Brain Ticklers

RESULTS FROM FALL

Perfect			
Couillard, J. Gregory	IL	A	'89
*Norris, Thomas G.	OK	A	'56
Norris, Thomas G., Jr.	PA	Γ	'79
Richards, John R.	NJ	B	'76
Other			
Alexander, Jay A.	IL	Γ	'86
Anderson, Kurt E.	KS	Γ	'90
Aron, Gert	IA	B	'58
Bohdan, Timothy E.	IN	Γ	'85
Brule, John D.	MI	B	'49
Handley, Vernon K.	GA	A	'86
Heierman, William E.	GA	A	'64
Josephson, William E.	AL	A	'87
Kimmel, Peter G.	Member's spouse		
Kimsey, David B.	AL	A	'71
Lalinsky, Mark A.	MI	Γ	'77
Marrone, James I.	IN	A	'61
Pinkerton, Audrey Smith	TX	A	'90
Pinkerton, Kate	Member's daughter		
Rentz, Peter E.	IN	A	'55
Roggli, Victor L.	TX	Γ	'73
Routh, Andre G.	FL	B	'89
Schmidt, V. Hugo	WA	B	'51
Spong, Robert N.	UT	A	'58
Stribling, Jeffrey R.	CA	A	'92
*Strong, Michael D.	PA	A	'84
Summerfield, Steven L.	MO	Γ	'85
Sutor, David C.	Member's son		
Svetlik, J. Frank	MI	A	'67
Thaller, David B.	MA	B	'93
Voellinger, Edward J.	Non-member		

*Denotes correct bonus solution

FALL REVIEW

Problem 3 (Cryptic Addition) was the hardest regular puzzler, as many respondents found the smallest two solutions, but not the subsequent two. The Bonus question, which required readers to find a solution to two simultaneous equations, received many correct solutions to the equations, but only two readers submitted an answer with the smallest *d*.

WINTER SOLUTIONS

The entries for the Winter Ticklers will be acknowledged in the Summer column. In the meantime, here are the solutions.

1 The total is **818,181**. It is clear that one of the digits must be 1, for otherwise the total would be a seven-digit number. Thus, the six positive integers must all start with 1, and the first digit of the sum must be 7, 8, or 9 (it can't be 6, which would require all six numbers to be 111,111), but it can't be 7 or 9, because there is no combination of m 1s and n 7s or 9s ($m + n = 6$) that sum to a number ending in 1, 7, or 9, which would be a requirement for the units column. Therefore, the second digit must be 8. Consider the 6 six-digit integers in the form of a 6x6 array, and let (i, j) be the number of 1s and 8s, respectively, in a column. The accompanying table lists the possible values for the sum of a column as a function of the number of 1s and 8s in the column plus the indicated carry from the column to its right. Since each column sum must end in 1 or 8, the values in the table are the only possibilities. Thus, the units column (which has no carry) must be (1, 5) or (0, 6), both of which give a carry of 4, so the tens column is (3, 3) or (2, 4). Since both give a carry of 3, the hundreds column is (0, 6). By similar reasoning, the thousands column is (6, 0) or (5, 1); the ten thousands column is (4, 2) or (3, 3); and the hundred thousands column is (6, 0). Since there are two possibilities for four of the columns, there are $2^4 = 16$ possible totals that can easily be read from the table. In a given column, a 1 and an 8 can be interchanged to eliminate duplicate numbers without affecting the sum. On examination, 12 of the 16 possibilities have duplicate numbers, and three have more than one set of values that give the same sum. Only 818,181 = 111,818 + 111,881 + 111,888 + 118,888 + 181,818 + 181,888 is the sum of six different numbers in only one way.

