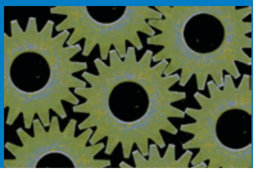

Brain Ticklers

RESULTS FROM WINTER 2009

Perfect

*Bachmann, David E.	MO B '72
*Beaudet, Paul R.	Father of member
*Couillard, J. Gregory	IL A '89
*De Vincentis, Joseph W.	TX Γ '93
*Gerken, Gary M.	Non-member
Griggs Jr., James L.	OH A '56
*Lints, Michael C.	NY Π '79
*Mayer, Michael A.	IL A '89
Rice, Thomas J.	Non-member
*Schmidt, V. Hugo	WA B '51
*Smith, Charles J.	CT Γ '09
*Thaller, David B.	MA B '93
*Weinstein, Stephen A.	NY Γ '96

Other

*Alexander, Jay A.	IL Γ '86
Aron, Gert	IA B '58
Barr, Adam D.	NJ Δ '88
*Berger, Toby	CT A '62
Bernacki, Stephen E.	MA A '70
Bertrand, Richard M.	WI B '73
*Bohdan, Timothy E.	IN Γ '85
Bradley, Michael J.	Non-member
Brule, John D.	MI B '49
Bukowski, Justin D.	OH A '90
*Doniger, Kenneth J.	CA A '77
Eckley, Paul L.	NV A '75
*Fenstermacher, T. Edward	MD B '80
Hamilton, Robert L.	OK A '57
*Hess, Richard I.	CA B '62
Jones, Donlan F.	CA Z '52
*Jones, Jesse D.	CA Γ '95
Kedari, Vishwanath R.	AL E '08
Kern, Peter L.	NY Λ '62
*Kimsey, David B.	AL A '71
Klein, Brian	Son of member
Lalinsky, Mark A.	MI Γ '77
Lamb, James A.	NY Γ '66
Lettau, Matthew K.	CA Y '03
Lew, Thomas M	TX Δ '84
*Mangis, J. Kevin	VA A '86
Marks, Lawrence B.	NY I '81
Marks, Benjamin	Son of member
Marks, Noah	Son of member
Marrone, James D.	IN A '87
*Midgley, James E.	MI Γ '56
*Quintana, Juan S.	OH Θ '62
*Rasbold, J. Charles	OH A '83
*Rentz, Peter E.	IN A '55
Rieger, Paul J.	KY A '66
Rubin, James D.	MI Γ '82
*Spong, Robert N.	UT A '58
*Stribling, Jeffrey R.	CA A '92
*Strong, Michael D.	PA A '84
*Voellinger, Edward J.	Non-member
Watry, Michael O.	CO B '86

* Denotes correct bonus solution

WINTER REVIEW

Problem 1 about a triangle inside another triangle and number 5 about the airplanes flying around the world got fewer correct answers than the bonus about the dice. For the Fall '08 Ticklers, the entry from Kenneth D. Marx, *OR A '61,* should have been listed as perfect.

SPRING SOLUTIONS

Readers' entries for the Spring problems will be acknowledged in the Fall BENT. Meanwhile, here are the answers:

1 This problem was poorly stated; the statement "ignore the change in gravity with distance" was meant to apply only to the spaceship, not to the escape velocity. Grading was lenient. Scott has about 20.3 min. to eject in his escape pod. The volume of Planet X is half that of earth; thus, $r_x = r_e/2^{1/3}$, where r is radius. Now, $g_x/g_e = M_x r_e^2/M_e r_x^2$ or $g_x = g_e/2^{1/3}$, where g is acceleration due to gravity and M is mass. Similarly, $v_{ex}/v_{ee} = (M_x r_e/M_e r_x)^{1/2}$ or $v_{ex} = v_{ee}/2^{1/3}$, where v_e is escape velocity. For earth, $g_e = 9.806$ m/s² and $v_{ee} = 11{,}186$ m/s. Therefore, $g_x = 7.783$ m/s² and $v_{ex} = 8{,}878$ m/s. The acceleration due to firing the engine is $T/m = 35{,}000/1{,}000 = 35$ m/s², where T is thrust and m is the mass of the spaceship, so the net acceleration is $g = 35 - 7.783 = 27.22$ m/s², and velocity at cutoff $v_c = 8{,}878/2 = 4{,}439$ m/s. Therefore, time to cutoff $t = 4{,}439/27.22 = 163.1$ s, and height, $S_c = gt^2/2 = 27.22(163.1^2)/2 = 362{,}000$ m. Now, $S = S_c + v_c t - g_x t^2/2$. On the surface, $S = 0$. Thus, $3.892t^2 - 4{,}439t - 362{,}000 = 0$. Solving for t gives 1,217 sec. or a little over 20 min.

