Brain Ticklers

RESULTS FROM FALL 2012

Perfect

*	Fenstermacher, T.Edward	MD	B	'80
*	Griggs, James L., Jr.	OH	A	'56
	Jones, John F.	WI	A	'59
	Spong, Robert N.	UT	A	'58
	Stribling, Jeffrey R.	CA	A	'92
	Voellinger, Edward J.	Non-member		

Other

	Alexander, Jay A.	IL	Γ	'86
	Andersen, Devin	6th grade		
	Andersen, Meaghan	6th grade		
	Andersen, Ryan	6th grade		
	Aron, Gert	IA	B	'58
	Bilodeau, Robert R.	ME	A	'91
	Bohdan, Timothy E.	IN	Γ	'85
	Bremner, Christopher J.	CA	M	'14
	Bremner, David	Father of member		
	Butler, Holly	6th grade		
	Couillard, J. Gregory	IL	A	'89
	DiRe, Peter	6th grade		
	Doniger, Kenneth J.	CA	A	'77
	Drango, Lindsay	6th grade		
	Fanelli, Anna	6th grade		
	Gaffney, Caitlin	6th grade		
	Grewal, Rashi	NJ	Γ	'09
	Grimes, Emma	6th grade		
	Gurin, Ilya V.	CA	A	'07
	Handley, Vernon K.	GA	A	'86
	Hedegard, Alan H.	IN	A	'64
	Hoyt, Joseph	7th grade		
*	Janssen, James R.	CA	Γ	'82
	Jenneman, Jeffrey H.	OK	A	'08
	Johnson, Roger W.	MN	A	'79
	Jones, Donlan F.	CA	Z	'52
	McMahon, Colin	6th grade		
	Marrone, James I.	IN	A	'61
	Medvecz, David J.	IN	A	'83
*	Melton, Walter C.	TX	A	'56
	Mitchell, Amanda	7th grade		
	Nevins, Russell T.	MA	B	'77
	Potter, Megan	6th grade		
	Prince, Lawrence R.	CT	B	'91
	Rawe, Richard A.	KY	A	'57
	Rentz, Peter E.	IN	A	'55
*	Routh, Andre G.	FL	B	'89
*	Schmidt, V. Hugo	WA	B	'51
	Schweitzer, Robert W.	NY	Z	'52
	Silver, Robert E.	NY	P	'80
	Slegel, Timothy J.	PA	A	'80
	Stein, Gary M.	FL	Δ	'04
*	Strong, Michael D.	PA	A	'84
	Summerfield, Steven L.	MO	Γ	'85
	Sutor, David C.	Son of member		

* Denotes correct bonus solution

FALL REVIEW

Problem 3 (about determining the bottles of gold with odd weights) was the hardest regular problem. No. 5 (about hosing down a wall) had the next fewest correct answers. On the bonus, many of our readers overlooked the fact that we asked for an exact answer and gave answers that were not exact. The 6th and 7th graders are students of **Donlan F. Jones**, *CA Z '52*. They correctly solved Fall No. 2.

Due to pressing personal business, **John L. Bradshaw,** *PA A '82*, has found it necessary to resign his position as a Brain Ticklers Judge. We thank John for his service and wish him well. As John's replacement, we are pleased to announce that **J. Charles "Chuck" Rasbold,** *OH A '83*, has accepted our invitation to become a Brain Ticklers judge. Chuck has been a faithful submitter of solutions for many years with a sound record of correct answers, and we look forward to working with him. His first column will appear in the Fall 2013 issue of The Bent.

WINTER SOLUTIONS

Readers' entries to the Winter Ticklers will be acknowledged in the Summer Bent. Meanwhile, here are the answers.

1 The 00 ticket is worth \$7.53, and the 88 ticket is worth \$0.06. The total number of data points is twice the number of quarters or 2(4)(46) = 368. The probability of the last digit of a score being 0 is $p_0 = 101/368 = 0.2745$, and the probability of 00 is $p_{00} = p_0^{\,2} = 0.07533$. Since a ticket can win more than one quarter, the expected value of the winnings from ticket 00 is $E_{00} = 4(25)p_{00} = \$7.53$. Similarly, $p_{88} = (9/368)^2 = 0.000598$ and $E_{88} = 4(25)p_{88} = \$0.06$.