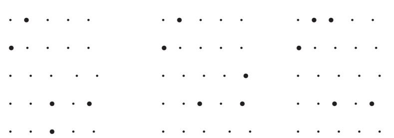
No. 1's	No. 8's	Column Sums					
Carry		2	1	5	3	4	0
6	0	8		11			
5	1			18			
4	2		21				
3	3		28			31	
2	4					38	
1	5						41
0	6				51		48

2 There are **377** ways to paint the totem pole so that no two adjacent animals are both blue. For a totem of N animals, at least $k = N/2$ (N even) or $(N-1)/2$ (N odd) must be yellow, so the number of blue animals must be in the range of 0 to $N-k$. For a totem with m yellow animals, there are $m+1$ places where blue animals can be inserted. To get the total number of different arrangements, sum over the range of blue animals, i.e., $\sum_{n=0}^{N-k} C(N+1-n, n) = C(13, 0)+C(12, 1)+C(11, 2)+C(10, 3)+C(9, 4)+C(8, 5)+C(7, 6) = 1+12+55+120+126+56+7 = 377$, where $C(i, j)$ is the number of combinations of i objects taken j at a time. Alternatively, find the answer for the first few totem sizes by inspection to get $T = 2, 3, 5, 8$ for $N = 1, 2, 3, 4$, respectively, and recognize these answers as part of the famous Fibonacci series. Note that $N_i = F_{i+2}$, where F_j is the jth Fibonacci number. Thus, the answer for a 12 animal totem is $F_{14} = 377$.

3 **FOURTEEN = 35617889.** From the first column of the cryptic for ELEVEN, we see that E = F + S + * + * + 1 (the 1 is to allow for a carry, which is very likely). Since F + S cannot be less than 3, we see that E = 6, 7, 8, or 9. Now, consider the cryptic for FOURTEEN. We see that E equals the units digit of N+N, so E cannot be 9, because this would require both E and N to equal 9. Try N = 9; then E = 8, and T = 7. Now, it is clear that U must be 6. Also, F = 2 or 3, and S = 1 or 2. Try S = 2; then O = 5, and F = 3. Finally, we see that R must be 1, so V = 0. This makes L = 4. So, FOUR = 3561, SEVEN = 28089, ELEVEN = 848089, and FOURTEEN = 35617889. Trying other values shows that this is the only solution for FOURTEEN. There are thousands of solutions to the cryptics when the *'s are considered.

4 There are exactly **148,362,637,348, 470,135,821,287,825** possible simple letter substitution codes (but the judges did not require this level of accuracy). Permutations in which no object remains in its normal position are called derangements. The formula for the number of derangements of n objects is $D_n = n![1 - 1/1! + 1/2! - 1/3! + ... + (-1)^n/n!]$. The logic for this expression is that there are $n!$ ways to arrange n objects. To get the number of derangements, we must subtract the permutations in which p_i is in its correct position. This is given by $C(n, 1)(n-1)! = [n!/1!(n-1)!](n-1)! = n!/1!$. But this overcorrects the derangements, so we must add back the cases where p_i and p_j are both out of place, which is given by $C(n, 2)(n-2)! = n!/2!$. But again this overcorrects, so we must subtract the cases where three objects are in correct positions, given by $C(n, 3)(n-3)! = n!/3!$, etc. Thus, we get $D_n = n![1 - 1/1! + 1/2! - 1/3! + ... + (-1)^n/n!]$. There are several ways to calculate the answer other than evaluating this expression. There are two recursion relationships that can be used: $D_n = nD_{n-1} + (-1)^n$ and $D_n = (n-1)(D_{n-1} + D_{n-2})$. Perhaps the easiest is to observe that the expression in square brackets, when $n = \infty$, is equal to $1/e$, so $D_n \approx n!/e$. For our case, $26!/e$, rounded to the nearest integer, gives the above answer.

5 There are three fundamentally different solutions: (1, 2)(2, 1)(4, 3)(4, 5)(5, 3); (1, 2)(2, 1)(3, 5)(4, 3)(4, 5); and (1, 2)(1, 3)(2, 1)(4, 3)(4, 5), where the first number in each pair is the row number and the second is the column number. All other solutions are rotations or reflections of these three solutions. The 10 different distances for the first two solutions are: 1, √2, 2, √5, 2√2, √10, √13, √17, 3√2, and 2√5. We gave credit for any correct solution. It is interesting to note that four of the points are the same for all three solutions, and only the position of the fifth point varies.