2 The order of scoring on the test was Don (highest), Al, Ed, Bill, Carl (lowest), and the corresponding room assignments were A, B, C, D, E. The key to the solution is that Ed's e2 comment cannot be true. Therefore, A finished ahead of E, and if e1 is true, B is ahead of E, but A is also ahead of E, which makes E third, and he would not tell the truth. Therefore, E scored higher than B, and we have A > E > B. Since we know A is not first, either C or D must be first with four possibilities for the other. This provides eight possibilities to try. Only the order D, A, E, B, C gives consistent results. From c2, A is in Room B, and from d1, C is in Room E. From b2, D is not in Room C, and from a2, he is not in Room D. Therefore, he must be in Room A. From d2, E is not in Room D; therefore, he is in Room C, which leaves B in Room D.

3 The solution to TBPI/AAAAA = .ONANDONAND… is 1784/33333 = .0535205352…. The problem can be rewritten as TBPI/AAAAA = ONAND/99999, which can be simplified to give 9(TBPI) = A(ONAND). A simple computer program then gives 1,784/33,333 = .0535205352… and 7,610/22,222 = .3424534245…. Since we want the largest PI, the first solution is the desired one.

4 In one day, a 24-hour clock exhibits 660 palindromes; the longest time between palindromes is 4 hr., 4 min., 11 sec.; the shortest time is 2 seconds; and the two palindromes closest to being 12 hr. apart are 1:33:31 and 13:33:31, which differ by exactly 12 hr. For hours less than 10, the first and fifth digits must agree, and the middle three digits must have the form *aba*, where *a* can be 0 to 5, and *b* can be 0 to 9. Therefore, there are 10(6)(10) = 600 possible five-digit palindromes. For hours 10 to 15 and 20 to 23, the seconds must be the reverse of the hour, and the minutes must be of the form *aa*, where *a* is 0 to 5. Therefore, there are 10(6) = 60 six-digit palindromes for a total of 660. No palindromes are possible for hours 16 to 19. The closest are 9:59:59 and 10:00:01, and the farthest apart are 15:55:51 and 20:00:02, determined either by computer or with a little logical thinking.

5 The probability of getting a 5-card royal flush in 7-card stud poker with a 53-card deck is 0.000169531. One must have either (1) A, K, Q, J, 10, in the same suit or (2) four of these five cards plus the joker. In case (1), there are 4 suits, C(5, 5) ways to get A, K, Q, J, 10, and C(48, 2) ways to get the

6th and 7th cards, where C(m, n) is the combinations of m things taken n at a time, for a total of 4(1)(48)(47)/2 = 4,512 hands. For case (2), there are 4 suits, C(5, 4) ways to get four of A, K, Q, J, 10, one way to get the joker, and C(47, 2) ways to get the 6th and 7th cards, for 4(5)(1)(47)(46)/2 = 21,620 hands. Total possible hands is C(53, 7) = 154,143,080. Therefore, P = (4,512 + 21,620)/154,143,080 = 139/819,910 = 0.000169531.

Bonus. The probability P that Alice took the right pills every day is $(n!)^k(k!)^n/(nk)!$, where n is the number of days and k is the number of different pills. Consider the first day. The probability of getting a different pill on the first draw is $nk/(nk)$; on the second draw it is $(nk-n)/(nk-1)$; on the third draw it is $(nk-2n)/(nk-2)$; etc., so the probability of getting k different pills on the first day is: $P_1 = nk(nk-n)(nk-2n)\ldots(n) / [nk(nk-1)(nk-2)\ldots(nk-k+1)] = n^k k!/[nk(nk-1)\ldots(nk-k+1)]$. It should be clear that we can get P_2 by replacing n with n-1 in the equation for P_1. Thus, $P_2 = (n-1)^k k!/[(nk-k)(nk-k-1)\ldots(nk-2k+1)]$, and finally, $P_n = 1^k k!/[k(k-1)\ldots1]$. Now, $P = P_1P_2\ldots P_n = n^k k!(n-1)^k k!\ldots1^k k!/(nk)! = (n!)^k(k!)^n/(nk)!$. Also, you can deduce the formula by calculating the probabilities for several low values of n and k and analyzing the results.

