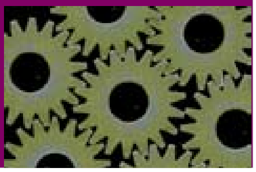

Brain Ticklers

RESULTS FROM WINTER 2007

Perfect

* Baines, Elliot "Chip" A., Jr.	NY	Δ	'78
* Brule, John D.	MI	B	'49
* Forest, Thomas M.	MI	Z	'87
* Griggs, James L., Jr.	OH	A	'56
* Hess, Richard I.	CA	B	'62
* Kimsey, David B.	AL	A	'71
Liu, Victor	CA	B	'07
* Mayer, Michael A.	IL	A	'89
Rasbold, J. Charles	OH	A	'83
* Schmidt, V. Hugo	WA	B	'51
* Stock, Daniel L.	OH	A	'80
* Stribling, Jeffrey R.	CA	A	'92
* Strong, Michael D.	PA	A	'84
* Venema, Todd M.	OH	E	'08
* Voellinger, Edward J.	Non-member		

Other

Adamo, Megan	Daughter of member		
Aron, Gert	IA	B	'58
Bachmann, David E.	MO	B	'72
* Beaudet, Paul R.	Father of member		
* Bower, Geoffrey	IL	A	'05
* Brana-Mulero, Francisco J.	PR	A	'74
Cole, Elizabeth	7th grader		
Corey, Patrick J.	WA	A	'58
* Couillard, J. Gregory	IL	A	'89
Dodini, Hana	CA	A	'08
* Foster, David E.	VA	B	'94
Jones, Donlan F.	CA	Z	'52
Mazeika, Daniel F.	PA	B	'55
Meerscheidt, Kyle	Husband of member		
* Nabutovsky, Joseph	Father of member		
* Pecsvaradi, Thomas	PA	Z	'64
* Quintana, Juan S.	OH	Θ	'62
Rentz, Peter E.	IN	A	'55
* Schleehauf, Martin W.	NY	N	'79
* Scholz, Gregory R.	PA	B	'00
* Spong, Robert N.	UT	A	'58
Van Houten, Karen	ID	A	'76

* Denotes correct bonus solution

WINTER REVIEW

The Winter Bonus (arranging seven points on a plane such that, for any three points, at least two points will be 1 cm apart) was solved by an unusually high percentage of entries. Looks like our readers excel at visualization.

SPRING SOLUTIONS

Readers' entries for the Spring problems will be acknowledged in the Fall BENT. Meanwhile, here are the answers:

1 The Pythagorean triangle is (693; 1,924; 2,045) with an area of 666,666. For a primitive Pythagorean triangle, $x=m^2–n^2$, $y=2mn$, and $z=m^2+n^2$, where m and n are relatively prime and of opposite parity. Therefore, $A=mn(m^2–n^2) = mn(m–n)(m+n)$. Now, for the area to be an integer consisting of the same digit, it must have a factor of 11 or 111 or 1,111 etc., and both m and n, as well as $m–n$ and $m+n$, must be factors of the area; that is m and n must be small multiples of the factors of an integer consisting of all 1s. A little trial shows that 11 (prime); 111 = 3×37; 1,111 = 11×101; and 11,111 = 41×271 don't work. Working with 111,111 = 3 × 7 × 11 × 13 × 37, we quickly find that 37–2(13)=11 and 37+2(13) = 63 = 7 × 9. Thus, m = 37 and n = 2(13) = 26 fit the requirements of the problem. Thus, $x=37^2–26^2=693$, $y=2(37)(26)=1,924$, $z=37^2+26^2=2,045$, and A = 693(1,924)/2 = 666,666.

