BRAIN TICKLERS

Results From Fall

Perfect Scores

Bachmann, David E.	MO	B	'72
*Berthold, Kristopher D.	TX	B	'04
*Chatcavage, Edward F.	PA	B	'80
*Couillard, J. Gregory	IL	A	'89
*Dechman, Don A.	TX	A	'57
Griggs Jr., James L.	OH	A	'56
*Gulian, Franklin J.	DE	A	'83
Gulian, Joseph D.	Son of member		
Janssen, James R.	CA	Γ	'81
McHenry, S. Dale	MO	B	'81
*Norris, Thomas G.	OK	A	'56
*Parks, Christopher J.	NY	Γ	'82
Rowe, Steven A.	ME	A	'81
Barr, David A.	Non-member		
Scott, Darrell J.	NC	Δ	'82
*Slegel, Timothy J.	PA	A	'80
Spong, Robert N.	UT	A	'58
*Voellinger, Edward J.	Non-member		

Other

*Bannister, Kenneth A.	PA	B	'82
Bertrand, Richard M.	WI	B	'73
Bohdan, Timothy E.	IN	Γ	'85
Garrard, Kenneth P.	NC	A	'82
Johnson, Roger W.	MN	A	'79
Jordan, R. Jeffrey	OK	Γ	'00
Kovalick, Albert W.	CA	H	'72
Lalinsky, Mark A.	MI	Γ	'77
Marks, Lawrence B.	NY	I	'81
Marks, Benjamin	Son of member		
Mitchneck, Adam	Non-member		
Pendleton III, Winston K.	MI	Γ	'62
Richards, John R.	NJ	B	'76
Riedesel, Jeremy M.	OH	B	'96
Roggli, Victor L.	TX	Γ	'73
Routh, Andre G.	FL	B	'89
Schmidt, V. Hugo	WA	B	'51
Schwartz, Beverly I.	MA	Δ	'85
Zison, Stanley W.	CA	Θ	'83

*Denotes correct bonus solution

Fall Review

All the regular problems had at least 80 percent correct answers, with number 3 (Path to Goddess) being the most missed. The Bonus (game of SETS) only had 55 percent correct answers.

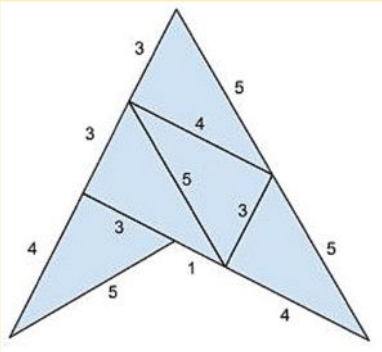
Winter Answers

1: The solution to the cryptic multiplication is **775 x 33 = 25575**. For this problem, we need only consider the digits 2, 3, 5, and 7. Examining options for the ones digit for both the multiplicand and the multiplier, it can be seen that at least one must be a 5, and the ones digit for the intermediate multiplication is a 5 that adds a carry of 1, 2, or 3. A carry of 3 forces the tens digit of the multiplicand to be even (2). The two potential choices for this condition both result in no valid intermediate multiplication. A carry of 2 initially looks promising as there are 4 potential choices for the tens digit of the multiplicand, but choosing a 2, 3, or 7 creates an invalid intermediate multiplication. Choosing the remaining digit 5 creates another carry of 2, and there are now 4 potential choices for the hundreds digit of the multiplicand. We can see that a choice of 5 or 7 for this digit will result in an intermediate multiplication of 2775 (555x5) or 3775 (755x5), respectively; each is a viable candidate that should be set aside for further analysis. Continuing with our study of an initial carry of 1, only a 7 in the tens digit of the multiplicand (with the 5 in the ones digit) will work. A moment of analysis will determine that 2325 (775x3) is the only valid intermediate multiplication for this configuration. Since all viable multiplicands are different, the two intermediate multiplications must be identical (the digits of the multiplier must be equal), and we check all three possibilities to determine the final answer. Only 775 x 33 = 25575 has all digits prime.