2 The area of the interface is 65.969 cm². Let r = initial radius of the soap bubbles, and R = radius after they coalesce. The volume of a spherical segment = $\pi h^2(3R - h)/3$, where h is the height of the segment, so $4\pi R^3/3 - \pi h^2(3R - h)/3 = 4\pi r^3/3$, or $4R^3 - 3Rh^2 + h^3 = 4r^3$. Now, $h = R(1 - \cos\theta)$, where θ is the angle between the radius through the center of the segment and a radius to the edge of the segment. Therefore, $R^3[4 - 3(1 - \cos\theta)^2 + (1 - \cos\theta)^3] = R^3(2 + 3\cos\theta - \cos^3\theta) = 4r^3$, so $R = 4^{1/3}r/(2 + 3\cos\theta - \cos^3\theta)^{1/3}$. Now, the curved surface area of a spherical segment $= 2\pi Rh = 2\pi R^2(1 - \cos\theta)$, and the area of the circular interface $= \pi h(2R - h) = \pi R^2\sin^2\theta$. Therefore, the area of the double bubble is: $S = 2[4\pi R^2 - 2\pi R^2(1 - \cos\theta)] + \pi R^2\sin^2\theta = 8\pi R^2 - 4\pi R^2 + 4\pi R^2\cos\theta + \pi R^2\sin^2\theta = \pi R^2(4 + 4\cos\theta + \sin^2\theta)$. $S/\pi = R^2(4 + 4\cos\theta + \sin^2\theta) = 4^{2/3}r^2(4 + 4\cos\theta + \sin^2\theta)/(2 + 3\cos\theta - \cos^3\theta)^{2/3}$. $S/(4^{2/3}\pi r^2) = (4 + 4\cos\theta + \sin^2\theta)/(2 + 3\cos\theta - \cos^3\theta)^{2/3}$. We want to minimize surface area; therefore, taking the derivative and setting it to 0 gives $dS/d\theta = 0 = (2+3\cos\theta-\cos^3\theta)^{2/3}(-4\sin\theta + 2\sin\theta\cos\theta) - (2/3)(4+4\cos\theta+\sin^2\theta)(2+3\cos\theta-\cos^3\theta)^{-1/3}(-3\sin\theta+ 3\sin\theta\cos^2\theta)/(2 + 3\cos\theta - \cos^3\theta)^{4/3}$, which reduces to $2\cos^3\theta + 3\cos^2\theta - 1 = 0$. Factoring gives: $(2\cos\theta - 1)(\cos\theta + 1)^2 = 0$. Therefore, $\cos\theta = 0.5$ and $\theta = 60^\circ$. The area of the interface $= \pi(R\sin\theta)^2$. Using the above formula for R gives $R = 5.29134$ cm, and $A = \pi(5.29134^2)(0.75) = 65.969$ cm².

3 The three coins are 1¢, 5¢, and 22¢. This is easily solved using a spreadsheet. Let the values of the three coins be X, Y, and Z (smallest to largest). Then, program the spreadsheet's columns as follows:

Col. A—Integers from 1 through 99
Col. B—FLOOR(Col. A/Z)
Col. C—FLOOR(MOD(Col. A, Z)/Y)
Col. D—MOD(Col. C, Y)
Col. E—Col. B + Col. C + Col. D

Cols. B, C, and D are the number of Z, Y, and X coins, respectively, needed to make change for an amount of N cents. Summing Col. E from 1 through 99 gives the total number of coins required to make change for amounts from 1¢ through 99¢. The approach is to try values for X, Y, and Z until a minimum is reached. The trial and error can be minimized based on the following observations. First, it is obvious that X must be 1. Second, it is probably good to have about the same number of each coin for the 99¢ case. Taking the cube root (since there are three coins) of 99 yields 4.6, which suggests a ratio of coin values of 5 to 1. Starting with (1, 5, 25) and trying close variations quickly shows that the minimum occurs for (1, 5, 22) and (1, 5, 23), both of which require a total of 526 coins. Since the king wants the smaller total, the answer is 1¢, 5¢, and 22¢.