2 The order of the grades was D (top) A E C B. Since no one predicted their own grade, then statement (a) was made by A or D; (b) by B or E; (c) by B or D; (d) by A or C; and (d) by D or E. Therefore, (d) must have been made by C, and (a) by A. Assume A is top; then A > C > B > E, but then no possibility for D gives us three FT and an FF for the other statements. Therefore, A cannot be top. A similar line of reasoning shows that C is not top, nor is either A or C bottom. Therefore, the only possibilities are B top, E bottom; E top, B bottom; and D top, B bottom. (Since a boy has the bottom grade, D can't be bottom.) A little trial and error shows that only the answer above gives a TT, three TFs, and an FF for the five statements.

3 There are 1,173 possible squares in the game of Quod. There are $(n\text{-}a+1)^2$ possible squares with a side of length a and sides parallel to the sides of an $n×n$ chessboard, and for each $a×a$ square there are a-1 possible squares, considering only squares with corners on its edges. Thus, the total number of squares is the sum from a = 2 to a = n of $(a\text{-}1)(n\text{-}a+1)^2$. This is easily evaluated using the well-known formulas for the sums of the powers of integers to give $n^2(n^2\text{-}1)/12$. From this, $4(n\text{-}2)+1$ must be subtracted to allow for the missing corners of the Quod board. With n = 11, N = 1,210 - 37 = 1,173.

4 The remainder is 2,006. Let N = 1(1!) + 2(2!) + 3(3!) + … + 222(222!) and let S = 1! + 2! + 3! + … + 222!. Adding and subtracting S from N gives N = (1 + 1)(1!) + (2 + 1)(2!) + (3 + 1)(3!) + … + (222 + 1)(222!) – S = 2! + 3! + 4! + … + 222! + 223! – S = 223! – 1. But, 2,007 = $3^2$223. Therefore, 2,007 divides 223!, which means that when 2,007 divides N, the remainder will be 2,006.

5 BANANA = 763636. Given 3 x APPLES > 2 × MELONS > 35 × PLUMS and 210 × PLUMS > 99 × LEMONS > 16 × APPLES. Multiplying the first of these by 6 and combining gives 18 × APPLES > 12 × MELONS > 210 × PLUMS > 99 × LEMONS > 16 × APPLES. Trying some partial inequalities, we see that 12 × MEL > 210 × PL. Since the max value of MEL is 987, we can deduce that PL < 56.4. Similarly, from 210 × PL > 99 × LEM, we find PL > 48.1. Thus, 56 > PL > 48. Using the value of 56 and the same relationship, we see that LEM < 118. Therefore, LEM = 10M and MEL = M01. Trying MEL = 801 shows this to be too low. Therefore, MEL = 901, and LEM = 109. A little thought will show that PL = 51. Using 210 × PL > 16 × APP and 18 × APP > 12 × MEL with the values just deduced shows that APP lies between 600 and 699. Therefore, A = 6, and APPLE = 65,510. Now, 12000 × MEL + 12 × ONS > 99000 × LEM + 99 × ONS. Solving shows that ONS < 241. Therefore, ONS = 23S, and MELON = 90123 and LEMON = 10923. From the relationship 12 × MELON > 210 × PLUM > 99 × LEMON, we find that PLUM = 5149. This makes S = 7 or 8; but only 8 works, making S = 8 and B = 7.

Bonus. Probability that the next ball is black = [55(6/6) + 154(6/7) + 280(6/8) + 420(6/9)]/909 = 677/909. [Refer to *Table A* below.]

Double Bonus. The best solution we have is N = 66. We have found thousands of solutions; one such solution is the four sets: [Refer to *Table B* below.]

NEW SUMMER PROBLEMS

1 The makers of Grunt, who claim that their aftershave is so appealing to women that users will have to fight them off, are puzzled that some men are using Phew, so they hired Judy to investigate. She boarded a bus to question the male passengers. The first man said, "I use Grunt." Then, the rest of the men in turn said, "I use Grunt, but the man you just interviewed uses Phew." Finally, the first man added, "The last man you interviewed uses Phew." Confused, Judy asked a few of the men how many of the male passengers used Phew. The tallest man said "19," and the man she had originally interviewed fourth after him said "24." The fattest man said "13," and the man she had originally interviewed fifth after him said "28." The nicest man said "24," and the man she had originally interviewed sixth after him said "13." If each man uses either Grunt or Phew, but only Grunt users tell the truth, how many men use Phew?