Bonus The total number of arrangements is $S_N = (N^4 + 6N^3 + 14N^2 + 15N + 6)/6$, where N is the sum of the number of envelopes on one side of the 3x3 array of pigeonholes. Let S_N be the number of solutions for N envelopes on a side. One approach is to find the value of S_N for several small values of N by inspection or computer (a spreadsheet is helpful) and then use this information to develop a polynomial in N. Counting ways to place the envelopes is made easier if you realize that once you specify the four corner values, the other values are fixed. The trick for a given N is to form all the different arrangements of the values of the four corners, where the value of a corner cell can be 0 to N and no two adjacent corners can have a sum greater than N. The notation we used for an arrangement is (a, b, c, d), where the four letters represent the values in the four corners, starting with the upper left and moving clockwise. When $N = 0$, the only solution is (0, 0, 0, 0), so $S_0 = 1$. For $N = 1$, we have (0, 0, 0, 0), (1, 0, 0, 0), (0, 1, 0, 0), (0, 0, 1, 0), (0, 0, 0, 1), (1, 0, 1, 0), and (0,1, 0, 1), so $S_1 = 7$. Similarly, for $N = 2$, 3, 4, and 5, you will find that $S_2 = 26$, $S_3 = 70$, $S_4 = 155$, and $S_5 = 301$. We can use the calculus of finite differences to find the general relationship for S_N as a function of N. A table of differences for the above values is shown below. The fact that the fourth difference is constant means that the desired equation is a quartic polynomial, which, in terms of differences, has the following form: $S_N = S_0 + N\Delta_1 + N(N-1)\Delta_2/2! + N(N-1)(N-2)\Delta_3/3! + N(N-1)(N-2)(N-3)\Delta_4/4!$. Using the leftmost values from the table gives $S_0 = 1$, $\Delta_1 = 6$, $\Delta_2 = 13$, $\Delta_3 = 12$, and $\Delta_4 = 4$. Substituting these values and simplifying gives: $S_N = N^4/6 + N^3 + 7N^2/3 + 5N/2 + 1$, or in terms of the least common denominator, $S_N = (N^4 + 6N^3 + 14N^2 + 15N + 6)/6$.

N	0	1	2	3	4	5
S_N	1	7	26	70	155	301
Δ_1		6	19	44	85	146
Δ_2			13	25	41	61
Δ_3				12	16	20
Δ_4					4	4

Computer Bonus The two numbers are 3,015,986,724 = $54{,}918^2$ and 6,714,983,025 = $81{,}945^2$. The approach is to pick a five-digit number, square it, and see if the square contains all 10 digits. If so, reverse the five-digit number and repeat the process. If this square contains all 10 digits, you have the answer. If not, go on to the next five-digit number. You can start your search at $1{,}023{,}456{,}789^{0.5}$ = 31,992, the square root of the smallest ten-digit number with all different digits, and end at 99,380, the square root of the largest such number. Also, since, by the rule of 9, any ten-digit number containing 10 different digits is divisible by 9, you only need to check five digit numbers divisible by 3. Starting with 31,992, successively add 3 to generate trial numbers. You only have to test 10 out of every 30 possibilities or a maximum of 22,462 trials to find the above unique answer.

NEW SPRING PROBLEMS

1 In the domino game Mexican Train, holding a double can be a problem, since when a double is played, a second domino of the same denomination must be played immediately, or else a domino from the bone pile must be drawn; and, if it doesn't match the double, you lose control of your train. The other night, I was playing Mexican Train with friends using a double 12 set (a double 12 set includes every two number combination from 0-0 through 12-12); and, in picking a hand of 11 dominos, I got four doubles. What is the probability of this?

—H.G. McIlvried III, *PA Γ '53*

2 Solve the following cryptic addition:

LETTERS
ALPHABET
SCRABBLE

All the usual rules apply. Each letter represents a different digit, and the same letter always represents the same digit. There are no leading zeros. This one is fairly easy, so, as

Hercule Poirot would say, give the little gray cells some exercise and try it without a computer.

