Brain Ticklers

The Pythagorean triangle is \((693;1,924;2,045)\) with an area of 666,666. For a primitive Pythagorean triangle, \(x^2 - m^2 = n\), when \(m\) and \(n\) are relatively prime and of opposite parity. Therefore, \(A = mn(m^2 - n^2) = mn(m - n)(m + n)\). Now, for the area to be an integer consisting of the same digit, it must have a factor of 11 or 111 or 1,111 etc., and both \(m\) and \(n\), as well as \(m - n\) and \(m + n\), must be factors of the area; that is \(m\) and \(n\) must be small multiples of the factors of an integer consisting of all 1s. A little trial and error shows that only \(b\) gives a TT, three FTs, and an FF for the five statements.

The order of the grades was D (top) A E C B. Since no one predicted their own grade, then statement (a) was made by A or D; (b) by B or E; (c) by B or D; (d) by A or C; and (d) by D or E. Therefore, (d) must have been made by C, and (a) by A. Assume A is top; then A > C > B > E, but then no possibility for D gives us three FT and an FF for the other statements.

The Pythagorean triangle is \((693;1,924;2,045)\) with an area of 666,666. For a primitive Pythagorean triangle, \(x^2 - m^2 = n\), when \(m\) and \(n\) are relatively prime and of opposite parity. Therefore, \(A = mn(m^2 - n^2) = mn(m - n)(m + n)\). Now, for the area to be an integer consisting of the same digit, it must have a factor of 11 or 111 or 1,111 etc., and both \(m\) and \(n\), as well as \(m - n\) and \(m + n\), must be factors of the area; that is \(m\) and \(n\) must be small multiples of the factors of an integer consisting of all 1s. A little trial and error shows that only \(b\) gives a TT, three FTs, and an FF for the five statements.

The order of the grades was D (top) A E C B. Since no one predicted their own grade, then statement (a) was made by A or D; (b) by B or E; (c) by B or D; (d) by A or C; and (d) by D or E. Therefore, (d) must have been made by C, and (a) by A. Assume A is top; then A > C > B > E, but then no possibility for D gives us three FT and an FF for the other statements.

There are 1,173 possible squares in the game of Quod. There are \((n-a+1)^2\) possible squares with a side of length \(a\) and sides parallel to the sides of an \(a\times a\) chessboard, and for each \(a\times a\) square there are \(a-1\) possible squares, considering only squares with corners on its edges. Thus, the total number of squares is the sum from \(a = 2\) to \(a = n\) of \((a-1)(a-a+1)^2\). This is easily evaluated using the well-known formula for the sums of the powers of integers to give \(n^2(n^2-1)/12\). From this, \(4(n-2)+1\) must be subtracted to allow for the missing corners of the Quod board.

The Winter Bonus (arranging seven points on a plane such that, for any three points, at least two points will be 1 cm apart) was solved by an unusually high percentage of entries. Looks like our readers excel at visualization.

Readers’ entries for the Spring problems will be acknowledged in the Fall BENT. Meanwhile, here are the answers:

\[\text{PLUMS} > 99\]
\textbf{B onus.} Probability that the next ball is black = \frac{155(6/6) + 154(6/7) + 280(6/8) + 420(6/9)}{909} = 0.677/909. [Refer to Table A below.]

\textbf{Double Bonus.} The best solution we have is \( N = 66 \). We have found thousands of solutions; one such solution is the four sets: [Refer to Table B below.]

**NEW SUMMER PROBLEMS**

1. The makers of Grunt, who claim that their aftershave is so appealing to women that users will have to fight them off, are puzzled that some men are using Phew, so they hired Judy to investigate. She boarded a bus to ask a few of the men how many of them use Phew. The male passengers used Phew. The female passengers did not. Judy was pretty sure that her fellow passengers were telling the truth, how many men use Phew?

