Brain Ticklers

RESULTS FROM FALL

Perfect

*Bohdan, Timothy E.	IN	Γ	'85
*Slegel, Timothy J.	PA	A	'80

Other

Alexander, Jay A.	IL	Γ	'86
Aron, Gert	IA	B	'58
*Couillard, J. Gregory	IL	A	'89
Doniger, Kenneth J.	CA	A	'77
*Ebersold, Dakota	Member's son		
*Gerken, Gary M.	CA	H	'11
*Gulian, Franklin J.	DE	A	'83
Gulian, William F.	Member's son		
Handley, Vernon K.	GA	A	'86
*Kimsey, David B.	AL	A	'71
Lalinsky, Mark A.	MI	Γ	'77
Lamb, James A.	NY	Γ	'66
*Mayer, Michael A.	IL	A	'89
*Norris, Thomas G.	OK	A	'56
*Prince, Lawrence R.	CT	B	'91
*Quan, Richard	CA	X	'01
Rentz, Peter E.	IN	A	'55
Rentz, Mark	Member's son		
*Richards, John R.	NJ	B	'76
*Schmidt, V. Hugo	WA	B	'51
*Shepperd, Stanley W.	MA	B	'70
Cantrell, Sage	Member's daughter		
Sigillito, Vincent G.	MD	B	'58
*Spong, Robert N.	UT	A	'58
Stadlin, Walter O.	NJ	Γ	'52
*Strong, Michael D.	PA	A	'84
*Voellinger, Edward J.	Non-member		

*Denotes correct bonus solution

FALL REVIEW

Problem 4 (books in a library) proved to be the most difficult, with fewer than 25% of submitters giving a correct answer, and Problem 2 (used car loan) was next hardest, with only 40% correct answers. Both were harder than the Bonus (security switches) for which 90% of the answers were correct.

WINTER ANSWERS

1 PUZZLE+PUZZLE+PUZZLE +PUZZLE=WINTER translates to **237716 + 237716 + 237716 + 237716 = 950864.** It is convenient to rewrite this Tickler as 4×PUZZLE = WINTER. To start, observe that P=1 or 2. Now, E ≠ 0, since R would also have to be 0, and E ≠ 1, 5, or 7 since an even carry from the units column to the tens column requires E to be even, and E ≠ 4 or 8 since an odd carry from units to tens column requires E to be odd, so E = 2, 3, 6, or 9. Try E=2, which implies that R=8, L=3, and P=1 (P can't be 2, since E=2). Only Z=7 works, leading to T=9 and N=0. This result has no valid solution for U. Try E=3 which implies that R=2 and L=8, and again P=1. Only Z=4 works, leading to T=9 and N=7. One of W, U, and I must be 0, but this is impossible. Try E=9 which implies that R=6 and L=4. Therefore, Z=8, leading to T=3 and N=5. One of W, U, I, and P must be 0, but this is impossible. Therefore, E=6, and R=4. This leads to L=1, so P=2. Only Z=7 works, leading to T=8 and N=0. There is only one possible way to distribute the remaining digits: U=3, I=5, and W=9, to give the solution shown above.

2 The ranking from first to last is Art, Dan, Bob, Eli, and Cai. C hears A state that he is not 2nd and B state that he is two places higher than D. C first intuits that only one of these statements is true. Then, after a pause, claims that he can announce the correct order of finish for all of them. Let's analyze C's reasoning. Being a Tau Bate, he realizes that, if A's statement is true and B's false, there are too many possibilities for the ranking (for example, ABDEC, ADEBC, and DBAEC) to be able to pick the correct one. Therefore, he decides to assume that it is B's statement that is true and A's that is false. This puts A 2nd and, together with C's knowing that he, himself, ranked last, leaves only BADEC, so he claims this is the ranking. Of course, he can't be sure, since it may be A's statement that is true, but he gambles that he is correct. Unfortunately for C, it turns out that the first three places are all wrong. This means that the true ranking must be either ADBEC or DBAEC, but we are told that A and B rank in order of truthfulness. Since DBAEC has B, whose statement is false, ranked ahead of A, whose statement is true, the correct ranking is ADBEC.