4 If $N = 2^mM$, then the number of

trapezoidal decompositions (TDs) is $\tau(M) - 1$, where $\tau(M)$ is the number of divisors of M. Let $n > 1$ be the length of a TD of N, and let a be the first term. Then, $N = a + (a+1) + (a+2) + \ldots + (a+n-1) = n(2a + n - 1)/2$. Rearranging gives $a = N/n - (n-1)/2$. For N an odd integer, there are two cases. If $N/n > (n-1)/2$, a is a positive integer which is the first term of a TD of length n. If $N/n \leq (n-1)/2$, a is 0 or negative. Letting A = -a, in this case the first $2A+1$ terms cancel, leaving a TD of length $n-2A-1$ starting with $1+A$. Thus, each divisor (except 1) leads to a TD. If $N = p_1^a p_2^b p_3^c \ldots p_n^k$, $\tau(N) = (1+a)(1+b)\ldots(1+k)$. If N is even, any even divisors of N do not give an integral a; only odd divisors are of interest. If $N = 2^m M$, the number of TDs is $\tau(M) - 1$. As an example, consider $90 = 2(3^2)(5)$. The number of TDs is $\tau(45) - 1 = (1+2)(1+1) - 1 = 5$. The divisors are 3, 5, 9, 15, and 45. For 3, a = 90/3 – (3–1)/2 = 30–1 = 29, which gives the TD 29+30+31. From the other factors we get: 16+17+18+19+20; 6+7+8+9+10+11+12+13+14; 2+3+4+5+6+7+8+9+10+11+12+13; and 21+22+23+24. Alternately, one can determine the number of divisors for small values of N (up to 50 or so) and deduce the relationship by analyzing the results.

5 The maximum number of cards in a Spot-It deck with eight symbols on each card is 57. Let C = number of cards in the deck; N = total number of different symbols; and n = number of symbols on each card. The statement that one, and only one, symbol in common appears on any pair of cards means that each pair of symbols can appear on only one card, for otherwise there would be two cards with two symbols in common. The total number of pairs of symbols is $C(N, 2) = N(N-1)/2$, where $C(i, j)$ is the combinations of i things taken j at a time, and the number of pairs of symbols appearing on each card is $C(n, 2) = n(n-1)/2$. Therefore, the number of cards $C = [N(N-1)/2]/[n(n-1)/2] = N(N-1)/[n(n-1)]$. If we choose $n(n-1) = N-1$, we get $C = N$, but N is also equal to $n(n-1)+1$, so $C = n(n-1)+1$. For the given problem, n=8; therefore, C = 8(7)+1 =57, although designing the 57 cards is not easy. You can also solve for n = 3, 4, and 5 and deduce the solution for n = 8.

Bonus Each package contains 22 red and 8 green N&Ns. Let R = number of red and G = number of green N&Ns in a package. First, consider Al's situation. The number of ways to get a red and a green N&N as the last two N&Ns is $(R+G-2)!/[(R-1)!(G-1)!]$, and the number of ways to get a red and two green N&Ns as the last three N&Ns is $(R+G-3)!/[(R-1)!(G-2)!]$. Since the ratio of these is 4, we have $(R+G-2)!(R-1)!(G-2)!/[(R+G-3)!(R-1)!(G-1)!] = (R+G-2)/(G-1) = 4$, or $R = 3G-2$. Now, consider Beth's situation. The probability of x greens left when the last red is eaten is $(R+G-x-1)!/[(R-1)!(G-x)!2^{R+G-x-1}]$. The probability of having $x+1$ greens left is $(R+G-x-2)!/[(R-1)!(G-x)!2^{R+G-x-2}]$. Since the ratio of these two values equals 4, then $(R+G-x-1)/[(G-x)2]=4$, or $R=7G-7x+1$. Equating these two values of R and solving for G gives $G=(7x-3)/4$. The only value that works is x=5, which gives G=8 and R=22. Another approach is to get the answer by trial and error construction of probability diagrams for various values of R and G.

Double Bonus The remainder on dividing $2^{4,700,063,497}$ by 4,700,063,497 is 3. Problems of this sort are most easily solved using modular arithmetic (see any book on number theory) on a spreadsheet. Let 4,700,063,497 = N. The approach is to first express N as the sum of powers of 2 i.e. $N = 2^0+2^3+2^8+2^9+2^{10}+2^{14}+2^{16}+2^{18}+2^{21}+2^{27}+2^{28}+2^{32}$. Therefore, $2^{4,700,063,497}$ = 2 to the power 2^0×2 to the power 2^3×2 to the power 2^8×…2 to the power 2^{32}. If we find the value of each of these terms (mod N) and then find their product (mod N), we will have the desired answer. This is easily accomplished on a spreadsheet set up as follows. In Col. A, list the integers from 0 to 32. In Col. B, calculate the corresponding powers of 2. Start col. C with a 2 (which equals 2 to the power 2^0 or 2^1); then in the 2nd row, calculate the square of the entry directly above (i.e., 2^2 = 2 to the power 2^1) and determine the remainder when divided by N (use the MOD function). Continuing down col. C will generate 2 to the power 2^n (mod N). Thus, the values in the three columns represent n, 2^n, and 2 to the power 2^n. Next, copy to col. D values from col. C corresponding to the powers of 2 in the expression for N. Now, starting with the first value in col. D, successively multiply by the next entry and, after each multiplication, find the remainder when divided by N. (Hint: Start by multiplying 2 by 256 to get 512. Since this is smaller than N, we now multiply by 3,573,049,424 and then find the remainder upon dividing by N to get 1,076,604,755, etc.) When you get to the end, your final remainder will be 3. Incidentally, this is the smallest value of N such that $2^N \equiv 3$ (mod N). Keep in mind that for this procedure to work, you need sufficient accuracy so that there are no rounding errors.