2: Anna is Alice, Dharma, and Darlene; Beth is Betty, Bianca, and Barbara; Cate is Carol, Claire, and Cristin; Dawn is Debra, Astrid, and Allison. Anna cannot be perfect or imperfect, because Cate's statements would be a contradiction in either case. So, one of Anna's statements is true and the other is false. If the first is false, then it would imply that Cate is perfect and, therefore, that Anna is imperfect, but we have shown this to be impossible. Thus, Beth is Betty and Cate must be imperfect, and Cate has the aliases Carol, Claire, and Cristin. It is, therefore, true that (like in real life) no one is perfect. This means Beth's first statement is false, and his second statement is clearly false as well, so Beth is imperfect and has the aliases Betty, Bianca, and Barbara. Since Carol is not Astrid, Dharma is Darlene and Dawn is not imperfect and must have the alias Debra. This matches Anna with Alice, Dharma, and Darlene, and matches Dawn with Debra, Astrid, and Allison.

3: The pressure seen at the bottom of the cylinder has **increased by 100 percent**. Assume the pressure inside the air bubble at the bottom is p, implying that the initial pressure on the bottom is $P_i = p$. Since the initial pressure seen at the top of the cylinder is zero, the air bubble pressure cancels out the pressure due to the weight of the water column (ρgh); that is, $p=\rho gh$. Because the water is incompressible and the volume is kept constant, the bubble must maintain its size as it reaches the top. No work is done on the air, so the temperature remains fixed. Δp is therefore also zero when the bubble reaches the top. The final pressure seen at the bottom of the cylinder is $P_f=\rho gh+p=2\rho gh$. This implies the pressure has doubled.

4: A solution to the problem is shown in the attached figure. Live long and prosper!



5: **9 drops are required** for a 100 story building, and for 14 drops, a **469 story building** can be covered. It helps to first consider the case of two bowling balls, as first presented in the Winter 2010 Ticklers. There, we saw the first partition breaks up the building into chunks of x stories, x–1, stories, and so on, down to one story. Then, once the first ball breaks, the second ball is used within the partition one story at a time until the appropriate floor is found. This method supports a solution of x drops in a building of up to $x(x+1)/2$ stories (triangular in value). Now, with an additional ball, we need to create a new set of first partitions. Within each of these partitions, we can apply the optimal technique for two balls previously described. A little recursive thinking shows that we need the first partition sizes to be equal to the elements in sequence of triangular numbers until the total sum of these elements equals (or exceeds) the number of floors N minus k attempts.

$\sum_{j=1}^{k-1} \sum_{i=1}^{j} i=N-k$, which can be expanded and terms consolidated to give $k^3/6+5k/6-N=0$. If N=100, we can find that k must be larger than ~8.24, so we require 9 drops. If k=14, we can solve to find N=469.

BONUS:

The roller coaster takes about **7.66 seconds** to complete the full loop. We wish to calculate the time elapsed for three separate sections: the lower curve, the vertical final answer will be twice the sum of these three components. Let R_1 be the radius of the lower curve, and R_2 be the radius of the upper curve. For the first circular section, $T_1=\int_0^{\pi/2} \frac{R_1}{V(\theta)} d\theta$. $V(\theta)$ can be found from conservation of energy concepts, with $(½)v_1^2=gy+(½)V(\theta)^2$.

A little geometry and some rearranging gives $V(\theta)=\sqrt{(v_1^2-2gR_1(1-\cos\theta))}$. Using conservation of energy, $v_1=\sqrt{(2gh)}$ for an initial height h. With h=100m, this yields v_1=44.29m/s. Plugging these values into the integral, we see this is an elliptic form that has no closed form solution. Numerical integration gives T_1=1.5475s. In the second section, the time elapsed is equal to the change in velocity over g, or $T_2=(v_3-v_2)/g$. The various velocities can also be found using conservation of energy, and we find $v_2=\sqrt{(v_1^2-2gR_1)}$=34.31m/s and $v_3=\sqrt{(v_1^2-2g(R_1+10))}$=31.32m/s. Plugging all this in gives T_2=0.3047s. In the final section, we repeat the method of the first section, this time with $T_3=\int_0^{\pi/2} \frac{R_2}{V(\theta)} d\theta$

$V(\theta)=\sqrt{(v_1^2-2g(R_2\sin\theta+R_1+10))}$, and numerical integration gives T_3=1.9767s. Adding all segments up and doubling gives about 7.66s for the total loop transit time. It should be noted that this is an ill-advised design, as riders experience 6 G at the bottom—most mortals would black out under these conditions!