3 The sides of the triangle are 3, 5, and 7. Let the sides of the scalene triangle be a, b, and c, with c opposite θ, the angle which is an integral number of degrees. The law of cosines states that $c^2 = a^2+b^2-2ab\cos\theta$, or $\cos\theta = (a^2+b^2-c^2)/(2ab)$; since a, b, and c are all integers, $\cos\theta$ is a rational number. Using Niven's theorem (I had never heard of it either, until Judge Jeff Stribling brought it to my attention), the cosine function is only rational when θ is a member of {60°, 90°, 120°} for $0 < \theta < 180°$. Since the triangle has no right angle, we need consider only $\theta = 60°$ and $\theta = 120°$, or $\cos\theta = 1/2$. This gives $c^2 = a^2 + b^2 \pm ab$. Trying some low values for a and b quickly shows $a = 3$, $b = 5$ and $c = 7$, giving a perimeter of 15. You can also use a spreadsheet.

4 The faucet's opening is 23.3 cm above the drain. Applying the equation of motion for the velocity of an object under free fall to the falling water gives $v = \sqrt{(v_o^2 + 2gy)}$, where v_o is the initial velocity, g is the gravitational constant, y is the distance of the drain from the faucet, and v is the velocity of the water at the drain. We are given r_o = 1 cm and r = 0.45 cm at the drain. The volumetric flow rate, F (given as 8.328 L/min), must be the same at every point along the stream. Thus we have: $v_o = F/(\pi r_o^2) = 138.8/\pi$ = 44.18 cm/s; and at the drain, $F/\pi r^2$ = $138.8/(0.45^2\pi)$ = 218.2 cm/s. Substituting for v and v_o in the equation for v gives $y = (218.2^2 - 44.2^2)/(2g)$ = 22828.8/980.7 = 23.3 cm, that is the faucet is 23.3 cm above the drain.

5 Allen received 101 votes and Bill got 99. Consider a square grid with a horizontal axis, and let a be the number of votes for A and b the votes for B. Any vote count is represented by a path starting at (1, 1) or (1, -1), depending on who got the first vote counted, to $(a+b, a-b)$. At any point in the count, an A-vote is indicated by an uptick from (p, q) to $(p+1, q+1)$, and a B-vote by a downtick from

(p, q) to $(p+1, q-1)$. Call a path "bad" or "good" depending on whether it touches or crosses the axis or not. Now, any path that starts with a B-vote is a bad path (because it eventually has to cross the axis); the number of such paths is $C(a+b-1, b-1) = (a+b-1)!/[a!(b-1)!]$, where $C(i, j)$ is the number of combinations of i objects taken j at a time. To find the number of bad A-paths, take any such path and consider the portion from (1, 1) to the point where the path touches the axis. Now, reflect this portion of the path across the axis. This converts the bad A-path into a B-path, which shows that there is a one-to-one correspondence between bad A-paths and B-paths, so the number of bad A-paths equals the number of B-paths. The total number of possible vote count paths is $C(a+b, a)$, and the number of good paths is $C(a+b, a) - 2C(a+b-1, b-1) = (a+b)!/(a!b!)-2(a+b-1)!/[a!(b-1)!] = (a+b-1)!(a+b-2b)/(a!b!) = (a+b)!(a-b)/[a!b!(a+b)]$. Dividing by the total number of paths gives the probability $P = (a-b)/(a+b)$ of A's staying strictly ahead of B in the count. If $P = 1/100$, then $100(a-b) = a+b$, or $99a = 101b$. With the restriction of only 350 members, the only integer solution is Allen got 101 votes and Bill got 99.

Bonus The i^{th} triangular number which is simultaneously the sum of two cubes and the difference of two cubes is $\boldsymbol{T_i = [3^6(2i-1)^{12} - 1]/8}$. We start with equations (1) and (2):

$$A^3 + B^3 = n(n+1)/2 \quad (1)$$
$$C^3 - D^3 = n(n+1)/2 \quad (2)$$

We need to determine n such that (1) and (2) are valid for some positive integers A, B, C, D. Multiplying (1) and (2) by 8 gives:

$$(2A)^3 + (2B)^3 = 4n(n+1) \quad (3)$$
$$(2C)^3 - (2D)^3 = 4n(n+1) \quad (4)$$

Let $2A = w-x$; $2B = y-z$;

$$2C = w+x;\ 2D = y+z \quad (5)$$

for some integers w, x, y, and z. To simplify the problem, it was given that $D=B+1$; so $(y+z)/2 = (y-z)/2+1$, which gives $z=1$. Expanding the cubic terms in (3) and (4) (with $z=1$) gives:

$$w^3 -3w^2x + 3wx^2 - x^3 + y^3 - 3y^2 + 3y - 1 = 4n^2 + 4n \quad (6)$$
$$w^3 +3w^2x + 3wx^2 + x^3 - y^3 - 3y^2 - 3y - 1 = 4n^2 + 4n \quad (7)$$

