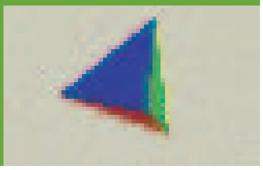

Brain Ticklers

RESULTS FROM FALL

Perfect

Bohdan, Timothy E.	IN	Γ	’85
Ebersold, Mareia T.	CA	A	’95
Ebersold, Dakota	Son of member		
*Gerken, Gary M.	CA	H	’11
Goodacre, Brian C.	NJ	B	’12
Lakocy, Alexander J.	IL	A	’14
Langenderfer, Brian A.	OH	Z	’85
*Mayer, Michael A.	IL	A	’89
*Norris, Thomas G.	OK	A	’56
Parks, Christopher J.	NY	Γ	’82
*Richards, John R.	NJ	B	’76
Schmidt, V. Hugo	WA	B	’51
Slegel, Timothy J.	PA	A	’80
Slegel, Dalton M.	Son of member		
*Stribling, Jeffrey R.	CA	A	’92
Strong, Michael D.	PA	A	’84
*Wing, Michael	Son of member		

Other

Aron, Gert	IA	B	’58
Benedict, Daniel H.	PA	H	’09
Chatcavage, Edward F.	PA	B	’80
Chowdhury, Suparna	AL	E	’15
Conway, David B.	TX	I	’79
Grewal, Rashi	NJ	Γ	’09
*Gulian, Frank J.	DE	A	’83
Gulian, William F.	Son of member		
Handley, Vernon K.	GA	A	’86
Harris, Kent	Non-member		
Henze, Edward D.	IL	Γ	’50
*Kimsey, David B.	AL	A	’71
Lloyd, Margaret M.	MA	B	’12
Marks, Lawrence B.	NY	I	’81
Marrone, James I.	IN	A	’61
Minton, Evelyn A.	NY	H	’73
Quan, Richard	CA	X	’01
Rentz, Mark	Son of member		
Rentz, Peter E.	IN	A	’55
Roggli, Victor L.	TX	Γ	’73
Sigillito, Vincent G.	MD	B	’58
Siskind, Kenneth S.	RI	A	’86
Siskind, Brian A.	Son of member		
*Spong, Robert N.	UT	A	’58
Sterns, Alan R.	OH	H	’88
Vinoski, Stephen B.	TN	Δ	’85
*Voellinger, Edward J.	Non-member		

*Denotes correct bonus solution

FALL REVIEW

Even though each of the regular Fall problems received more than 85% correct answers, incorrect solutions to problems 2 (ages of men), 3 (kings) and/or 4 (officers) kept several entrants from getting perfect scores.

While the regular problems may have been a little easier than usual, the Fall Bonus (pi resistance using 1-ohm resistors) appears to have been a little harder, as only about a quarter of the entrants tried to solve it.

WINTER SOLUTIONS

Readers’ entries for the Winter Ticklers will be acknowledged in the Summer *Bent.* Meanwhile, here are the answers.

1 A = 3, B = 1, C = 4 and D = 2. We are given the information in the table, where the a-statements are derived from the given statements based on the principle of transposition: if p → q then ~q → ~p.

No.	Statement
S1	If A is 1, then B is not 3
S2	If B is not 1, then D is 4
S2a	If D is not 4, then B is 1
S3	If B is 1, then C is 4
S4	If C is 3, then D is not 2
S5	If C is not 2, then D is 2
S5a	If D is not 2, then C is 2
S6	If D is 3, then A is not 4

Since D appears most often, start by considering its value. D can’t be 1 or 3, because if D ≠ 4, then B = 1 (S2a), and B = 1 implies C = 4 (S3), but if D ≠ 2, then C = 2 (S5a), but C = 4, a contradiction. Next, test D = 4. If D ≠ 2, then C = 2 (S5a), and A and B are 1 and 3 or 3 and 1. Now, if A = 1, B ≠ 3 (S1), but B must be 3, so A = 3 and B = 1, but then C = 4 (S3), but D = 4, another contradiction. Therefore, D must be 2, with B = 1 (S2a), C = 4 (S3), and A = 3. Checking, we find this is consistent with all the statements. This approach, which uses neither S4 nor S6, is only one line of reasoning. There are others.

