Brain Ticklers

RESULTS FROM FALL

Perfect

Bohdan, Timothy E.	IN	Γ	’85
*Celestino, James R.	NJ	B	’00
*Eanes, Robert Sterling	TX	Γ	’67
*Gerken, Gary M.	CA	H	’11
*Griggs Jr., James L.	OH	A	’56
*Gulian, Franklin J.	DE	A	’83
Gulian, Joseph D.	Son of member		
Harvey, Arthur J.	OH	A	’83
*Heske III, Theodore	PA	A	’86
Janssen, James R.	CA	Γ	’82
Johnson, Mark C.	IL	A	’00
Marks, Lawrence B.	NY	I	’81
Marks, Benjamin	Son of member		
*Norris, Thomas G.	OK	A	’56
Richards, John R.	NJ	B	’76
Riedesel, Jeremy M.	OH	B	’96
*Sarasohn, Max A.	NJ	B	’17
Schmidt, V. Hugo	WA	B	’51
*Slegel, Timothy J.	PA	A	’80

Other

Aron, Gert	IA	B	’58
*Celani, Paul E.	MD	Γ	’97
Chamness, Mark F.	TN	Δ	’77
*Couillard, J. Gregory	IL	A	’89
*Gaston, Charles A.	PA	B	’61
Handley, Vernon K.	GA	A	’86
Johnson, Roger W.	MN	A	’79
Jones, John F.	WI	A	’59
Jones, Jeffrey C.	Son of member		
Lalinsky, Mark A.	MI	Γ	’77
*Mayer, Michael A.	IL	A	’89
Quan, Richard	CA	X	’01
Rao, Sandesh S.	MI	B	'17
Rentz, Mark	Son of member		
Spong, Robert N.	UT	A	’58
*Voellinger, Edward J.	Non-member		

*Denotes correct bonus solution

FALL REVIEW

The Fall column appears to have been an easy one, as all five regular problems had better than 80% correct answers, while the Bonus had almost 60% correct answers. We received more Perfect than Other scores.

WINTER ANSWERS

1 SLEET + SNOW = FROSTY translates into **98552 + 9374 = 107926**. By inspection, there must be a carry of one into the ten thousands place to ensure FROSTY has six digits, so it follows directly that S=9, R=0 and F=1. Since there cannot be a carry into the thousands place, L cannot be 2. L can therefore potentially be any number from 3 to 8, inclusive (and O = L-1), and E+N must equal either 8 or 9 based on the carry in the hundreds place. Assume L=3, so O=2. This forces E and N to be 4 and 5 in some order and there can be no carry in the hundreds place. This implies T is 6 or 7 with no carry in the tens place, but this is not possible given the valid values remaining for W and Y. Assume L=4, so O=3. Two pairs of possibilities exist for E and N. If E=2 and N=7, this implies T is 5 with no carry in the tens place, but this is not possible given the valid values remaining for W and Y. If E=6 and N=2, no valid T can be found. Assume L=5, so O=4. Three pairs of possibilities exist for E and N. If E=2 and N=7, this implies T=6 and if E=3 and N=6, this implies T=7, both with no carry in the tens place. Neither is viable given the valid values remaining for W and Y. Assume L=6, so O=5. This forces E=2 and N=7, and then T=8 with a carry needed in the tens place. No valid solutions remain for W and Y. Assume L=7, so O=6. This forces E=5 and N=3, and then T=2 with a carry needed in the tens place. So it must be that L=8 and O=7. If E=3 and N=6 with no carry in the hundreds place, there is no solution for T. Similarly, if E=6 and N=2 with a carry in the hundreds place, there is no viable solution for the T, W, Y values amongst the remaining digits 3, 4 and 5. So it must be that E=5 and N=3, with a carry in the hundreds place. It follows that T=2 with no carry in the tens place, and W=4 and Y=6.

2 **The solution is given in the following table:**

Student	Color Worn	Toy Made	No. Toys
Gail	Mauve	End tables	6
Hal	Navy	Chairs	3
Irene	Lilac	Beds	5
Jill	Purple	Lamps	9
Kyle	Orange	Desks	7

We know that the sum of the toys was 30, the minimum possible number of toys made by an individual was 3, one of the numbers must be a 5, there must be three consecutive numbers represented, and one of the numbers is a third of another number. The only possible combinations of number that fit these constraints are 3,4,5,6,12 and 3,5,6,7,9. For the first combination, it follows that Jill made 12 toys (lamps) and that the student who wore navy made 4. It must be Gail that made 4 toys, and the student who wore lilac made 3 toys, and there were 5 desks made (by Irene). Kyle (wearing orange) could only have made 7 toys, so Hal made 3 chairs. Unfortunately, there is no way for mauve end tables to be constructed in this scenario. So the number of different toys made must be distributed 3,5,6,7,9. It follows that Jill made 9 toys (lamps) and that the student who wore navy made 3. It must be Gail that made 6 toys, and the student who made 5 toys (Irene) wore lilac. Kyle (wearing orange) could only have made 7 toys (desks), so Hal made 3 chairs, and Irene must have made beds. The mauve-wearing student who made end tables must have been Gail, and Jill must have worn purple.