COMPUTER BONUS:

The longest sequence is of length **11**, and the first number in such a sequence is **2520**. Note that SD(n) is an infinitely repeating sequence of the digits 1,2,...9. It is trivial to note that SD(n)=1 always divides n, and with the fact that any n with a sum of digits divisible by 9 is itself divisible by 9, we can see that SD(n)=9 will also always divide n. Because the repeat sequence is of size 9, every other value of n resulting in an even SD(n) will be odd, and n/SD(n) will not result in an integer. For n=3 or n=7, it is not readily apparent if n will divide evenly by SD(n). Regardless, these facts show that the longest sequence that can be made must start with an n that results in SD(n)=9, wraps through some magic set of SD(n)=1 through 9 that works, wraps again to another SD(n)=1 that will obviously divide n. The next SD(n)=2 will not divide, and thus the sequence length can be at most 11. A sequence can be found by starting with an integer which is lcm(1,2,3,4,5,6,7,8,9) = 2520, having an SD(2520)=9. This ensures that the numbers a distance i away from 2520 will have an SD(2520+i)=i mod 9 and will be divisible by 2520+i. A quick computer program can verify that this is indeed the first such sequence of maximal length.

New Spring Problems

1: Cryptic Addition

Solve the following cryptic addition in a base less than 10:

ALL + ALONE + AT = NIGHT

The usual rules apply: each letter represents a different digit and there are no leading zeros.

—*Journal of Recreational Mathematics*

2: Urn Probability

An urn contains 10 counters marked with the numbers 1-10. A random counter is drawn, the number on the counter is noted, and the counter is returned to the urn. This process is repeated for a total of 10 draws.

What is the exact probability that the sum of the numbers drawn is 50?

—de Moivre, c. 1717

3: Find the Temperature

The local radio station gives the temperature in degrees Fahrenheit and Celsius. The Fahrenheit value represents the actual temperature rounded to the nearest degree. The Celsius value is derived from the Fahrenheit value by consulting a chart which gives, for each integer

BTs continue on page 40.

Spring Problems *Continued*

degree Fahrenheit temperature, the nearest integer degree Celsius temperature. What is the exact probability that the reported Celsius temperature is wrong, i.e. not the actual temperature rounded to the nearest degree Celsius? Assume that the actual temperature is equally likely to be anywhere in the range 32 to 104 degrees Fahrenheit.

—*Technology Review*

4: Magic Square

A magic square is an array of different positive numbers arranged such that the sums of each row, the sums of each column, and the sums of the two main diagonals are all equal.

Find a 4x4 magic square that continues to be magic if the digits of all 16 numbers are reversed. For example: 2 reversed is 2, 20 reversed is also 2 (i.e. 02) but 21 reversed is 12.

—*Numbers: Fun & Facts*
by J. Newton Friend

5: Cereal Prize

A breakfast cereal company decides to include a figure of one of Snow White's seven dwarfs in each cereal box.

A) If the same number of each dwarf is distributed among the boxes, what is the expected number of boxes one must buy to get a complete set?

B) If there are only half as many Dopey figures as there are of the other dwarfs, what is the expected number of boxes one must buy to get a complete set?

For each scenario, provide the expected number of boxes purchased to within +/- 0.1 boxes.

—*Classic Mathemagic*
by R. Blum, A. Hart-Davis, B. Longe, and D. Niederman

BONUS:

It's the first day of summer and I'm drinking a margarita while lounging in my backyard in Albuquerque. I notice that the shadow cast by my vertical flagpole onto the level yard is changing direction. The time is local solar noon (when the sun is directly to the south). To the nearest degree per hour, what is the instantaneous rate of change in the shadow's azimuth? Assume that Albuquerque is at 35 degrees north latitude and the Earth's axis is tilted 23.5 degrees relative to its orbital plan.