2 The length of the side of the square is $(10 + 3\sqrt{7})^{0.5}$. Set up a coordinate system with the origin at the corner of the square 3 cm from the point and x and y axes along the sides of the square. Then, the coordinates of the point are (x, y), and the equations of the squares of the distances to the three corners are:

E1: $(L - x)^2 + y^2 = 2^2$

E2: $x^2 + y^2 = 3^2$

E3: $x^2 + (L - y)^2 = 4^2$

Subtracting E2 from E1 and solving for x gives: $x = (L^2 + 5)/(2L)$, and subtracting E2 from E3 and solving for y gives: $y = (L^2 - 7)/(2L)$. Substituting these results into E2 gives: $[(L^2 + 5)/(2L)]^2 + [(L^2 - 7)/(2L)]^2 = 9$. Expanding, combining terms, and simplifying gives: $L^4 - 20L^2 + 37 = 0$. By the quadratic equation, $L^2 = 10 + 3\sqrt{7}$, so $L = \sqrt{(10 + 3\sqrt{7})} = 4.235$.

3 Eleven of the 35 different hexominoes can be folded into unit cubes. Start by analyzing the 12 different pentominoes to find that 8 can be folded to form a 4-sided, open-topped box. For each of these, there are 4 places (the top edges) where a unit square can be added to serve as a lid for the box, giving 32 hexominoes that can be folded into a cube. Discarding the duplicates leaves 11 hexominoes that can be folded into a cube. In the following list, each hexomino is represented by a code word. For each code word, blacking in the squares in the grid corresponding to the letters in the code word will display one of the hexominoes: abchij; abghil; abghim; abghin; abghmn; afghik; afghil; afghim; afghin; bfghil; and bfghim.

a	b	c	d	e
f	g	h	i	j
k	l	m	n	o

4 The maximum number of cards in a 5-spot deck with 5 letters on each card is 21. Consider the general case with n letters on each card. The same letter can appear on at most n cards, for if this letter appears on $(n + 1)$ cards, then in a maximal deck, each letter will not occur the same number of times in the deck. Thus, a random card can match a given letter with $(n - 1)$ other cards, and since each card has n letters, a card can match $n(n - 1)$ cards. When the card itself is added to the total, the maximum deck size is $n(n - 1) + 1$ cards. For $n = 5$, this is 21 cards. One possible deck of cards is shown in the table.

ABCDE	BHLPT	DGMPR
AFGHI	BIMQU	DHJOU
AJKLM	CFKPU	DIKNT
ANOPQ	CGJQT	EFMOT
ARSTU	CHMNS	EGLNU
BFJKL	CILOR	EHKQR
BGKOS	DFLQS	EIJPS

5 There are **845** different 10-digit integers in which the sum of adjacent digits is a prime. This problem can be solved using a very large decision tree, but we used a computer.

Bonus The probabilities are **0.4574** for a pair and **0.4048** for a bust. There are C(52,5) = 2,598,960 possible five card poker hands, where C(*i*, *j*) is the number of combinations of *i* things taken *j* at a time. The cards in a bust (no pair or higher rated hand) must be of five different ranks, which can occur in C(13, 5) = 1287 ways; also, each card can be one of four suits, so there are 4^5 = 1024 possible suit distributions, for a total of 1,317,888 hands, but this total must be corrected for straights and flushes; there are 10 possible straights (*AKQJT* through *5432A*) and 4 possible flushes, one for each suit, giving [C(13,5) – 10][$C(4,1)^5$ – 4] = 1,302,540 possible bust hands, *if there are no wild cards.*

If one-eyed jacks (OEJs, the jacks of spades and hearts) are wild, no bust hand can have an OEJ, as any such hand is, at worst, a pair. To correct for OEJs, first calculate the number of hands with no jacks. By reasoning similar to that above, except that, with no jacks, there are only 12 ranks and possible straights are reduced by 4 (*AKQJT* to *JT987*), we get [C(12,5) – 6][$C(4,1)^5$ – 4] = 801,720 hands, so the number of hands with a jack is 1,302,540 – 801,720 = 500,820, but only half will have an OEJ (the other half will have a two-eyed jack), so the number of bust hands is 1,302,540 – 500,820/2 = 1,052,130, or a probability of 1,052,130/2,598,960 = 0.4048.

To count the number of hands with a pair, consider three cases with three subcases: (1) hands that have a natural pair with 0, 1, or 2 two-eyed jacks, but no OEJ; (2) hands that have one OEJ but no two-eyed jack, and (3) hands with a OEJ and a two-eyed jack.