3 **$f(x) = e^{(3-\sqrt{5})x/2}$.** Consider a function of the form $f(x) = ce^{ax}$. Given the initial condition that $f(0) = 1$, $c = 1$. Plugging $f(x) = e^{ax}$ into the differential equation gives: $f = af + 2a^2f + 3a^3f + 4a^4f + \ldots$. Dividing by f gives: $1 = a + 2a^2 + 3a^3 + 4a^4 + \ldots = a(1 + a + a^2 + a^3 + a^4 \ldots) + a^2(1 + a + a^2 + a^3 + a^4 \ldots) + a^3(1 + a + a^2 + a^3 + \ldots) + \ldots = (a + a^2 + a^3 + a^4 + \ldots)(1 + a + a^2 + a^3 + a^4 + \ldots) = [a/(1 - a)][1/(1 - a)] = a/(1 - a)^2$. With the condition that $|a| < 1$, this gives: $1 - a/(1 - a)^2 = 0$. Rearranging yields the quadratic: $a^2 - 3a + 1 = 0$. Of its roots, only $a = (3-\sqrt{5})/2$ has an absolute value less than 1, leading to $f(x) = e^{(3-\sqrt{5})x/2}$.

4 **The product of the *n*-1 chords is *n*.** One approach is to consider the n points as representing the n nth roots of 1 on the unit circle in the complex plane. Consider the identity $z^n-1 = (z-r_1)(z-r_2)\ldots(z-rn)$, where the r's are the n nth roots of 1. Let $r_1 = 1$, which is the first of the n nth roots of 1. Replacing z with z+1 and r_1 with 1, the identity can be written as $(z+1)^n - 1 = (z+1-1)(z+1-r_2)\ldots(z+1-r_n)$. The binomial expansion of the left side gives: $z^n + nz^{n-1} + n(n-1)z^{n-2}/2 + \ldots + nz + 1 - 1 = z(z+1-r_2)\ldots(z+1-r_n)$. After dividing both sides by z, this simplifies to: $z^{n-1} + nz^{n-2} + n(n-1)z^{n-3}/2 + \ldots + n = (z+1-r_2)\ldots(z+1-r_n)$. Since this identity holds for all values of z, picking z=0 results in $n = (1-r_2)\ldots(1-r_n)$. Since $|1-r_n|$ signifies the distance in the complex plane between the point $r_1 = 1$ and the point r_i (*i.e.*, the chord length between vertices in the polygon), it follows directly that the product of all chords with one end fixed at a given vertex is simply ***n***. An alternative approach is to use the law of cosines to calculate the chord lengths between point 1 and the other points. Consider the triangle formed by radii from the center of the unit circle to points 1 and i and the chord between these points. The law of cosines gives $c^2 = 2 - 2\cos\theta$, or $c = \sqrt{2}\sqrt{(1 - \cos\theta)} = 2\sqrt{[(1 - \cos\theta)/2]} = 2\sin(\theta/2)$, where $\theta = 2\pi/n$. Then the product, $P = 2^{n-1}\sin(\theta/2)\sin(2\theta/2)\sin(3\theta/2)\ldots$ From a table of products, we find that $\prod_{k=1}^{n-1}\sin(k\theta/n) = n/2^{n-1}$. Then, the product of the chords equals $2^{n-1}(n/2^{n-1}) = n$.

5 It takes Brother Adrian, using the following sequence, a minimum of 28 moves to visit the Chapel: **L-K-I-H-F-E-C-B-A-B-C-A-E-F-H-A-I-K-L-A-L-K-I-H-F-E-C-B.**

Bonus The probability of the players winning is 103/429. Consider the dealer's cards as she completes her hand. She will continue to draw until she either has a count between 17 and 21 (she wins) or goes over 21 (she loses). Note that an ace drawn as the first card will have a value of 11 (a win with 20), but other times has a value of 1. A little thought and a lot of careful counting will show that the possible sequences of draws can be organized into groups, a group consisting of all the sequences that are identical except for the final card. For example, one possibility is 6-A-2, and another is 6-A-4. These two would be in the same group, since they differ only in the last card. There are 28 such groups, but the number can be reduced somewhat by noting, for example, that 4-3- and 3-4- are essentially the same and can be placed in the same group. Depending on how you combine the sequences, you can have 18 to 28 groups. It doesn't matter as long as you keep your accounting straight. The following table is one way of assigning groups. The table shows the various (win for dealer) sequences. Cards in parentheses are used only one a time. Thus, 5-(3,4,6,7) stands for four sequences: 5-3; 5-4; 5-6; and 5-7. The second column shows the probability of the dealer's winning with the indicated hand, where $P_{13} = 1/13$; $P_{12} = 1/12$; $P_{11} = 1/11$; and $P_{10} = 1/10$. A common denominator for all the probabilities is 13×12×11×10 = 17,160. The last column gives the numerators of all the probabilities adjusted to the common denominator; the total is 13,040, so the probability of the dealer winning is 13,040/17,160 = 326/429, and the probability of your winning is 1 – (326/429) = 103/429.