—*Classic Puzzles* by Gyles Brandreth

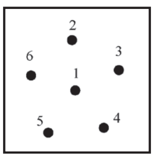
3 Suppose the moon had a hole along its axis and an astronaut dropped a golf ball in at the north pole. What would be the ball's speed (km/h) as it passed the center of the moon on its way to the South Pole? Assume the moon is a perfect sphere with a radius of 1,737 km, a uniform specific gravity of 3.346, and frictionless travel of the ball. Use G = 6.674×10^{-11} $m^3/kg\ s^2$.

—R.W. Rowland, *MD B '51*

4 The door to Prof. Adams laboratory has one of those keypad locks that requires entering five digits to open. Unfortunately, he has a hard time remembering the combination, but he has figured out a way to determine it. The five digits are all different, and he has observed that the first two digits form a perfect square, while the last two digits form a smaller perfect square. Also, the middle digit is the smallest. If he arranges the five digits to form all possible five-digit integers (leading zeros allowed) and adds all these numbers, the sum is a palindrome, with each of its digits a multiple of three. What is the combination?

—Adrian Somerfield in *New Scientist*

5 In the accompanying figure, connect nine pairs of points with straight line segments in such a way that no three of the six points are the vertices of a triangle. Present your answer as nine two-digit integers, where the two digits are the numbers of the points being connected, for example, 14, 16, 46, … would indicate 1 and 4 connected, 1 and 6 connected, 4 and 6 connected, etc. Of course, this is not an allowed arrangement, because 146 is a triangle.



Math Puzzles for the Clever Mind by Derrick Niederman

Bonus Select a random point in a unit square. What is the expected distance of this point from the lower left-hand corner of the square? We want an exact answer expressed in terms of common mathematical functions, such as log, sine, etc.

—Stephen E. Bernacki, *MA A '70*

Double Bonus A magic square is an NxN array of numbers such that the sum of each row, column, and main diagonal is the magic sum. The most famous magic square is the *lo shu* (shown here), which is claimed to be thousands of years old.

2	9	4
7	5	3
6	1	8

What we want is a 3x3 magic square that, in addition to a magic sum, has the added property that the sum of the products of the three numbers in each row equals the sum of the products of the three numbers in each column, and this also equals the sum of the products of the three numbers in each of the two main diagonals, i.e., if the numbers in the rows are (a, b, c), (d, e, f), and (g, h, i), then $abc + def + ghi = adg + beh + cfi = aei + ceg$. Although the nine numbers in the magic square must be nine different positive integers, they do not need to be consecutive. It is relatively easy to find such a square by computer, but we want an algorithm that generates all fundamentally different squares, that is, squares in which the nine entries have no common factor.

—*A Gardner's Workout* by Martin Gardner

Send your answers to any or all of the Spring Brain Ticklers to **Curt Gomulinski, Tau Beta Pi, P. O. Box 2697, Knoxville, TN 37901-2697** or email to *BrainTicklers@tbp.org* as plain text only. The cutoff date for entries to the Spring column is the appearance of the Summer *Bent* in early July. The method of solution is not necessary, unless you think it will be of interest to the judges. We welcome any interesting problems that might be suitable for the column. The Double Bonus is not graded. Curt will forward your entries to the judges who are **F.J. Tydeman,** *CA Δ '73*; **D.A. Dechman,** *TX A '57*; **J.C. Rasbold,** *OH A '83*; and the columnist for this issue, **H.G. McIlvried III,** *PA Γ '53*.

Letters to the Editor

(Continued from page 9)

The Bent, I was saddened when his last article was published in 2012. To my pleasant surprise, a republication of Lyle's law #5, was included in the Summer issue of *The Bent.*

My question is, why not republish one of Lyle's 40 laws in each future issue of *The Bent*? It should stimulate sales of his Book, increase donations to Tau Beta Pi, and make readers, like my self, happy. I'm sure there are hundreds of new members that have not seen these articles, and thousands more that would like to read them again.

Arnold Eusebio, MI E '52

[Editor's Note: We realize that Lyle's Laws continues to be a favorite among our readers. However, we very rarely reprint material in the magazine as space is at a costly premium in the magazine, and there is a great deal of new information to share with our members. There may be a time when Lyle and I agree to reprint another law, but we have no plans to so on a regular basis. We do hope to make all of the past laws available to subscribers through the website in the near future.]