Double Bonus. Ten is the maximum number of primes having a common difference of 210. Let N be the common difference, and let p_{n+1} be the smallest prime that does not divide N, and $m = p_{n+1}-1$. Then, p_{n+1} divides one term of the sequence P, $P+N$, $P+2N$, …, $P+mN$, and none of the terms are divisible by a prime smaller than p_{n+1}. To prove this, let r_0, r_1, …, r_m, be the remainders upon dividing the terms of the sequence by p_{n+1}. Assume that $r_i = r_j = r$. Then, $P+n_iN \equiv r \pmod{p_{n+1}}$ and $P+n_jN \equiv r \pmod{p_{n+1}}$. Subtracting gives $(n_i - n_j)N \equiv 0 \pmod{p_{n+1}}$, which means $(n_i - n_j)N$ is divisible by p_{n+1}, but both n_i and n_j are $< p_{n+1}$, so p_{n+1} cannot divide $(n_i - n_j)$ and must divide N, but this contradicts the condition that p_{n+1} does not divide N. Therefore, all the r's are different, and since there are p_{n+1} of them, one must equal 0. Thus, in a sequence of p_{n+1} terms of the form $P + nN$, p_{n+1} divides one term, so the maximum length of the sequence is p_{n+1} - 1, unless $P = p_{n+1}$, in which case the maximum length is p_{n+1}. For N = 210 = 2*3*5*7, p_{n+1} = 11, so 10 is the maximum sequence length. To have a 10-prime sequence, the next term, P + 2,100, must be divisible by 11. This means P must be of the form $11k + 1$. The first few such primes are 23, 67, 89, 199, 331, and 353, of which 199 starts a ten-prime sequence ending with 2,089.

NEW SUMMER PROBLEMS

1 The five offices of the Mudbridge Women's Guild are held by Mesdames Virtue, Wellbeloved, Xerxes, Youngblood, and Zenith. These powerful ladies are also very partial to a game of contract bridge and, on each weekday morning, a different one of the five acts as hostess to three of the others. The resulting fours are the same each week, and each lady has just one morning off each week—the one on which she is not part of the four. All those whom Mrs. Virtue invites to her morning game invite Mrs. Wellbeloved to theirs. Mrs. Wellbeloved invites two of the ladies whom Mrs. Xerxes invites and, as the fourth member, a lady not among those invited by Mrs. Youngblood. Because she is president, Mrs. Zenith's invitations confer a certain distinction on the recipients. Who, besides herself, does not receive one? This problem is a bit tricky. Be sure to think outside the box.

—Tantalizer by Martin Hollis in *New Scientist*

2 Three sharpshooters fire (not simultaneously) at a rapidly spinning sphere. If each of them hits the sphere, what is the probability that all three shots land in the same hemisphere for the following two cases: (1) the great circle dividing the sphere into two hemispheres can have any orientation and (2) the great circle dividing the sphere into two hemispheres must pass through the sphere's poles? Assume each point on the sphere has an equal chance of being hit.

—Daryl Cooper

3 Solve the following cryptic addition: SEVEN + THREE + TWO = TWELVE. We want the solution in which TWO is the product of two primes. No leading zeros; different letters are different digits; same letter is same digit. A computer is not necessary to solve this.

—*Mathematical Puzzling* by A. Gardiner

4 Al and Ben play a game that uses a supply of marbles marked 1, 2, 3, etc., and five saucers. Al puts marble 1 into a saucer. Ben puts marble 2 into a saucer. Al puts marble 3 into a saucer, and so on. The only restriction is that the differences between any pair of numbers in a saucer must be different from the differences between any other pair in that saucer. For instance, a saucer must not contain 1, 5, 14, and 18. If a player cannot place his number in any saucer, he loses. The game can go on for a long time, but what is the shortest possible game, and what are the numbers in the saucers when the game stops? Hint: When the game stops, all saucers have fewer than 4 marbles.

—Enigma by Stephen Ainley in *New Scientist*

5 Find the value of:
(√(2 √(2 √(2 √(2 √(2 . . .)))))

—*Crucible*

Bonus. N (an odd number) people own a bank. What is the minimum number of different locks that must be put on a safe so that when duplicate keys to these locks are distributed to the different owners, every majority contains a complete set of keys but no minority does? For example, 3 people need 3 locks with keys distributed as: 1-2, 2-3, 3-1; no single person can open the safe, but all pairs can. Express your answer as a function of N.

—Puzzle Corner by Allan Gottlieb in *Technology Review*

Double Bonus. In 2006, a teacher said to his class, an American hero was born X years ago and died at age Y, where Y minus its reverse equals the square of the cube root of (X - Y). If half of this hero's birth year is a

(Continued on page 47.)

BRAIN TICKLERS

(Continued from page 45.)

prime number, whose birthday was the teacher referring to, and what anniversary was 2006? —Aziz S. Inan

Postal mail your answers to any or all of the Brain Ticklers to Jim Froula, Tau Beta Pi, P.O. Box 2697, Knoxville, TN 37901-2697, or email to *BrainTicklers@tbp.org* plain text (no HTML, no attachments). The cutoff date for entries to the Summer column is the appearance of the Fall BENT during early October. The method of solution is not necessary, unless you think it will be of interest to the judges. We also welcome any interesting new problems that may be suitable for use in the column. The Double Bonus is not graded. Jim will forward your entries to the judges, who are: **H.G. McIlvried III,** *PA Γ '53*; **D.A. Dechman,** *TX A '57*; **J.L. Bradshaw,** *PA A '82*; and the columnist for this issue, —**F.J. Tydeman,** *CA Δ '73*.