Values of Cards Drawn	Prob. win by dealer	No. of 17,160th
(A,8,9,10,J,Q,K)	$7P_{13}$	9,240
7-(A,2,3,4,5)	$5P_{13}P_{12}$	550
6-(2,3,4,5)	$4P_{13}P_{12}$	440
6-A-(2,3,4,5)	$4P_{13}P_{12}P_{11}$	40
5-(3,4,6,7)	$4P_{13}P_{12}$	440
5-A-(2,3,4,6)	$4P_{13}P_{12}P_{11}$	40
(5,2)-(2,5)-(A,3,4)	$2{\times}3P_{13}P_{12}P_{11}$	60
4-(5,6,7,8)	$4P_{13}P_{12}$	440
(4,2)-(2,4)-(3,5,6)	$2{\times}3P_{13}P_{12}P_{11}$	60
(4,3)-(3,4)-(A,2,5)	$2{\times}3P_{13}P_{12}P_{11}$	60
4-A-(3,5,6,7)	$4P_{13}P_{12}P_{11}$	40
4-(A,2)-(2,A)-(3,5)	$2{\times}2P_{13}P_{12}P_{11}P_{10}$	4
2-(A,4)-(4,A)-(3,5)	$2{\times}2P_{13}P_{12}P_{11}P_{10}$	4
3-(5,6,7,8,9)	$5P_{13}P_{12}$	550
3-A-(4,5,6,7,8)	$5P_{13}P_{12}P_{11}$	50
(3,2)-(2,3)-A-(4,5,6)	$2{\times}3P_{13}P_{12}P_{11}P_{10}$	6
3-(A,2)-(2,A)-(4,5,6)	$2{\times}3P_{13}P_{12}P_{11}P_{10}$	6
(3,2)-(2,3)-(4,5,6,7)	$2{\times}4P_{13}P_{12}P_{11}$	80
2-(6,7,8,9,10,J,Q,K)	$8P_{13}P_{12}$	880
2-A-(5,6,7,8,9)	$5P_{13}P_{12}P_{11}$	50
	Total	13,040

Double Bonus The missing mascot is Joe Miner, of Missouri University of Science and Technology (MO Beta). To solve this problem, first find the school and (as the problem alluded to) the associated mascot based on the logos provided. The schools are in alphabetical order which may help to facilitate identification. The reference to "Greek" in the problem statement should have keyed you into the specific TBP chapters for each school.Intriguingly, every Greek letter from alpha to nu is uniquely represented by the thirteen schools. This should suggest arranging the schools in alphabetical order by their TBΠ Greek chapter designations (see table). The numbers under the logos are indexes to the letters in the names of the schools' mascots. Selecting the indexed letter from each mascot's name reveals: HAS A SLIDE RULE, and the only slide rule totin' mascot is none other than **Joe Miner, MO B.** Astute puzzlers may know that the slide rule was referenced in the original TBΠ yell, adopted in 1908!

Alpha	Lehigh Univ.	CLUTCH	6	H
Beta	Michigan Tech	BLIZZARD	6	A
Gamma	Ohio State Univ.	BRUTUS	6	S
Delta	University of S. California	TRAVELER	3	A
Epsilon	Trine	STORM	1	S
Zeta	Kettering Univ.	GENERAL DET-ERMINATION	7	L
Eta	City College of NY	BENNIE	5	I
Theta	Univ. of Mass., Lowell	ROWDY	4	D
Iota	S. Methodist Univ.	PERUNA	2	E
Kappa	W.Michigan Univ.	BUSTER	6	R
Lambda	Univ. of CA, Davis	GUNROCK	2	U
Mu	Penn State, Erie	NITTANY LION	8	L
Nu	Univ. at Buffalo	VICTOR E BULL	7	E

NEW SPRING PROBLEMS

1 I have found five nine-digit integers such that, among them, the digit 1 appears once, the digit 2 appears twice, 3 appears three times, and so forth to 9 which appears nine times. The first and second integers, both of which are exact fifth powers, between them, contain all nine different digits.The third integer is an exact fourth power. The fourth integer, which consists of only two different digits, is a palindrome divisible by 33 and 111. The digits of the fifth integer form an increasing sequence in which each digit is equal to, or greater than, the preceding digit. What are the five nine-digit integers?

