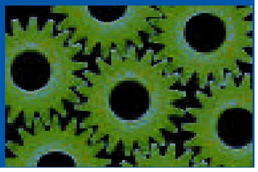

BRAIN TICKLERS

Brain Ticklers Celebrates 50th Year!

WINTER REVIEW

For the double bonus problem, Jeffrey R. Stribling, *CA* Α *'92*, submitted an alternate construction for finding the midpoint between two points. Whereas our method required eight steps, he did it in seven.

FIFTIETH ANNIVERSARY

We wrap up the observance of the 50th anniversary of this column by mentioning those who have made it possible. The column is managed by a panel of four judges who generate, select, and edit the problems and solutions and grade the submitted answers. Fourteen Tau Bates have served as judges:

1951-52	Robert H. Nagel	NY Δ '39
1952-55	Richard B. Moyer	PA Α '41
1953-58	John E. Lauer	MI Γ '50
1953-58	Warren E. Mathews	MA Β '44
1956-82	John W. Langhaar	PA Α '33
1959-63	Robert L. Patton	TX Δ '33
1959-	Howard G. McIlvried III	PA Γ '53
1963-75	Lawrence Israel	NY Α '56
1975-82	Daniel Steinhorn	MA Δ '56
1976-	Frederick J. Tydeman	CA Δ '73
1983-93	Byron R. Adams	TX Α '58
1983-	R. Wilson Rowland	MD Β '51
1995-96	Kevin M. Stewart	NJ Δ '77
1997-	Don A. Dechman	TX Α '57

Note that Dr. McIlvried has served 43 years, much longer than anyone else. Since 1982 he has been the head judge. He is to be especially honored for his long and faithful brain tickling. The other incumbents have set records also—Tydeman was the earliest-after-graduation to start, Dechman was the latest-after-graduation to start, and Rowland is the latest-after-graduation to serve. Last but not least, the staff of The Bent assembles and prints it on time.

SPRING ANSWERS

Here are the solutions for the Spring problems. Spring entries will be acknowledged in the Fall issue.

1. Pam's house number is 9. First assume C says that the number, N, has two digits; then if it is a square, it must be 16, 25, 36, 49, 64, or 81. If it is divisible by 3, D's answer would not help, and if it is divisible by 5, R would know it is 25 and not need Pam's answer. If it is not a square, then there are so many possibilities that knowing whether it was divisible by 3 or 5 wouldn't help. Therefore, N must be a single digit. Now, if it is a square, D's answer would not help, and if it is divisible by 3, it would have to be 9, and other answers would not be needed. Thus, N is a non-square. Now, if N is divisible by 3, D's answer would not help. Therefore, N is not divisible by 3 and must be 5, 7, or 8. After D's answer, R concludes that N is 7 or 8, which P's answer clarifies. However, since A and B both lied, we know that N is a single digit square divisible by 3. Thus, it must be **9.**

2. There must be 2 five-ovatnec, 2 ten-ovatnec, 31 twenty-ovatnec, and 7 fifty-ovatnec coins with a total value of 10 oseps. We have $5N + 10D + 20Q + 50H = 1{,}000$ where N, D, Q, and H are all prime. Since the total is even, N must be 2, the only even prime. Also, $H < 20$. Therefore, $10D + 20Q = 990 - 50H$, or $D + 2Q = 99 - 5H$. Since we want the smallest number of coins, we start with $H = 19$ and try successively smaller primes. Assuming H is an odd prime, we see that $D + 2Q$ must be even; D must be 2. $H = 7$ is the first value that gives a prime Q. $Q = [99 - 5(7) - 2]/2 = 31$, for a total of **42 coins.**

3. YELLOW + YELLOW + RED = ORANGE
143329 +143329 +846 = 287504

4. The 1,234,567,890th permutation in the alphabetical listing is
CHMDBELIJGKFA.
A 13-letter sequence starting with A would contain 12! = 479,001,600

permutations. Thus, the sequence starts with C and has already used 958,003,200 permutations, leaving a remainder of 276,564,690 permutations. A 12-letter sequence starting with A would contain 11!= 39,916,800 permutations. Thus, the second letter of the sequence is H and has used an additional 239,500,800 permutations, since A has previously been used. This leaves a remainder of 37,063,890 permutations. This process can be repeated to yield the full sequence listed above.

5. Let r = length of red candle and w = length of white candle. Then, $r_0 = w_0 + 2$, where the subscript 0 refers to the initial length. We know that the red candle burned for 8 hrs. and the white one for 8.75 hrs. Since each candle burned at its own constant rate, we have $r = r_0 - at$ and $w = w_0 - b(t - 0.25)$, where $t = 0$ at 4 p.m., $r = 0$ at $t = 8$, and $w = 0$ at $t = 9$. Therefore, $a = r_0/8$, $b = w_0/8.75$, $r = r_0(1 - t/8)$, and $w = 4w_0(9 - t)/35$. Setting the equations for r and w equal to each other at $t = 4$, we get $r_0 = 8w_0/7$. But, $r_0 = w_0 + 2$. Solving these two equations simultaneously, we find that initially the red candle was **16 cm** long and the white candle was **14 cm** long.