For Case 1, count the ways to get a natural pair in a hand with 0, 1 or 2 two-eyed jacks. For no jacks, there are C(12,1) ways to pick the rank of the pair; C(4,2) ways to pick two cards of that rank; C(11,3) ways to pick the ranks of the other three cards, with $C(4,1)^3$ possible suit distributions, for: C(12,1)C(4,2)C(11,3) $C(4,1)^3$ = 760,320 hands. With one two-eyed jack, everything is the same except that there are C(2,1) ways to pick the jack, and only C(11,2) ways to pick the ranks of the 2 non-jack, non-pair cards for: C(12,1)C(4,2)C(2,1)C(11,2)$C(4,1)^2$ = 126,720 hands. If both two-eyed jacks are in the hand, the hand has a pair, and there are C(12,3)$C(4,1)^3$ = 14,080 ways to pick the other three cards. Total for hands with a natural pair is: 760,320+126,720+14,080 = 901,120.

For Case 2, there are two ways of picking the OEJ; C(12,4) ways of picking the ranks of the other four cards, minus 28 ways of making a straight with an OEJ (for example, a OEJ plus *9876*, *9875*, *9865*, *9765*, *etc.*); and $C(4,1)^4$ possible suit distributions minus 4 ways to make a flush for: 2[C(12,4)-28][$C(4,1)^4$-4] = 235,368 hands with an OEJ/non-jack pair.

For Case 3, there are two ways of picking the OEJ; two ways of picking the two-eyed jack; C(12,3) ways of picking the other three ranks, less 13 ways to make a straight with a two-eyed jack and a OEJ, and $C(4,1)^3$ possible suit distributions, less one way to make a flush for: 2(2)[C(12,3)-13][$C(4,1)^3$-1] = 52,164 hands.

In total, there are 901,120+ 235,368+52,164 = 1,188,652 hands with a pair, so the probability of getting exactly one pair is 1,188,652/2,598,960 = 0.4574. Thus, with one-eyed jacks wild, it is easier to get a pair than it is to get a bust hand, which violates the spirit of poker that higher ranked hands should be harder to get.

Computer Bonus There are 18 four-digit mirror-image integer pairs with mirror-image squares. The smallest pair is 1,002 and 2,001 with squares 1,004,004 and 4,004,001, and the largest pair is 2,022 and 2,202 with squares 4,088,484 and 4,848,804. Each pair has at least one repeated digit

NEW SPRING PROBLEMS

Now, here are some new Ticklers for your entertainment.

1 We start with a special Tickler. Tau Beta Pi is launching a new initiative, the Chapter Endowment Initiative, and in recognition of that effort, here is a new cryptic created specifically to call attention to the initiative. Solve the following addition/subtraction cryptic:

DONATE – MONEY – TO – THE + NEW = ENDOW + MENT.

Of course, we want the answer with the largest ENDOWMENT. The usual rules apply. Each different letter stands for a different digit, and each different digit is always represented by the same letter; there are no leading zeros. This cryptic is a little trickier than usual, but it can be solved without a computer. Give it a try.

—Howard G. McIlvried III, Ph.D., *P A Γ '53*

2 If Bill's age in years is appended to Alice's age, the resultant four digit integer is a perfect square. The same thing will be true thirteen years from now. What are Bill's and Alice's ages?

—Mathematics Teacher

3 Three married couples, Al and Xenia, Bill and Yvette, and Carl and Zoe, are out for a hike when they come to a river. The good news is they find a boat; the bad news is the boat holds, at most, two people at a time. The problem is that the men are very amorous and will kiss any wife who is separated from her husband. The husbands are also very jealous, so how can the three couples cross the river without any illicit kissing occurring? Present your answer as a sequential list of the occupants of the boat as it makes multiple crossings of the river. To make grading easier, when there is an option for whom to put in the boat, choose the one or two eligible persons whose names come first alphabetically.

—*The Chicken from Minsk* by Yuri Chernyak and Robert Rose

4 You have a bag containing five marbles—three are red and two are blue. You agree to share them with your friends. One friend reaches into the bag, grabs a marble and leaves without your seeing the color. You then reach in and remove a marble and see that it is blue. What is the probability that the marble your friend took is also blue?