—Adapted from an Enigma by Adrian Somerfield in *New Scientist*

2 At a recent bridge game, Don thought his luck couldn't get any worse, until he was dealt a hand that not only didn't have any aces or face cards, but included exactly two 2's, three 3's, and four 4's. What is the exact probability of being dealt such a hand? A bridge hand consists of 13 cards dealt from a well shuffled standard deck of 52 cards.

—Don A. Dechman, *TX A '58*

3 At Sam's Soup, Salad & Sandwich Deli, patrons can order from a menu consisting of ten different items.They can order as many different items as they wish, but they can order at most two of any given item. To minimize data entry, Sam has devised an approach that requires sending only one number per order to the kitchen. He did this by assigning a code number consisting of a positive integer to each item on the menu. The waiter then enters only the sum of the codes for the items ordered. (If two of the same item are ordered, the item's code is added twice.) If the codes have been chosen so that every possible order results in a unique number, what is a simple set of codes for the ten items on the menu?

—Adapted from *Doctor Ecco's Cyberpuzzles* by Dennis E. Shasha

4 In the famous Tower of Hanoi puzzle, you start with three pegs (P_1, P_2, and P_3). On P_1 are n discs of different sizes, increasing in diameter from the smallest at the top to the largest at the bottom. The objective is, in as few moves as possible, to transfer the stack from P_1 to P_3 by moving the discs, one at a time, from peg to peg to peg while observing the rule that no disc can ever be placed on top of a smaller disc. For three pegs and n discs, the process takes $2^n - 1$ moves. For example with three pegs and $n = 4$, 15 moves are required; and three pegs with n = 18 requires 262,143 moves and (at a rate of one move per second) requires 72 hr, 49 min, and 3 sec. For Super-Hanoi, you can have more than three pegs. For p pegs, the objective is to transfer an n disc stack from P_1 to P_p in the fewest possible moves. For example, with 4 pegs, 4 discs can be transferred in 9 moves instead of 15. What is the minimum number of pegs needed to be able to transfer 18 discs from P_1 to P_p in a minute or less at the rate of one move per second? Also, what is the arrangement of discs on the pegs immediately before the largest disc is moved from P_1 to P_p? Transferring a single disc from one peg to another constitutes a move.

—An Enigma by Stephen Ainley in *New Scientist*

5 In a right triangle, inscribe a semicircular arc, such that the arc is tangent to both legs of the right triangle, and the center of the semicircle and its two endpoints lie on the hypotenuse of the triangle. These three points divide the hypotenuse into four segments. Starting at one end of the hypotenuse and moving toward the other end, the lengths of these segments are x cm, 120 cm, 120 cm, and y cm. Given that x and y are integers, what are the maximum and minimum lengths the hypotenuse can have?

—Adapted from *The Wall Street Journal* by **Robert W. Schweitzer,** *NY Z '52*

Bonus Suppose that, in a distant galaxy, there is a solar system in which, instead of being spheres, the planets are right circular cones (with heights equal to the diameters of their bases). Suppose one of these planets has the same total volume and mass as our Earth, but a uniform density; what would be the gravitational acceleration on a person standing in the center of the circular base, and what would be the gravitational acceleration on a person standing at the apex? Assume the Earth is a perfect sphere with a radius of 6,370 km and an average density of 5,518 kg/m^3. Use a value of 6.673×10^{-11} N m^2/ kg^2 for G. Express your answers correct to three significant figures.

—*Higher Mathematics for Engineers and Physicists* by I.S. ans E.S. Skolonikoff

Double Bonus Solve the following cryptic addition in base 13.

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 HOWIE
  FRED
 CHUCK
  JEFF
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 JUDGE
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Although the base is an unlucky number, the following circumstances negate this. The panel of JUDGEs is, of course, prime, as are FRED, CHUCK, and JEFF, while HOWIE is the product of 7 (lucky number), not necessarily different, primes. The usual rules apply. Use a, b, and c to represent the digits 10, 11, and 12.

—H.G. McIlvried, III, *PA Γ '53*

Postal mail your answers to any or all of the Spring Brain Ticklers to **Curt Gomulinski, Tau Beta Pi, P.O. Box 2697, Knoxville, TN 37901-2697** or email to *BrainTicklers@tbp.org* as plain text only. The method of solution is not necessary, and the Double Bonus is not graded. The cutoff date for entries to the Spring column is the appearance of the Summer *Bent* in mid-June (the electronic version is distributed a few days earlier). We welcome any interesting problems that might be suitable for the column. Curt will forward your entries to the judges who are **F.J. Tydeman,** *CA Δ '73*; **J.C. Rasbold,** *OH A '83*; **J.R. Stribling,** *CA A '92*, and the columnist for this issue,

H.G. McIlvried, III, *PA Γ '53*