   —Martin Hollis

2. On his trips to Monte Carlo, Sam always gambles exactly \$50 each night on roulette. Usually, he just plunks all his chips down on black (result is double or nothing) and quits after all his chips down on black (result is black = \[55(6/6) + 154(6/7) + 280(6/8) + 420(6/9)\]/909 = 677/909. [Refer to Table A below.]

   The probability of winning is \[ \frac{18}{37} \]. One night he decides to change his strategy by making \$1 bets on black until he either loses the \$50 or gains another \$50. What is his probability of winning with his new strategy?

   —\textit{Technology Review}

3. Different letters are different digits; same letter is same digit throughout.

   \[
   \begin{array}{cccc}
   - & - & a & - \\
   d & - & - & x \\
   k & - & - & - \\
   t & - & - & - \\
   m & - & - & - \\
   m & - & - & - \\
   x & - & x & - \\
   h & - & - & - \\
   - & - & - & k
   \end{array}
   \]

   —Eric R. Emmet

4. On the island of Approxima, all entries are not graded. Jim will forward your entries to the judges, who are: H.G. McIlvried III, D.A. Dechman, and J.L. Bradshaw, PA A '82; and the columnist for this issue, —F.J. Tydeman, CA A '73.

5. You have eight objects of eight different weights, which have been partially ordered. That is, the order of the first four is known among themselves, and the order of the second four is known among themselves. What is the minimum number of weighings on a two-pan balance to ensure that the eight weights can be ordered in all cases?

   —\textit{Super-Puzzles}, Jean-Claude Baillif

6. A student pilot, initially located at coordinates \((x_0,y_0)\), rashly decides to fly under the Golden Gate Bridge, the ends of which are located at \((-1.0,1.0)\) and \((1.0,1.0)\). These end points subtend a visual angle \(\Theta\) (vertex at the student's airplane), and all movement and measurements lie in the same \(X-Y\) plane. He flies along a path that maximizes the instantaneous rate of increase of \(\Theta\). What is the equation of his flight path? Assume the \(X-Y\) plane is parallel to a flat ocean.

   —\textbf{Byron R. Adams, TX A '58}

---

**Table A**

<table>
<thead>
<tr>
<th>Assumed Number of White Balls</th>
<th>Probability of drawing 3 White Balls in a Row</th>
<th>Probability that Next Ball is Black</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>( \frac{3(9)(2/8)(1/7)}{6} = 18/84 = 55/4,620 )</td>
<td>6/6</td>
</tr>
<tr>
<td>4</td>
<td>( \frac{4(10)(3/9)(2/8)}{6} = 1/30 = 154/4,620 )</td>
<td>6/7</td>
</tr>
<tr>
<td>5</td>
<td>( \frac{5(11)(4/10)(3/9)}{6} = 233/280 = 420/4,620 )</td>
<td>6/8</td>
</tr>
<tr>
<td>6</td>
<td>( \frac{6(12)(5/11)(4/10)}{6} = 1/11 = 420/4,620 )</td>
<td>6/9</td>
</tr>
<tr>
<td>Total</td>
<td>909/4,620</td>
<td></td>
</tr>
</tbody>
</table>

---

**Table B**

<table>
<thead>
<tr>
<th>Set</th>
<th>Members of Set</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1, 2, 4, 8, 11, 16, 22, 25, 40, 43, 53, 66</td>
</tr>
<tr>
<td>B</td>
<td>3, 5-7, 19, 21, 23, 34, 35, 50-52, 63-65</td>
</tr>
<tr>
<td>C</td>
<td>9, 10, 12-15, 17, 18, 20, 54-62</td>
</tr>
<tr>
<td>D</td>
<td>24, 26-33, 36-39, 41, 42, 44-49</td>
</tr>
</tbody>
</table>

---

\textit{Brain Ticklers to:} Jim Froula, Tau Beta Pi, PO Box 2697, Knoxville, TN 37901-2697 or email plain text to: BrainTicklers@tbp.org.