BRAIN TICKLERS

Results From Fall

Perfect Scores

Berthold, Kristopher D.	TX	Β	'04
Bohdan, Timothy E.	IN	Γ	'85
*Couillard, J. Greg	IL	Α	'89
*Gibbs, Kenneth P.	MO	Γ	'76
Gulian, Franklin J.	DE	Α	'83
Gulian, Joseph D.	Son of member		
Johnson, Mark C.	IL	Α	'00
*Kimsey, David B.	AL	Α	'71
*Norris, Thomas G.	OK	Α	'56
Norris Jr., Thomas G.	PA	Γ	'79
Scott, Darrell J.	NC	Δ	'82
Slegel, Timothy J.	PA	Α	'80
Spong, Robert N.	UT	Α	'58
Zison, Stanley W.	CA	Θ	'83

Other

Bannister, Kenneth A.	PA	Β	'82
Bertrand, Richard M.	WI	Β	'73
Capelli, Ronald B.	MI	Γ	'73
Chatcavage, Edward F.	PA	Β	'80
Costantino, John T.	NJ	Α	'79
Crouse, John M.	PA	Β	'74
Gaston, Chuck A.	PA	Β	'61
Grewal, Kalwant S.	TX	Η	'73
Griggs Jr., James L.	OH	Α	'56
Handley, Vernon K.	GA	Α	'86
Janssen, Jim R.	CA	Γ	'81
Jordan, R. Jeffrey	OK	Γ	'00
Klaver, Naftali	Son of member		
Marks, Lawrence B.	NY	Ι	'81
Hertz, Caryn M.	NY	Ι	'81
Marks, Benjamin	Son of member		
McHenry, S. Dale	MO	Β	'81
Parks, Christopher J.	NY	Γ	'82
Riedesel, Jeremy M.	OH	Β	'96
Roggli, Victor L.	TX	Γ	'73
Schmidt, V. Hugo	WA	Β	'51
Shamblin, G. Richard	FL	Α	'72
Spring, Gary S.	MA	Ζ	'82
Spring, Mitchell G.	Son of member		
Summerfield, Steven L.	MO	Γ	'85
Voellinger, Edward J.	Non-member		

*Denotes correct bonus solution

Fall Review

The Fall puzzles were harder than usual. Less than 1/4 of the Bonus answers (space ship) submitted were correct. Only 2/3 of the answers about policy numbers (#2) were correct. The remaining problems had 84 percent or better correct answers.

Winter Answers

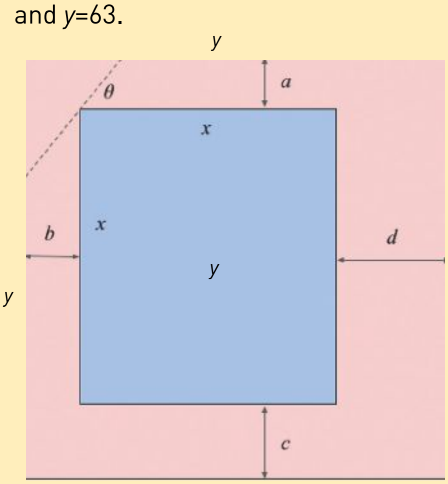
1: The solution to the cryptarithm is **12642325–9760233=2882092.** Writing the problem as CORNELL+ESSENCE=BERKELEY, it is obvious that B=1 and C=9, since C+E+c1=10+E only works if we have a carry and C=9. In the middle column, we have either N+E+c4=10+E or N+E+c4=E. The former will not work since N≠9, so N=c4=0. c5=1 because E≠L, so now 1+E=L. L+C+c6=10+E, and since C=9, c6=0. This implies that L+E<9, and the only viable (E,L) pairs are (3,4) and (2,3). If E=3 and L=4, then Y=7. From the third column, R+S=10+K or R+S=K, and with the numbers 2,5,6,8 available, only R=2, S=6, and K=8 work, which forces O=5. Since O+S=10+R implies 11=12, this is not a legal (E,L) pair. We choose E=2 and L=3, so Y=5. Again, R+S=10+K or R+S=K, this time with the numbers 4,6,7,8 available. Only R=6, S=8, and K=4 work, which forces O=7. O+S+c2=10+R implies c2=1 and we have a consistent solution.