—Marilyn Savant in *Parade Magazine*

5 The new One World Trade Center (OWTC) building has an unusual shape—it has a square base and a same-size square top, rotated 45° relative to the base, with sides consisting of eight isosceles triangles which extend the entire height of the building. (This shape is called a square antiprism.) Thus, a horizontal cross section of the building at any height is an octagon, although only the cross section halfway up is a regular octagon. Let us idealize the situation and suppose that the OWTC has exactly 100 evenly spaced stories with the base square being the floor of the first story and the top square being the ceiling of the 100th story. Assuming that the entire floor space of each story is usable, what is the ratio R of the total floor space of the 100 stories to the area of the base? (The ceiling of the 100th story is not considered to be floor space.) Now, having considered a specific case, consider the general case of a building with N stories, and find a simple expression for R in terms of N, that is express R as a function of N. (Remember, the ceiling of the top story is not floor space.) An expression involving a summation is not acceptable.

—**Robert N. Spong,** *UT A '58*

Bonus Find a 4×4 magic square consisting of 16 different positive semiprimes less than 100. A semiprime is a positive integer with exactly two, not necessarily different, prime factors. (For example, both $21 = 3\times7$ and $25 = 5^2$ are semiprimes.) In a magic square, the entries in each column, row, and major diagonal have the same sum (known as the magic sum). We want a magic square for which, in addition, the four center entries, the four corners, and each of the four quadrants also exhibit the same magic sum, making a total of 16 ways to get the magic sum. We want the square with the smallest magic sum for such a square. Present your answer as four rows of four integers each.

—**Howard G. McIlvried III,** Ph.D., *PA Γ '53*

Computer Bonus Let $S(N)$ equal the sum of all the divisors of N that are less than N. For example, $S(8) = 1 + 2 + 4 = 7$, and $S(30) = 1 + 2 + 3 + 5 + 6 + 10 + 15 = 42$. Define $S^2(N) = S(S(N))$ and $S^3(N) = S(S^2(N)) = S(S(S(N)))$, or in general, $S^n(N) = S(S^{n-1}(N))$. Call N a sociable number of degree n if $S^n(N) = N$ and there is no $m < n$ such that $S^m(N) = N$. An example of a sociable number of degree 5 is 12,496, i.e., $S^5(12{,}496) = 12{,}496$ (12,496 → 14,288 → 15,472 → 14,536 → 14,264 → 12,496). Find the smallest sociable number of degree 28.

—*Madachy's Mathematical Recreations* by Joseph S. Madachy

Send your answers to any or all of the Spring Brain Ticklers to Curt Gomulinski, Tau Beta Pi, P. O. Box 2697, Knoxville, TN 37901-2697 or email to *BrainTicklers@tbp.org* plain text only. The cutoff date for entries to the Spring column is the appearance of the Summer *Bent* in mid-June. Note that the electronic version of *The Bent* can be available to its subscribers considerably before some copies are received by regular mail. It is not necessary to include the method of solution. The Computer Bonus is not graded. The judges welcome any interesting problems that might be suitable for the column. Curt forwards your entries to the judges who are **F. J. Tydeman,** *CA Δ '73;* **D. A. Dechman,** *TX A '57;* **J. C. Rasbold,** *OH A '83;* and the columnist for this issue, **H. G. McIlvried III,** Ph.D., *PA Γ '53.*

LETTERS TO THE EDITOR

(Continued from page 9)

reported that following their recent school massacre, Pakistan now encourages teachers to carry guns.

Misconstruing data to justify ill-conceived gun laws (that multiple, scientifically proper studies show actually increase violent crime) does not show an intelligent 'commitment to diminish gun violence' as Mr. Wolf stated in his letter. Mr. Bloomberg not only misuses data, his upcoming gun control seminar for the press won't allow opposing presentations. His regular misuse of data and squelching intelligent opposition, not politics, is why I reject Mr. Bloomberg as worthy of mention by Tau Beta Pi.

James R. Hahn Jr., FL B '61

[Editor's Note: This started out as what was thought to be an innocuous 85 word note about the activities of one of our members has turned into a multi-page discussion about the merits and drawbacks of gun control. It is clear that with over 550,000 members, there is some disagreement regarding this issue! We could continue printing letters back and forth for decades and neither side would be satisfied. While there is undoubtedly more that could be said about this matter, we will not be including any additional letters to the editor on this topic.

An astute member noted that we do not include information on where to send Letters to the Editor. As noted at the bottom of the table of contents, correspondence (including Letters to the Editor) can be sent to tbp@tbp.org or to The Bent of Tau Beta Pi / P.O. Box 2697 / Knoxville, TN 37901.]