Brain Ticklers

RESULTS FROM WINTER 2010

Perfect

*Bachmann, David E.	MO	B	'72
*Beaudet, Paul R.	Father of member		
Bush, James L.	TN	Γ	'01
*Campbell Jr., H. Grady	TN	B	'59
*Fenstermacher, T. Edward	MD	B	'80
*Fuemmeler, Jason A.	OH	Θ	'00
*Gaston, Charles A.	PA	B	'61
Jenneman, Jeffrey H.	OK	A	'08
*Kaliski Jr., Burton S.	MA	B	'84
Kaliski, Stephen	Son of member		
*Kimsey, David B.	AL	A	'71
Kutcher Jr., John F.	MD	A	'86
*Mangis, J. Kevin	VA	A	'86
*Mayer, Michael A.	IL	A	'89
*McBride, John J.	WA	A	'57
*Norris, Thomas G.	OK	A	'56
Rasbold, J. Charles	OH	A	'83
Schleehauf, Martin W.	NY	N	'79
*Schmidt, V. Hugo	WA	B	'51
Silver, Robert E.	NY	P	'80
*Stribling, Jeffrey R.	CA	A	'92
*Strong, Michael D.	PA	A	'84
*Stubler, William G.	NY	N	'07
*Wendling, D. Greg	IL	A	'79

Other

Achterberg, Karl J.	WI	A	'84
*Alexander, Jay A.	IL	Γ	'86
Aron, Gert	IA	B	'58
*Berger, Toby	CT	A	'62
Bernacki, Stephen E.	MA	A	'70
Bertrand, Richard M.	WI	B	'73
Bukowski, Justin D.	OH	A	'90
*Couillard, J. Gregory	IL	A	'89
*De Vincentis, Joseph W.	TX	Γ	'93
deVitry, David M.	PA	H	'97
Harris, Kent	Non-member		
*Harvey, Arthur J.	OH	A	'83
Jordan, R. Jeffrey	OK	Γ	'00
Klinger, Allen	NY	I	'57
*Midgley, James E.	MI	Γ	'56
Minnick, Michael V.	SC	A	'81
Palmer, William A.	ND	A	'65
Rentz, Peter E.	IN	A	'55
Robillard, David J.	MD	Γ	'88
Routh, Andre G.	FL	B	'89
Sigillito, Vincent G.	MD	B	'58
*Spong, Robert N.	UT	A	'58
Stepanian, Shant P.	NJ	A	'06
Summerfield, Steven L.	MO	Γ	'85
Sutor, David	Son of member		
Svetlik, J. Frank	MI	A	'67
*Voellinger, Edward J.	Non-member		
*White Jr., Warren N.	LA	B	'74
*York, Jeffrey A.	NC	A	'85
*Zelinka, Samuel L.	WI	A	'05

* Denotes correct bonus solution

In Memory

We sadly report the death on March 7, 2009, of **John W. Langhaar,** *PA A '33.* John served as a Brain Ticklers judge for 29 years, 1955-83. He was one of the finest recreational mathematicians we have known. Some of his problems are still in our files. In fact, one of his original problems is new problem No. 5 in this column.

WINTER REVIEW

Problems 1 (bowling balls) and 2 (donuts) were the ones that presented the most difficulty of the regular problems. Many readers found the Computer Bonus easy.

SPRING SOLUTIONS

Readers' entries for the Spring problems will be acknowledged in the Fall BENT. Meanwhile, here are answers:

1 This 3x3 magic-square puzzle is easier to solve by starting at the end and working backwards. A 3x3 magic square is fully defined by three numbers (m, x, y) as shown by:

$m+x$	$m-(x+y)$	$m+y$
$m-(x-y)$	m	$m+(x-y)$
$m-y$	$m+(x+y)$	$m-x$

Since the number of letters in the spellings of the numbers one to seventy-two range from 3 to 11, the middle square must be 7, and the final magic square is the numbers 3 to 11 (which is unique except for rotations and reflections).

Final			*Original*		
(# of letters)			*(numbers)*		
8	3	10	18	2	25
9	7	5	22	15	8
4	11	6	5	28	12

A list of the numbers 1 to 72, sorted by number of letters in their names, makes it easy to find the limited choices for each square. Seven letters only comes from 15, 16, and 70, while four letters comes from 4, 5, and 9. There are many choices for ten letters. The only combinations of 4-7-10 letters that works is 5-15-25. Three letters only comes from 1, 2, 6, and 10, while 11 letters has many choices. The only combinations of 3-7-11 letters that works is 2-15-28. Having five of the numbers in the original square allows the rest to be found from simple math from the sum of any row, column, or diagonal being 3m. That result is the original square shown above.