Let $x^3 = 3y$; $3wx^2 = 3y^2$; $3w^2x = y^3$ (8)

Substituting these values into (6) and (7) and canceling like terms reduces both (6) and (7) to:

$$w^3 -1 = 4n^2 + 4n \quad (9)$$

The three equations (8) are dependent equations, so we can select an arbitrary value of x to ensure w, x, y are all integers. One choice is:

$$x=3k \quad (10)$$

which makes:

$$w=9k^4;\ y=9k^3 \quad (11)$$

This gives:

$$2A=3^2k^4 - 3k;\ 2B=3^2k^3-1;\ 2C=3^2k^4+3k;\ 2D=3^2k^3+1 \quad (12)$$

Substituting into (9) gives:

$$(3^2k^4)^3 - 1 = 3^6k^{12} - 1 = 4n^2 + 4n \quad (13)$$

Dividing by 8 gives:

$$T=A^3+B^3= C^3 - D^3=n(n+1)/2 =(3^6k^{12} - 1)/8. \quad (14)$$

The relationships (14) confirm that there are triangular numbers which are both the sum and difference of cubes. Since $(3^6k^{12} - 1)$ must be even (it is divisible by 8), then k must be odd. The i^{th} such triangular number is,

$$T_i = n(n+1)/2 = [3^6(2i-1)^{12}- 1]/8 \quad (15)$$
$$A=(3^2k^4 -3k)/2;\ B=(3^2k^3 - 1)/2;\ C=(3^2k^4 + 3k)/2;\ D=(3^2k^3 + 1)/2 \quad (16)$$

We can determine n from:

$$n(n+1)/2= (3^6k^{12}-1)/8 \quad (17)$$

Rearranging gives:

$$4n^2 + 4n + 1= (2n + 1)^2 = (3^3k^6)^2 \quad (18)$$

solving for n, with $k=2i-1$, gives

$$n = [3^3(2i-1)^6- 1]/2 \quad (19)$$

Letting $i=1$ confirms the given solution of 91 with $n=13$, and A, B, C, D = 3; 4; 6; 5, respectively; and $i=2$ gives a triangular number of 48,427,561 with $n=9841$ and A, B, C, D = 360; 121; 369; 122.

Computer Bonus The smallest x is 693,901,529. For this value of x, the value of the prime counting function, $\pi(x)$ = 35,953,285, and the value of the semi-prime counting function, $\pi_2(x)$ = 112,950,577. The ratio of $\pi_2(x)/\pi(x) \cong 3.14159268\ldots$, as close to π as you can come.

NEW SPRING PROBLEMS

1 At summer camp, John has just returned to his tent with two 10-liter containers full of water, when two of his friends show up, each asking for 2 liters of water. Al has a 4-liter pail, and Fay has a 5-liter pail. No other containers are available. How can John give Al and Fay each exactly 2 liters of water in their pails without spilling any water? On each transfer, one of the containers must be completely filled or totally emptied. Present your answer as a table with four columns labeled 10a, 10b, 5, and 4. The first line in the table will be 10 10 0 0, the initial situation. The following rows will be the situation after a transfer has occurred. For example, if John's first move was to fill the 4-liter pail from the first 10-liter container, the second line of the table would be 6 10 0 4. We want the shortest sequence.

—*Mathematical Puzzles of Sam Loyd*, Volume Two, edited by Martin Gardner

2 The first four triangular numbers are 1, 3, 6, and 10. Triangular numbers are numbers generated by the formula, $n(n + 1)/2$, where n is a positive integer. If you take a triangular number of dots, they can be arranged in a triangle, similar to the ten pins in a bowling alley. ONE, THREE, SIX, and TEN are each a triangular number. What are their values? The usual rules for cryptics apply.

—Richard England in *New Scientist*

3 A manufacturer of paving blocks makes all sizes of rectangular blocks that have the following properties: the length, width, and thickness are each an integral number of cm, and the sum of the lengths of the 12 edges of the block is equal to two thirds the block's volume. What sizes does the manufacturer produce? Present your answer as a series of three integers in parentheses, such as (L, W, T) with L ≥ W ≥ T.