Bonus. For a satellite in Earth orbit, total energy $E = K + U$ (where K is kinetic energy and U is potential energy) remains constant at all points in the orbit. For a circular orbit, $K = mv^2/2$, $U = -GMm/r$, and centripetal force mv^2/r is equal to gravitational force GMm/r^2 (where v is velocity, r is the radius of the orbit, $M = 5.983\_10^{24}$ kg is the Earth's mass, m is the satellite's mass, and $G = 6.67\_10^{-11}$ m^3/kg s^2 is the universal gravitation constant). Therefore, $K = GMm/2r$ and $E = -GMm/2r$.

For an elliptical orbit, we can replace r with a, where a is the major semi-axis, without changing the validity of the equation for E. For any orbit, $K = E - U$. Substituting the above values and solving for v gives $v = [GM(2/r - 1/a)]^{0.5}$, which reduces to $v = (GM/r)^{0.5}$ for a circular orbit.

Subscripts used in the following analysis are: i is the initial orbit, g is the geosynchronous orbit, t is the transfer orbit, a is apogee, and p is perigee. We first find $v_i = (GM/r_i)^{0.5}$. Since, $r_i = 400 + 6{,}378 = 6{,}778$ km, $v_i = 7{,}673$ m/s. We next find the radius of the geosynchronous orbit. Note, that for a circular orbit, $v = 2\pi r/T$, where T is the period. Substituting this into the equation for v gives $r = [GMT^2/(4\pi^2)]^{1/3}$. For a geosynchronous orbit, $T = 86{,}164$ s; therefore, $r_g = 42{,}181$ km and $v_g = 3{,}076$ m/s.

Finally, we examine the elliptical transfer orbit. For this orbit, $a_t = (6{,}778 + 42{,}181)/2 = 24{,}480$ km.

We can now calculate that $v_{tp} =$ 10,072 m/s and $v_{ta} = 1{,}619$ m/s.

Therefore, at the low orbit the satellite's velocity must be increased by 10,072 - 7,673 = **2,399 m/s** to boost it into the transfer orbit, and then at apogee, it must be increased by 3,076 - 1,619 = **1,457 m/s** to put it into geosynchronous orbit.

Double Bonus. Standard analysis leads to difficulties with the coin-toss game, because of the unsymmetrical outcomes for A and B and because any given game can potentially involve an infinite number of coin flips. Thus, a different approach is needed. Consider that every time B flips a coin, he has a 0.5 probability of winning \$1 and a 0.5 probability of losing \$1. Therefore, the expected value of his winnings is 0.5(1) + 0.5(-1) = 0; so as far as B is concerned, the game is fair. If the game is fair for B it must also be fair for A. Thus, the conclusion is that the game is **fair**.

NEW SUMMER PROBLEMS

1. In June 2001, my wife and I celebrated our golden wedding anniversary. To our surprise, the anniversary occurred on the same day of the week as the wedding 50 years ago. I wondered if this is a common occurrence. Consider the 52 years from 1900 through 1951. What is the exact probability that, for a randomly selected wedding date in this period, the golden anniversary fell on the same day of the week?

— R. Wilson Rowland, *MD* B *'51*

2. Solve this cryptic addition, with ELEVEN divisible by 11:

THREE + THREE + FIVE = ELEVEN

— Howard G. McIlvried III, *PA* Γ *'53*

3. As of 1999, 71 movies won Academy Awards for best picture. Of these, 33 appeared in the American Film Institute's list of 100 best pictures of all time, celebrating the centennial of the industry. These facts can be represented by a Venn diagram, in which sets are represented by circles with areas proportional to the number of members in the sets. If the 33 films in both categories are represented by the overlapping area of two such circles, what is the distance between centers of the two circles? Assume that the larger circle has a diameter of 10 cm.

— Craig K. Galer, *MI* A *'77*

4. As each of three perfect logicians enters a room, a letter is whispered into his or her ear. When all three are in the room, they are told that their three letters spell one of the following words: HOE, OAR, PAD, TOE, or VAT. The first logician is asked, "Do you know the word?" and replies, "Yes." The second is asked the same question and also replies, "Yes." Finally the third is asked and also says, "Yes." What is the word?

— *Mind Teasers* by G.J. Summers

5. What is the smallest positive integer that has at least 1,000 different integer factors? For example, 12 has six factors: 1, 2, 3, 4, 6, and 12.

— Frederick J. Tydeman, *CA* Δ *'73*

Bonus. A horizontal water pipe, 1 m in diameter, runs through a vertical manhole that is 2 m in diameter, in such a way that the center line of the pipe intersects the center line of the manhole. What volume of the pipe lies within the manhole?

— *Mathematical Visitor*

Computer Bonus. Find the largest four-digit number that, when squared, yields an answer containing only digits in the original number.

— Don A. Dechman, *TX* A *'58*

The judges are: H.G. McIlvried III, F.J. Tydeman, D.A. Dechman, and the columnist for this issue,

— R. Wilson Rowland, *MD* B *'51.*