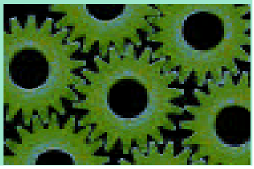

Cannonballs & Other Brain Ticklers

RESULTS FROM WINTER 2003

Perfect	
*Anderson, Paul MacLean	son
*Baines, Elliot A., Jr.	NY Δ '78
Benson, Gary R.	WA A '60
*Biggadike, Robert H.	AR A '58
*Bradshaw, John L.	PA A '82
*Brzezinski, Mark A.	OH B '00
Capelli, Ronald B.	MI Γ '73
*Cole, Todmund E.	CA Σ '93
*Creutz, Michael J.	CA B '66
Creutz, Edward C.	PA Γ '36
Duker, Rachayl Novoseller	NY A '97
*Easwar, Siddharth S.	GA A '04
*Fletcher, David Q.	CA Λ '67
*Fuemmeler, Jason A.	OH Θ '00
*Garnett, James M.	MS A '65
Griggs, James L., Jr.	OH A '56
*Jones, Donlan F.	CA Z '52
*Kimsey, David B.	AL A '71
Lawes, C. Peter	MA B '58
McNeill, John A.	husband
*Mayer, Michael A.	IL A '89
Minnick, Michael V.	SC A '81
*Mrozek, Paul M.	MI H '91
Peithman, Harlan W., Jr.	IL A '53
Rasbold, J. Charles	OH A '83
*Santee, Steven G.	VA A '87
Santee, Connie	wife
*Schmidt, V. Hugo	WA B '51
Segars, Roy, Jr.	GA A '70
Shelton, William T.	GA A '93
*Spong, Robert N.	UT A '58
*Strong, Michael D.	PA A '84
*Thaller, David B.	MA B '93
Tom, Danny T.	CA A '99
White, R. Dudley	VA A '76
Other	
*Alexander, Jay A.	IL Γ '86
Alderson, William S.	MI H '43
Allred, Marjorie H.	UT Γ '79
Alverson, Robert L.	OH B '85
Anderson, Kurt E.	KS Γ '90
Anidi, Chibueze	non-member
Aron, Gert	IA B '59
Bachmann, David E.	MO B '72
Balleisen, Charles E.	PA Z '33
Bhave, Prakash V.	CA A '98
Chai, Bob Being Sho	NY A '97
*Couillard, J. Gregory	IL A '89
Davis, Edward T.	AZ Γ '91
Erman, Andrew M.	NJ B '92
Ford, Robert A.	CA K '94
*Garside, Jeffrey J.	WI E '90
Gillis, Peter P.	RI A '53
Ginat, Daniel T.	NY A '01
Handley, Vernon K.	GA A '86
*Henry, Jerry R.	MO A '63
Henze, Edward D.	IL Γ '50
Hewitt, Sara T.	MD B '01
Kaczynski, Daniel R.	WI B '88
Kade, Jon G.	MI H '04
Kern, Peter L.	NY Λ '62
Kopp, Robert W.	WI A '51
Kronenfeld, Scott W.	WI A '03
Zeppenfeld, Johannes	non-member
La Budde, C. Donald	non-member
Lalinsky, Mark A.	MI Γ '77
*Langenderfer, Brian A.	OH Z '85
Levitch, Roy N.	NY Γ '61
Lew, Thomas M.	TX Δ '84
Lipson, Shane M.	PA Δ '98
Marks, Lawrence B.	NY I '81
Marrone, James D.	IN A '87
Marrone, James I.	IN A '61
*Mitchell, Donald B.	CA Γ '59
O'Keeffe, Claudia Marx	MD B '92
*Prince, Lawrence R.	CT B '91
Pugh, Kenneth H.	NC Γ '70
Quintana, Juan S.	OH Θ '62
*Rentz, Peter E.	IN A '55
Roggli, Victor L.	TX Γ '73
Roggli, Linda	wife
*Shinham, C. Robert	MD A '71
Smith, Timothy A.	MO A '98
Snyder, M. Duane	IA B '63
*Stribling, Jeffrey R.	CA A '92
Valko, Andrew G.	PA Λ '80
Verser, Rocke	KS Γ '77
*Wakeley, Thomas A.	PA Z '71
*Weinstein, Stephen A.	NY Γ '96

*denotes correct Bonus solution

WINTER REVIEW

Trial and error was used for part of the solution given in the Spring BENT for Winter Brain Tickler No. 4 (concerning the weights of five schoolgirls). Steven R.VanShaar, *UT Γ '00*, has submitted a more elegant solution: After finding that $w_3 = 95$, realize that the sum of the weights of the lightest and third-lightest girls corresponds to the second-lowest measurement—similarly for the third lightest and the heaviest girl. This leads directly to $w_1 = 91$ and $w_5 = 101$. The lowest and highest measurements then yield $w_2 = 92$ and $w_4 = 99$.