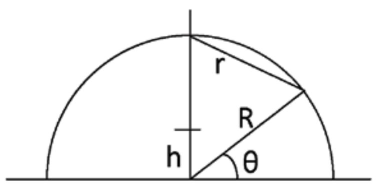
2 Zack lives at 55, Ann at 81, and Beth at 64. Let the house numbers be N_A, N_B, and N_Z. Z must have answered both A's questions yes. That would mean that N_Z is 64 or 81, and the only way A could think she knows N_Z is if one of these numbers is hers, so the other is Z's. But only Z's second answer was truthful, so N_Z is a non-square > 50. Again, for B to think she knows N_Z, Z must have answered both her questions yes, which would mean that N_Z is 27 or 64, and since one is her number, she thinks the other is Z's. But again only the second answer is correct. Now, since A and B live in different houses and their numbers are > 50, N_A must be 81, and N_B must be 64. Since the sum of the house numbers is twice a square and $50<N_Z<64$, twice the square must be 200, and Z's number is 55.

3 \$1.76 is the amount the player should wager. Since the 'adds' and 'takes' from the pot are symmetrical about \$10, they cancel each other, and all that matters is the odds of exactly \$10 in the pot. So, expected payout is: \$10 x comb(20, 10)/2^{20} = \$10 x 184,756 / 1,048,576 = \$1.76. Since there are 10 throws of 2 coins, it is the 'same' as 20 throws of 1 coin with the requirement that 10 of the throws be heads. This can also be solved by calculating and adding the probabilities of the six possible outcomes with #HH = #TT and the number of draws even.

4 100,000! is 456,574 digits long. Stirling's approximation of $n! \approx (n/e)^n \sqrt{(2\pi n)}$ implies $\log_{10}(n!) = (n)\log_{10}(n/e) + (0.5)\log_{10}(2\pi n)$. So, for log(100,000!), we get 456,573.45; hence, there are 456,574 digits in 100,000! This can also be solved by adding $\log_{10}(j)$ for j=1 to 100,000.

5 57/12 + 96/384 = 5 is the solution to AB/CD + EF/GHJ = A. Since GHJ is a multiple of CD with all digits being different, there are a limited number of trials to do by hand or computer.

B onus. One is the ratio of the period of the oscillation of the half hoop to that of the whole hoop. The moment of inertia I of the hulahoop [HH] about its center of gravity [c.g.] (the center of the HH) is MR^2. I about the suspension point is I about its c.g. plus ML^2, where L is the distance between the c.g. and the suspension point. For the HH, $L = R$, so $I = MR^2 + MR^2 = 2MR^2$. For a physical pendulum, the period $T = 2\pi\sqrt{(I/(MgL))}$. Therefore, for the HH, $T = 2\pi\sqrt{(2MR^2/(MgL))} = 2\pi\sqrt{(2R/g)}$. For the half HH, the c.g. is on the vertical center line at a distance h above the center (see figure). Now $\int_0^\pi (R\sin\theta - h)dM = 0$, but $dM = \rho R d\theta$, where ρ = mass/unit length. Therefore, $\rho R^2\int_0^\pi(\sin\theta - h/R)d\theta = \rho R^2[-\cos\theta - h\theta/R]_0^\pi = \rho R^2(1 - \pi h/R + 1) = \rho R^2(2 - \pi h/R) = 0$. Therefore, $h/R = 2/\pi$. I of the half HH about its suspension point is $\int r^2 dM$ (see figure). Now, $r^2 = (R\cos\theta)^2$



$+ (R - R\sin\theta)^2 = R^2(\cos^2\theta + \sin^2\theta) + R^2 - 2R^2\sin\theta = 2R^2(1 - \sin\theta)$ and $dM = \rho ds = \rho R d\theta$. Therefore, $I = \int_0^\pi 2R^2(1 - \sin\theta)\rho R d\theta = 2\rho R^3\int_0^\pi(1 - \sin\theta)d\theta = 2\rho R^3[\theta + \cos\theta]_0^\pi = 2\rho R^3\pi(1 - 2/\pi)$. But, $M = \pi\rho R$. Therefore, $I = 2MR^2(1 - 2/\pi)$, and $T = 2\pi\sqrt{(2MR^2(1 - 2/\pi)/(MgL))} = 2\pi\sqrt{[2MR^2(1 - 2/\pi)/(MgR(1 - 2/\pi))]} = 2\pi\sqrt{(2R/g)}$. Thus, the periods of the HH and half HH are the same.

C omputer Bonus. The thousandth sequence of consecutive primes ending in 1, 3, 7, 9 starts with 11,740,031. One approach would be to start with 11 and increment by 10, test for a sequence of four primes (P_1, P_3, P_7, P_9), and continue this process, incrementing by 10 each time. However, a little analysis can reduce computing time. Consider a prime P_1 that is the first of a series, and use casting out 9s to reduce the sum of its digits to a single digit S_1. S_1 cannot be 3, 6, or 9, because then P_1 would be divisible by 3. Also, S_1 cannot be 1, 4, or 7, since then P_3 would be divisible by 3. Therefore, S_1 must be 2, 5, or 8. But if we increment by 10, then S_1 will change to 3, 6, or 9, meaning P_1 will not be prime. Now try incrementing by 20. S_1 will then become 4, 7, or 1, and S_3 will be 6, 9, or 3, so P_3 will not be prime. If we increment by 30, S_1 becomes 5, 8, or 2, the same values. Thus, in the computer search, it is only necessary to increment by 30.

