*

3 In a game of 7-card stud poker with deuces wild, what is the probability of getting five of a kind?

—**Howard G. McIlvried III,** *PA Γ'53*

4 Twenty-six identical coins lie on a table—ten heads and 16 tails. Blindfolded, you are told to divide the coins into two groups with the same number of heads in each group. You may move the coins and turn them over, but have no way of telling whether a coin is heads or tails. How do you accomplish this?

—*All-Star Mathlete Puzzles*
by **Richard I. Hess,** *CA B'62*

5 Any number, real or complex, may be represented in the form $r(\cos\theta + i\sin\theta)$. Considering values of θ only in the range -π to +π radians, determine whether i to the ith to the ith to the ith power, ad infinitum, approaches a limit, and, if so, provide r and θ to 4 digits. As a reminder of standard practice, note, for example, that 3 to the 3rd to the 3rd means 3^{27}, and not 27^3. That is, the continued exponential is evaluated from the top down, not from the bottom up.

—**John W. Langhaar,** *PA A'33*

Bonus. Given a 2x4 rectangle of squares (a map) with the front marked with the digits

1	2	3	4
5	6	7	8

and the back marked with letters:

d	c	b	a
h	g	f	e

(that is, the Nth letter of the alphabet is on the back of the square with the digit N).

This 2x4 map can be folded (which includes tucks) along the boundaries between squares in many ways to end with a 1x1 by 8 layers configuration with the '1' square on top of the other seven and the '1' visible. For example, abcd folded onto efgh, 78 folded onto 65, and 3 folded onto 4, ends up with the configuration: 1e8d3g6b. How many unique final configurations can be achieved, and what are they? To make grading easier, please provide the configurations, one per line, sorted

(Continued on page 47

New York Mu

Dr. Thomas C. Sheahan, P.E., **'81,** has been named as senior associate dean for academic affairs at Northeastern University's college of engineering. He has been on the faculty there since 1991 and is the 2007 Tau Beta Pi-McDonald Mentor. He has also been active in organizations including ASME and Engineers Without Borders.

New York Nu

Mark A. Noblett, P.E., **'70,** has retired as a civilian from the U.S. Air Force at Andrews AFB. He served as chief of engineering with the civil engineer squadron. He previously served with the U.S. Army Corps of Engineers, Baltimore district, supporting projects in Bosnia and Afghanistan.

New York Upsilon

Erin E. Anthony, '11, continued to solidify her position as a top student-athlete when she repeated as a first team Capital One academic all-America basketball selection for 2011. A two-time defending Patriot League scholar-athlete of the year, Anthony was also a first team academic all-America honoree in 2010. She is the fourth honoree in back-to-back years in her league's 20-year history and first to make the top team twice.

Ohio Lambda

Jerad R. Shuster, '00, and his wife Jennifer welcomed their third child on April 7, 2011. Sydney Julia joins sister Alexis Lily and brother Logan Andrew. Jerad is the supervisor of the industrial engineering lab at Diebold, Inc., in North Canton, OH. The family resides in Canfield, OH.

Pennsylvania Alpha

Catherine G. Gorzkowski, '00, and **Dr. Edward P. Gorzkowski III, '99,** have welcomed Edward James Gorzkowski IV, born at a healthy 10 lb. 2 oz. and 21.5 inches. Abigail is loving being a big sister. Catherine is an Engineering Futures Facilitator, and Ed is a District 3 Director.

Texas Delta

Dr. Dale A. Cope, '82, has been elected an associate fellow of the American Institute of Aeronautics and Astronautics. He is a program manager in Southwest Research Institute's mechanical engineering division. Cope has more than 20 years of experience in aircraft-structure technology. Prior to joining his firm in 2007, he was a laboratory director at the National Institute for Aviation Research. Cope is a retired U.S. Air Force officer.

Texas Iota

Nita K. Patel, P.E., **'95,** has been named 2011 New Hampshire engineer of the year by the NH Joint Engineering Societies and selected as a candidate for 2012 IEEE-USA president elect. She is the current institute VP of communications and public awareness, an engineering manager at L-3 Insight Technology, and lives in Bedford, NH.

Wisconsin Delta

Austin A. Meier, '12, has been named Capital One men's basketball college division academic all-America of the year. An architectural engineering and construction management major with a G.P.A. of 3.9, he is a senior forward for the MSOE Raiders. The previous season, Meier was selected to the CoSIDA academic all-America first team.

BRAIN TICKLERS

(Continued from page 45.)

in ascending order (digits less than letters and using only the marks on the upper surfaces) using the above numbering system.

—**Fred J. Tydeman,** *CA Δ '73*

Double Bonus. Consider a quadrilateral. Construct a square on each of its sides. Prove that the two line segments joining the centers of opposite squares are equal in length and mutually perpendicular.

—**Jan Smit,** *CA Δ '41*

Postal mail your answers to any or all of the Brain Ticklers to Jim Froula, **Tau Beta Pi, P. O. Box 2697, Knoxville, TN 37901-2697,** or email to *BrainTicklers@tbp.org* plain text (no HTML, no attachments). The cutoff date for entries to the Summer column is the appearance of the Fall Bent during early October. The method of solution is not necessary, unless you think it will be of interest to the judges. We also welcome any interesting new problems that may be suitable for use in the column. The Double Bonus is not graded. Jim will forward your entries to the judges, who are: **H.G. McIlvried III,** *PA Γ '53;* **D.A. Dechman,** *TX A '57*; **J.L. Bradshaw,** *PA A '82*; and the columnist for this issue,

—**F.J. Tydeman,** *CA Δ '73*

Write Your Own Alumnus Note!

Your fellow Tau Bates are interested in news about you. Send items about civic activities, honors won, weddings, births, promotions, changes in address, etc. to Tau Beta Pi, Box 2697, Knoxville, TN 37901-2697 or to *alumnote@tbp.org*. Material for publication must be received for the **Fall** issue by August 1, **Winter** issue by November 1, **Spring** issue by February 1, and **Summer** issue by May 1. Include name, address, chapter, class year, and email address or phone no. Thank you!