SPRING ANSWERS

Here are the solutions for the Spring problems. Spring entries will be acknowledged in the Fall issue.

1. You were given some clues to solve a right triangle. For a Pythagorean right triangle, x = $k(m^2 - n^2)$, $y = 2kmn$, and $z = k(m^2 + n^2)$, where are k, m, and n (and thus x, y, and z) are integers. Therefore, perimeter $P = 2km(m + n)$ and area $A = k^2mn(m^2 - n^2)$. Let ratio $R = A/P$; therefore, $R = kn(m\text{-}n)/2$ or $2R = kn(m\text{-}n)$. For R to be an integer, we get solutions: $n = 1$, $m = 2$, $k = 2R$, $x = 6R$, $y = 8R$, $z = 10R$; and $n = 2$, $m = 3$, $k = R$, $x = 5R$, $y = 12R$, $z = 13R$. As long as $R \leq 7$, this gives different triangles with sides less than 100. For $R = 9$, we get $x = 54$, $y = 72$, $z = 90$; but we also have the solution $k = 3$, $m = 5$, $n = 2$, $x = 63$, $y = 60$, $z = 87$. Therefore, only $R = 8$ gives a unique solution and $x =$ **48,** $y =$ **64,** and $z =$ **80**.

2. This involves a couple of syzygies of Venus, Earth, and Mars. Let T = time in Earth days until the planets realign. For this to occur, each planet must make an integral number of half revolutions around the Sun plus the same fraction of a revolution. Let $2T/225 = n_v + f$, $2T/365 = n_e + f$, and $2T/687 = n_m + f$. From these we see that $2T/225 - 2T/365 = 280T/(225 \times 365) = n_v - n_e = p$ and $2T/365 - 2T/687 = 644T/(365 \times 687) = n_e - n_m = q$, where p and q are integers. Solving for T gives $T = 225(365)p/280 = 365(687)q/644$. Therefore, $p/q = 458/345$. Since p and q must be integers, we have $p = 458$ and $q = 345$. Therefore, $T = 225(365)(458)/280$ Earth days or $10{,}305/28 =$ **368 1/28** Earth years.

3. "Find a 10-digit integer containing the digits 0 through 9 each once such that, for n equal to 1 through 10, the integer formed by the first n digits is divisible by n." For a number to be divisible by 10, the last digit must be 0. Since the first five digits must be divisible by 5, the 5th digit must be 5. Also, the 2nd, 4th, 6th, and 8th digits must be even. Since this exhausts the even digits, the number must have alternating odd and even digits, starting with an odd digit. Since the sum of the digits is divisible by 9, the 1st and 9th digits can be any odd digit. Other requirements are that the sum of the first three digits is divisible by 3, the sum of the 4th, 5th, and 6th digits must be divisible by 3, the two-digit number formed by the 3rd and 4th digits must be divisible by 4, and the three-digit number formed by the 6th, 7th, and 8th digits must be divisible by 8. Thus, the 4th digit can't be 4, and the 8th digit can't be 8. When we add the requirement that the first seven digits be divisible by 7, there is only one possibility: **3,816,547,290.**

4. This deduction problem uses VIER and NEUN to represent encrypted squares. From a table of squares, we see that there are only nine possible (VIER, NEUN) solutions: (4761, 5625), (1764, 5625), (7056, 1521), (4356, 1521), (6241, 9409), (1369, 4624), (1369, 5625), (7569, 1681), and (7569, 4624). The requirement that VIER uniquely determine NEUN eliminates the last four of these solutions, and the requirement that NEUN uniquely determine VIER eliminates the first four. Therefore, VIER = **6241** and NEUN = **9409**. Incidentally, vier is 4 and neun is 9 in German.