2: The smallest value in each expression for N is $\mathbf{2^{105}3^{70}5^{126}7^{120}}$. If we let $N/2=a^2$, $N/3=b^3$, $N/5=c^5$, and $N/7=d^7$, we want to find values of a, b, c, and d such that $2a^2=3b^3=5c^5=7d^7$. In each expression for N, there must be terms of $2^{3\cdot5\cdot7\cdot i}$, $3^{2\cdot5\cdot7\cdot j}$, $5^{2\cdot3\cdot7\cdot k}$, and $7^{2\cdot3\cdot5\cdot l}$, that is, 2^{105i}, 3^{70j}, 5^{42k}, and 7^{30l}, for the smallest possible integers i, j, k, and l such that one less than each of the exponent terms is divisible by the base. For i=1, 105–1=104 is divisible by 2. For j=1, 70–1=69 is divisible by 3. For k=1, 42–1=41 is not divisible by 5, so we must choose k=3. Similarly, we choose l=4, leading to an answer of $N=2^{105}3^{70}5^{126}7^{120}$.

3: Beth was Honduran, wore ivory, and left at 9AM; Carol was English, wore khaki, and left at 10AM; Ann was German, wore lavender, and left at 11AM; Doris was French, wore jade, and left at 12 noon. We are told that it must be Carol or Doris at 10AM, wearing khaki or lavender. So, this cannot be the time slot one hour after the girl (Ann or Beth) who was not wearing khaki and was not English or French left. The only possible time for this is 11AM, meaning that at 12 noon the girl was not wearing ivory or jade. At 12 noon, the girl must be Ann, Beth, or Doris, and on the tabulation, only the name of the girl at 9AM or 12 noon can be correct. If it is Ann at 9AM, then Beth and Doris must be at 11AM and 12 noon respectively, which is impossible. So, Doris is at 12 noon, Carol is at 10AM, Beth is at 9AM, and Ann is at 11AM. Beth is from Honduras, and Ann must be from Germany, which is correct on the tabulation. Then it follows that Carol and Doris are English and French, respectively. Since it is impossible for any of the final three times to have the correct color on the tabulation, Beth must wear ivory, and it follows that Doris wears jade, Carol wears khaki, and Ann wears lavender.

4: There are **703,098,107,712,000** unique passcodes. One approach is to consider the number of times a given digit 0-9 appears in the passcode, and break the problem into groups based on the patterns of the quantities of various digits. In this way, we can create 11 groups of passcodes. Those where 1 digit appears 7 times and the rest once, another group where 1 digit appears 6 times, another digit appears 2 times and the rest once, etc. For succinctness, if we suppress digits which appear only once, the eleven groups can be expressed as (7), (6)(2), (5)(3), (4)(4), (5)(2)(2), (4)(3)(2), (3)(3)(3), (4)(2)(2)(2), (3)(3)(2)(2), (3)(2)(2)(2)(2), (2)(2)(2)(2)(2)(2). The first group has $16!/7! \times C(10,1)$ passcodes. The second group has $16!/(6!2!) \times C(10,1)C(9,1)$ passcodes. Continuing each subsequent group contributes $16!/(5!3!) \times C(10,1)C(9,1)$, $16!/(4!4!) \times C(10,2)$, $16!/(5!2!2!) \times C(10,1)C(9,2)$, and so forth. Summing these 11 quantities together gives the desired result above.

5: The original rug measures **63 feet** on a side. For corner cuts (as exemplified by the attached diagram) at an angle θ from horizontal, the clipped area will be of the form $\frac{1}{2}(b+a\cot\theta)(a+b\tan\theta)$. Using calculus to minimize this area as a function of θ leads to the not-surprising result that $\tan\theta = a/b$ and one corner's clipped area of $2ab$; this can be extended to the other four corners with appropriate exchange of variables. The ratio of the total red area over the clipped red area will be 10, and is given by the expression $(x(a+b+c+d)+ab+ad+bc+bd)/(2ab+2ad+2bc+2bd)=10$. Noting that $a+c=b+d$ allows us to see the total clipped area can be written as $2(a+c)^2$ and simplifying the previous expression gives $x/(a+c) + \frac{1}{2} = 10$, or $x = 9.5(a+c)$. If the clipped area is an integral number of square yards, then $a+c$ must be a multiple of 3, so $x = 28.5k$. The smallest integer k which makes x (and also y) an integer is $k=2$, yielding dimensions of $x=57$ and $y=63$.



BONUS: It takes $(\pi/2)\sqrt{(L/\mu g)}$ seconds to come to a stop at the desired position when the initial velocity is $\sqrt{(L\mu g)}$. $F=\mu N=ma$ where we consider only the portion of the rod $x(t)$ above the rough surface to contribute to the equation, and assuming a uniform rod we have $\mu g(-x/L)m=ma$. Cancelling the masses gives a familiar differential equation $\ddot{x}=-x(\mu g/L)$. With the initial conditions that $x(0)=0$ and $\dot{x}(0)=v_0$, the solution is of the form $x(t) = v_0\sqrt{(L/\mu g)}\sin(t\sqrt{(\mu g/L)})$ and $\dot{x}(t)=v_0\cos(t\sqrt{(\mu g/L)})$. To stop with the end of the rod at the beginning of the rough patch requires $0 = v_0\cos(t\sqrt{(\mu g/L)})$ so $t=(\pi/2)\sqrt{(L/\mu g)}$ and $L = v_0\sqrt{(L/\mu g)}\sin(\pi/2) = v_0\sqrt{(L/\mu g)}$ so $v_0=\sqrt{(L\mu g)}$.