—*Puzzle Corner* by Allan Gottlieb in *Technology Review*

4 A, B, C, and D represent four different digits that can be combined to yield 24 different four-digit integers. These 24 integers have the following properties:

- **4** are primes
- **7** are the products of two different odd primes
- **1** is the square of a prime
- **8** are divisible by **2** but not by **4**
- **2** are divisible by **4** but not by **8**
- **1** is divisible by **8** but not by **16**
- **1** is divisible by **16**

What are the values of A, B, C, and D?

—*Classic Puzzles* by Gyles Brandreth

5 A standard die is tossed repeatedly until the total of the numbers thrown exceeds 12. What is the most likely final total?

—*Book of Curious and Interesting Puzzles* by David Wells

Bonus In the Power Ball lottery, five balls are drawn from among 69 white balls, numbered 1 to 69, and one Power Ball is drawn from among 26 red balls, numbered 1 to 26. If a player's ticket matches all five white balls and the Power Ball, he wins the jackpot, but there are eight other ways to win, as shown in the table. Tickets cost $2, but there is the power play option for an additional $1. At the end of the regular drawing, a power play number is drawn (from 42 balls, 24 of which are 2's, 13 are 3's, 3 are 4's, and 2 are 5's); and, if the player has paid the extra dollar, his winnings (except the jackpot, which isn't affected, and the $1,000,000 prize, which only goes to $2,000,000, regardless of the power play number) are multiplied by the power play number. How big must the jackpot be for the game to be fair for a $3 ticket, i.e., for the expected value of the player's winnings to equal $3? Ignore taxes, the fact that the money is given as an annuity, and the possibility of multiple winners.

Matches	Winnings
5 W + PB	Jackpot
5 W	$1,000,000
4 W + PB	$50,000
4 W	$100
3 W + PB	$100
3 W	$7
2 W + PB	$7
1 W + PB	$4
PB	$4

—*A Mathematician at the Ballpark*, by Ken Ross

Double Bonus Starting with a standard double-six set of dominos, arrange the 28 dominos into the perimeter of a square, such that the pip values on each of the four sides of the square sum to the same value. At each corner of the square, there is a 90^0 turn, so that each side is 7½ dominos long. The usual rules apply; that is, where two dominos meet, the ends must match (have the same number of pips). Place the double value dominos lengthwise, not crosswise. Present your answer as four strings of 7½ dominos each, starting at the upper left corner and proceeding clockwise around the square. For example, the top edge might be: 0-3|3-3|3-5|5-5|5-0|0-2|2-4|4 (sum = 44), and the right edge might be: 4-6|6-2|2-2|2-1|1-5|5-4|4-0|0 (sum = 44). There are multiple answers, and any valid solution will be accepted.

—Howard G. McIlvried III, *PA Γ '53*

The method of solution is not necessary, and the Double Bonus is not graded. Postal mail your answers to any or all of the Spring Brain Ticklers to Curt Gomulinski, Tau Beta Pi, P. O. Box 2697, Knoxville, TN 37901-2697 or email to Brain-Ticklers@tbp.org as plain text only. The cutoff date for entries to the Spring column is the appearance of the Summer *Bent* in mid-June (the electronic version is distributed a few days earlier). We welcome any interesting problems that might be suitable for the column. Curt will forward your entries to the judges who are **F. J. Tydeman,** *CA Δ '73*; **J. C. Rasbold,** *OH A '83*; **J. R. Stribling,** *CA A '92*, and the columnist for this issue,

H. G. McIlvried, III, *PA Γ '53*

LETTERS TO THE EDITOR

(Continued from page 5)

order for the university to operate.

NCEES has stated "engineering education has fallen behind other professions in preparing students for practice." There has been a persistent decrease in the content of education for years.

ASCE, ASME, IEEE, and other technical societies have expressed concern as well.

One solution to the problem of the strength of the engineering education program is to implement the NSPE Policy across the U.S. That policy states in part "NSPE recommends the establishment of professional schools of engineering which provides formal education beyond the bachelor degree—and operates under the direction of qualified practitioners."

That policy would require significant changes in the laws governing the practice, which would primarily be the responsibility of the various states. To my knowledge, no state has made any effort to establish a professional school of engineering. If the "dual system," as suggested in my full article were used, only one professional school would be required in each state. NSPE could be most helpful by providing "model laws" and guidelines as well as working with state legislators, as they have in the past.

If you are interested in reading the full article I have developed on this subject as well as submitting any comments or questions, please email me at *chipincorp@aol.com.*

Tom L. Underwood, P.E., OH Γ '52