5. You are to take most of the marbles from an urn that initially contains 100 black and 100 white ones. Since each turn reduces the number of marbles in the urn, there must eventually be fewer than three marbles in the urn. The only move that decreases the number of white marbles in the urn is drawing three white marbles and replacing one. Thus, the number of white marbles is always even and must eventually be reduced to two. At this point in the process, if any black marbles are left, only black marbles can be eliminated, and this will continue until no black marbles are left. Thus, the last two marbles must be white with **100%** probability.

Bonus. The task is to minimize the area between a parabola and a perpendicular to it. The equation of the parabola is $y = x^2$. The slope at point P is $dy/dx = 2x_P$. The slope of the perpendicular at the same point is $-1/(2x_P)$, and the equation of the perpendicular is $y = -x/(2x_P) + x_P^2 + 1/2$. Solving this simultaneously with the equation for the parabola gives the other point Q of intersection as $x_Q = -1/(2x_P) - x_P$. Now the area we are trying to minimize equals the area of the trapezoid bounded by the four points $(x_P, 0)$, (x_P, x_P^2), (x_Q, x_Q^2), and $(x_Q, 0)$ minus the area under the parabola between Q and P. The area of the trapezoid is given by $A_T = (x_P - x_Q)(x_P^2 + x_Q^2)/2$, and the area under the parabola is equal to the integral of x^2 between Q and P or $A_C = (x_P^3 - x_Q^3)/3$. Combining these results, we get $A = A_T - A_C = (x_P - x_Q)^3/6$. Differentiating gives $dA/dx_P = (x_P - x_Q)^2(1 - dx_Q/dx_P)/2$. Setting $dA/dx_P = 0$ gives $dx_Q/dx_P = 1$, leading to $-1 + 1/(2x_P^2) = 1$. Therefore, point P is at **(1/2, 1/4).** The other point Q of intersection is (–3/2, 9/4), and the area bounded by the perpendicular and the parabola is 4/3.

Computer Bonus. This is the smallest, and reported to be the only, nonpalindromic integer whose cube is a palindrome: 2201^3 = **10662526601**.

NEW SUMMER PROBLEMS

1. Two park rangers at a Civil War battle site are assigned the job of counting a pile of cannonballs arranged in the shape of a regular tetrahedron. The first ranger says, "We will have to disassemble the pile to count the cannonballs in the middle." The second ranger, a Tau Bate, replies, "That won't be necessary. Just tell me the number n of balls along one side of the bottom layer, and I can calculate the total number in the stack." What closed-form formula did he use?

—John R. Sellars

2. An urn contains ten balls, numbered 1 through 10. Three balls are drawn at random without replacement. What is the probability that the values on the three balls can be sides of the same triangle?

—Unknown

3. Solve the following cryptics. Each letter has the same value in both cryptics, and each letter also represents a different digit.

FIVE – FOUR = ONE
FIVE + FIVE = EVEN

—**M. Yagi,** *CA* I *'96*

4. Given a one-meter cube, drill a hole of diameter 0.25m with its axis along a diagonal of the cube. What is the volume of the material removed?

—SW Missouri State U. Math. Dept.

5. If the Earth's movement matched the Gregorian calendar perfectly, how many degrees per day would the Earth rotate relative to the "fixed" stars? Give the answer exactly.

—**Byron R. Adams,** *TX* A *'58*

Bonus. This is a variation of a problem that appeared in the Winter 2003 column. A package-handling company will ship a package with a length plus girth of up to 108 inches. A customer wishes to mail a right-circular cone of maximum volume consistent with the given limitation. If the girth is defined as the shortest loop of string through which the cone will pass and the length is defined as the maximum dimension of the cone, what are the altitude and base diameters of the cone?

—**R. Wilson Rowland,** *MD* B *'51*

Computer Bonus. What are the smallest and largest perfect squares that use each of the digits 0 to 9 once and only once. A leading 0 is not allowed.

—Technology Review

Send your answers to any or all of the Summer Brain Ticklers to:
Jim Froula, Tau Beta Pi
P. O. Box 2697
Knoxville, TN 37901-2697
If your answers are text only, they may be emailed to: *BrainTicklers@tbp.org.*

Details of your calculations are not required, and the Computer Bonus is not graded. The cutoff date for entries is the appearance of the Fall issue in early October. Jim will forward your entries to the judges:
H.G. McIlvried III, *PA* Γ *'53,*
F.J. Tydeman, *CA* Δ *'73,*
D.A. Dechman, *TX* A *'57,* and the columnist for this issue,

—**R. Wilson Rowland,** *MD* B *'51.*