NEW SUMMER PROBLEMS

1 Find all primes of the form A^4+4B^4, where A and B are positive integers.

—Puzzle Corner by Allan Gottlieb in *Technology Review*

2 Algernon, Bertie, and Clarence had so often expressed their opinion about Professor Popoff that when he was found murdered (stabbed with a dagger, but in a thoroughly gentlemanly way) it was natural that they should be suspected. In fact, for reasons into which we need not now go, it may be taken as certain that one of them is guilty. They made statements as follows:

Algernon:
1: I hadn't seen Popoff or had any contact with him for a week before his unfortunate demise.
2: Everything Bertie says is true.
3: Everything Clarence says is true.

Bertie:
1: I have never handled a dagger.
2: Everything Algernon says is false.
3: Everything Clarence says is false.

Clarence:
1: Algernon was talking to Popoff just before he was killed.
2: Bertie has handled a dagger.
3: I have for a long time thought more of Popoff than is generally realized.

Looking back on the tragic event now, it is interesting to see that Algernon and Bertie both made the same number of true statements. (This number can be anything from 0 to 3, inclusive).

Who killed Popoff?

—*Brain Puzzler's Delight* by E.R. Emmet

3 At its main base, the Patagonian navy had 16 Class I gunboats with 5 guns apiece, some Class II gunboats with 4 guns apiece, and some Class III gunboats with 3 guns apiece, when the admiral was ordered to crush a revolt that had broken out simultaneously on seven scattered islands. After some thought, he split his fleet into seven flotillas of 6 ships each, with each flotilla having at least one gunboat of each class but a different number of guns. How many Class II gunboats were there?

—Tantalizer by Martin Hollis in *New Scientist*

4 I dreamed that I lived in a large, vertical, rectangular painting, where everything was located by its x-z coordinates. My mail always landed right at my house, at coordinates (1,0), from an airplane flying at a constant speed of 1 m/sec along the path z = x². The falling mail was subjected to an acceleration due to "gravity" of 1 m/sec/sec downward, perpendicular to the x-axis. What was the x coordinate of the release point when the plane was flying in (1) the positive direction and (2) the negative direction? Ignore "air" resistance. Assume the initial velocity of the mail when it is dropped is the same as the airplane.

—**Byron R. Adams,** *TX A '58*

5 A vertical belt sander is being used to polish rolling pins. The rolling pin, with a diameter of 5 cm and a length of 100 cm, is rotated at 100 rpm about a horizontal fixed axis while being pressed against the belt with a force of 10 Newtons perpendicular to the belt, which is traveling at 2 m/s. If the coefficient of friction is 0.5, how many watts are being dissipated as friction when (1) the rotation of the rolling pin is opposite to the direction of the belt and (2) the rotation of the rolling pin is in the same direction as the belt.

—**John W. Langhaar,** *PA A '33*

B onus. George is building a rectangular patio, which will be covered with one-foot-square concrete slabs of seven different colors. He has divided the patio into seven rectangular zones, each to be covered by slabs of a single color, with five different colors appearing around the perimeter of the patio and four different colors at the corners. The seven zones are all different sizes, but all have the same perimeter, which is less than 60 feet. What are the dimensions of the patio, and what are the dimensions of the seven zones?

—Enigma by Colin Singleton in *New Scientist*

(Continued on page 53.)

Double Bonus. Given the lengths of the *N* sides of an irregular polygon, how should the sides be arranged to maximize the enclosed area? Prove your answer.
—Puzzle Corner by Allan Gottlieb in *Technology Review*

Send your answers to any or all of the Summer Brain Ticklers to Jim Froula, Tau Beta Pi, P.O. Box 2697, Knoxville, TN 37901-2697 or email to: *BrainTicklers@tbp.org* only as plain text. The cutoff date for entries to the Summer column is the appearance of the Fall Bent. The method of solution is not necessary. We welcome any interesting problems that might be suitable for the column. The Double Bonus is not graded. Jim will forward your entries to the judges who are **H. G. McIlvried III,** *PA Γ '53;* **J.L. Bradshaw,** *PA Α '82;* **D.A. Dechman,** *TX Α '57.* and the columnist for this issue,
F. J. Tydeman, *CA Δ '73*

125TH ANNIVERSARY CLUB

2010 celebrates the 125th anniversary of the founding of Tau Beta Pi. To commemorate this historic occasion, a one-time recognition club has been established for donors contributing $125 or more in 2010. (Members of existing clubs will still be listed in those clubs.) Donors of $125 or more will receive a striking memento acknowledging their special support during the Quasquicentennial celebration. This limited-edition item features the Association's 125th anniversary logo etched onto a polished 3" x 3" black marble paperweight. For more information, contact Patricia McDaniel—*pat@tbp.org*, 865/546-4578.

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BRAIN TICKLERS *(Continued from page 51.)*

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