—Martin Hollis

2 On his trips to Monte Carlo, Sam always gambles exactly $50 each night on roulette. Usually, he just plunks all his chips down on black (result is double or nothing) and quits after one spin, regardless of the outcome. The probability of winning is 18/37. One night he decides to change his strategy by making $1 bets on black until he either loses the $50 or gains another $50. What is his probability of winning with his new strategy?

—*Technology Review*

3 Different letters are different digits; same letter is same digit throughout.

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             - - a -
     - -  )d - - - - x
           k - -
           -----
               - -
               t -
               ---
               m - -
               m - -
               -----
                 x - x
                 h - -
                 -----
                   k -
```

—Eric R. Emmet

4 On the island of Approxima, all measurements are only to the closest inch. Alex, who makes yardsticks, has realized that he doesn't have to put marks at each inch to still be able to measure all integral number of inches from 1 through 36, inclusive. Distances are measured between two marks, or between a mark and an end of the ruler, or between the two ends of the ruler. For example, a foot ruler only needs marks at 1, 2, 6, and 9 inches in order to measure all distances from 1 inch through 12 inches, inclusive. Using eight marks, where does Alex put them on his yardsticks?

—*The Numerology of Dr. Matrix* by Martin Gardner

5 You have eight objects of eight different weights, which have been partially ordered. That is, the order of the first four is known among themselves, and the order of the second four is known among themselves. What is the minimum number of weighings on a two-pan balance to ensure that the eight weights can be ordered in all cases?

—*Super-Puzzles*, Jean-Claude Baillif

Bonus. A student pilot, initially located at coordinates (x_0,y_0), rashly decides to fly under the Golden Gate Bridge, the ends of which are located at (-1,0) and (1,0). These end points subtend a visual angle Θ (vertex at the student's airplane), and all movement and measurements lie in the same X-Y plane. He flies along a path that maximizes the instantaneous rate of increase of Θ. What is the equation of his flight path? Assume the X-Y plane is parallel to a flat ocean.

—**Byron R. Adams**, *TX A '58*

Double Bonus. If A, B, and C are the measures of the angles of a plane triangle, what are the values of L and H in $L \leq \sin(3A) + \sin(3B) + \sin(3C) \leq H$ when equality holds? Prove your answers.

—Tenth U.S.A. Mathematical Olympiad (1981)

Postal mail your answers to any or all of the Brain Ticklers to: **Jim Froula, Tau Beta Pi, PO Box 2697, Knoxville, TN 37901-2697** or email plain text to: *BrainTicklers@tbp.org*. The cutoff date for entries to the Summer column is the appearance of the Fall Bent during early October. The method of solution is not necessary. We also welcome any interesting new problems that may be suitable for use in the column. The Double Bonus is not graded. Jim will forward your entries to the judges, who are: **H.G. McIlvried III**, *PA Γ '53*; **D.A. Dechman**, *TX A '57*; **J.L. Bradshaw**, *PA A '82*; and the columnist for this issue,

—**F.J. Tydeman**, *CA Δ '73*.

Table A

Assumed Number of White Balls	Probability of drawing 3 White Balls in a Row	Probability that Next Ball is Black
3	(3/9)(2/8)(1/7) =1/84 = 55/4,620	6/6
4	(4/10)(3/9)(2/8) = 1/30 = 154/4,620	6/7
5	(5/11)(4/10)(3/9) = 2/33 = 280/4,620	6/8
6	(6/12)(5/11)(4/10) = 1/11 = 420/4,620	6/9
Total	909/4,620	

Table B

Set	Members of Set
A	1, 2, 4, 8, 11, 16, 22, 25, 40, 43, 53, 66
B	3, 5-7, 19, 21, 23, 34, 35, 50-52, 63-65
C	9, 10, 12-15, 17, 18, 20, 54-62
D	24, 26-33, 36-39, 41, 42, 44-49