COMPUTER BONUS:

The first ten 6th cousins of Giuga are: **14, 21, 182, 1463, 14098, 18802, 24695930, 49305333, 55388487, and 1925224630**.

New Spring Problems

On September 3, 2021, friend and former judge **Don Dechman,** *TX A '57*, passed away. In Don's memory, the first and last puzzles of this issue are his.

1: Cable Length Dip

A flexible cable was hung across the Black Canyon between two points that were exactly 1 km apart at the same elevation. During the cool night, the cable length contracted by a small amount, causing the dip (the height difference between the highest and lowest points) to decrease by the same amount. What was the length of the cable?

—**Don A. Dechman,** *TX A '57*

2: Tricky Cryptarithm

In the following base 12 cryptic addition, what is the minimum number of TRICKYs that result in a solution? The usual rules of cryptic addition apply.

TRICKY + TRICKY + ... + TRICKY + BRAIN = TICKLERS

—**Howard G. McIlvried,** *PA Γ '53*

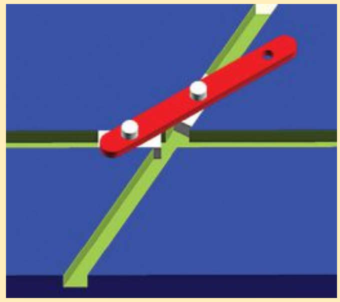
3: Find the Ace

Starting with a standard 52 card deck and drawing cards randomly, what is the expected number of cards that must be drawn to get an ace?

—*The Theory of Gambling and Statistical Logic* by Richard A. Epstein

4: Trammel Curve

In the trammel below, the holes in the red rod are spaced 10 cm apart, center to center. What is the equation of the curve traced by the unoccupied hole as the rod makes a complete orbit in the apparatus? Assume the x-axis is along the horizontal slot and that the origin is at the intersection of the two slots, which are separated by an angle of 60 degrees.



—Puzzle Corner by Allen Gottlieb in *Technology Review*

5: Toss the Twos & Fives

In the game "Drop Dead," you roll a number of six-sided dice. If a roll does not include any 2s or 5s, you add the sum of the dice to your score and roll all of the dice again. If your roll does include 2s or 5s, you receive no points for that roll, the dice with 2s or 5s are discarded, and the remaining dice are rolled again. You repeat this procedure until all dice have been discarded. If you start with five dice, what is your expected score by the time you have discarded all of your dice?

—*MathWithBadDrawings.com*

BONUS: Syzygy A solar system has three small coplanar planets orbiting in the same direction about a large central sun.

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Spring Problems: Brain Ticklers

Continued from page 23

BONUS: Syzygy The planets' orbits are circular and have orbital periods of 3, 5, and 7 Earth years, respectively. The planets and the sun are currently collinear. Not counting their present state, how many times in the next 105 Earth years will the three planets be collinear (not necessarily collinear with the sun)?

—*FiveThirtyEight.com*

COMPUTER BONUS

Consider all ten-digit integers composed of the digits 0 through 9 each used exactly once, such that the first five digits are all odd and the last five digits are all even. Of these numbers, find the one that is closest to being a perfect square, that is, find N such that $|N - \text{round}(\sqrt{N})^2|$ is a minimum. For example, 1,357,902,468 differs from a perfect square by $|1{,}357{,}902{,}468 - 36{,}850^2| = 20{,}032$, but this is not the smallest difference.

—**Don A. Dechman,** *TX A '57*

Email your answers (plain text only) to any or all of the Spring Brain Ticklers to *BrainTicklers@tbp.org* or by postal mail to **Dylan Lane, Tau Beta Pi, P.O. Box 2697, Knoxville, TN 37901-2697**. The method of solution is not necessary. The Computer Bonus is not graded. Where possible, exact answers are preferable to approximations. The cutoff date for entries to the Spring column is the appearance of the Summer *Bent* which typically arrives in mid-June (the digital distribution is several days earlier). We welcome any interesting problems that might be suitable for the column. Dylan will forward your entries to the judges who are **F.J. Tydeman,** *CA Δ '73;* **J.C. Rasbold,** *OH A '83;* **J.R. Stribling,** *CA A '92;* and the columnist for this issue,

— **G.M. Gerken,** *CA H '11*