—**James L. Griggs Jr.,** *OH A '56*

DOUBLE BONUS:

In the two-handed game of Gin Rummy, which is played with a standard 52-card bridge deck, "Gin" is a particular type of hand in which all ten cards are "matched cards." A "matched card" is one which belongs to a set (or meld) of 3 or 4 cards of the same rank or a set of 3 to 10 cards in sequence in the same suit. A "Gin" hand may contain both kinds of sets; however, a particular card cannot be used simultaneously in a rank and in a sequence. Further, an ace in sequence can only be used in an ace-2-3-4- . . . sequence, never with the king-queen sequence.

Of the $C(52,10)$ possible deals, how many are Gin?

—*Journal of Recreational Mathematics*

Email your answers (plain text only) to any or all of the Spring Brain Ticklers to *BrainTicklers@tbp.org* or by postal mail to **Dylan Lane, Tau Beta Pi, P.O. Box 2697, Knoxville, TN 37901-2697**. The method of solution is not necessary. The Double Bonus is not graded. Where possible, exact answers are preferable to approximations. The cutoff date for entries to the Spring column is the appearance of the Summer *Bent* which typically arrives in mid-June (the digital distribution is several days earlier). We welcome any interesting problems that might be suitable for the column. Dylan will forward your entries to the judges who are **F.J. Tydeman,** *CA Δ '73;* **J.C. Rasbold,** *OH A '83;* **J.R. Stribling,** *CA A '92;* and the columnist for this issue,

— **G.M. Gerken,** *CA H '11*

CHAPTER ETERNAL

Continued from page 39

VIRGINIA

ALPHA VA A

Addington Jr., Joseph, *'48*, Nov. 16, 2004.
Spicer Jr., Garland Hanes, *'48*, no details.
Blake Jr., Edward F., *'50*, Sept. 21, 2013.
Zapata, Richardo Nakin, *'55*, May 10, 2008.
Peters, Eric Gordon, *'62*, no details.
Cairns, Natalie Bates, *'74*, Dec. 22, 2010.
Le, Loc Tan, *'81*, April 10, 2007.

BETA VA B

Kingrea, Charles Leo, *'43*, Feb. 19, 2000.
Greenlee Jr., Art Bland, *'44*, no details.
Musser Jr., Harry Plaine, *'44*, Sept. 28, 2005.
Hall, Christopher H., *'49*, August 6, 2004.
Ewing, Hugh Culfbert, *'52*, July 25, 2016.
Knibb, Donald Eugene, *'53*, Jan. 29, 2017.
Brawley Jr., Andrew V.B., *'61*, no details.
Peele, Billy Keech, *'65*, March 15, 2019.
Wilbourn, Edward Gray, *'67*, Nov. 10, 2007.

DELTA VA Δ

Chandler Jr., Webster M., *'46*, July 14, 2012.
Taylor Jr., Arthur C., *'47*, July 22, 2016.
Newsom Jr., James Holt, *'48*, July 28, 2004.
Thompson Jr., Forest C., *'55*, Sept. 14, 2020.
Hammond, Leroy Dow, *'57*, Sept. 13, 2005.
Erchul, Ronald Anton, *'61*, October 8, 2011.

WASHINGTON

ALPHA WA A

Gribskov, Jon Russel, *'84*, Nov. 21, 2006.
Hartman, Christopher D., *'94*, Oct. 6, 2020.

BETA WA B

Hansen, Hans Edward, *'52*, January 7, 2015.
Hendricks, Lowell Eugene, *'59*, Dec. 7, 2003.

WEST VIRGINIA

BETA WV B

Teshome, Meaza, *'97*, May 9, 2011.

WISCONSIN

ALPHA WI A

Manteufel, Robert John, *'44*, Feb. 6, 2012.
Wendt Jr., William R., *'45*, Dec. 22, 2011.
Canute, Daniel Arthur, *'47*, March 14, 2014.
Gray Jr., Walter Erwin, *'47*, Dec. 3, 2001.
Gundersen, Sidney, *'49*, no details.
Te Beest, Roger Wayne, *'60*, July 14, 2001.

BETA WI B

Probert, Walter Leslie, *'50*, Feb. 23, 2017.
Miotke, Thomas Francis, *'57*, May 29, 2005.
Donovan, Dennis Michael, *'67*, Oct. 2, 2020.
O'Connor, Todd Brian, *'86*, January 7, 2013.

WYOMING

ALPHA WY A

Johnson, Bradley D., *'95*, March 15, 2001.