NEW SPRING PROBLEMS

Here are the new Spring Ticklers to keep the little gray cells active during Spring break. For the most part they can be solved without a computer.

1 Joe has wired 100 bulbs, labeled 1 to 100, into an electrical circuit along with a button switch. He starts with all the bulbs unlit. When he pushes the button, every light lights. When he pushes it a second time, every second light (i.e., lights 2, 4, 6, etc.) goes off. On the third push, every third light changes status, that is, if it is off, it turns on, and if it is on, it turns off. On the fourth push the same thing occurs for every fourth light, and so on for the fifth through hundredth pushes. At this point, how many lights are lit?

—*The Electric Toilet Virgin Death Lottery and Other Outrageous Logic Problems*, by Thomas Byrne and Tom Cassidy

2 As part of an experiment to study the alertness of students, the letters A to F were permuted and briefly shown (in the form of a six letter "word") to seven students. After an hour, the students were asked to write down the order of the letters. What the students wrote was: Greg, BCDAEF; Hal, DAEFBC; Ivan, ABEFDC; Jill, BCFDEA; Kate, AEBDFC; Lila, CFEABD; and Mel, DCAEFB. The students were then asked a series of questions that each answered based on what he or she had written as his or her recollection of the order of the letters. For each possible pair of letters, they were asked if, reading from left to right, one letter came before another. A typical question was, "Does A come before D?" (The letters in the questions were always in alphabetical order.) Each student got a different even number of questions correct. (No one had them all wrong.) What was the correct order of the letters?

—Susan Denham in *New Scientist*

3 A typical roll of NECCO® wafers contains 40 wafers with a random distribution of eight different flavors. Suppose NECCO® decides to make sample rolls containing only 12 wafers. What is the probability that a twelve-wafer roll will have at least one wafer of each flavor? Assume that the rolls are made up from batches of wafers that contain equal numbers of each flavor and that the wafers in each roll are selected at random.

—Howard G. McIlvried III, *PA Γ '53*

4 Consider a double row of regular hexagons, ten in the top row and nine in the bottom row, arranged like the cells in a honeycomb. Starting at the upper left hexagon and zigzagging down and up, label the hexagons A through S. How many different paths are there, starting at A and ending at S, where a path consists of a series of ten to 19 letters indicating the order in which the hexagons are visited? You can only move from one hexagon to another hexagon that shares a common edge. No hexagon can be visited more than once for a given path.

—*Why Do Buses Come in Threes?* by Rob Eastaway and Jeremy Wyndham

5 Find three different nonzero digits such that each of the six permutations of the digits, read as six three-digit integers, is a semiprime. A semiprime is an integer that is the product of two, not necessarily different, primes. For example, 121 = 11×11 and 143 = 11×13 are semiprimes, while 153 = 3×3×17 and 105 = 3×5×7 are not.

—Richard England in *New Scientist*

Bonus Al, Beth, Carl, and Dawn are sitting around a table at a bar, as Al tries to guess Beth's age. They all know she is at least 21, or she wouldn't have been allowed into the bar. Al asks Beth five questions, pausing for contemplation after each question:

1. Is your age a multiple of 17?
2. Is your age a multiple of 3?
3. Is your age a prime number?
4. Are you older than I am? (Beth knows Al's age.)
5. Have you celebrated your 51st birthday?

At this point, Al announces that he has deduced Beth's age, but Beth tells him he is wrong. Carl, whose age is a prime number, has been listening to this conversation and is able to correctly deduce Al's age. From his knowledge of Beth, he surmises that she has not answered all the questions truthfully and guesses that she has alternated correct and incorrect answers. He knows that Beth is older than he is, and although he has guessed correctly how many of Beth's answers are incorrect, he has assumed the wrong ones. So, when he announces what he has deduced as Beth's age, Beth tells him he is also wrong. Finally, Dawn, who has also been listening in and is sharper than Carl, guesses correctly which of Beth's answers are incorrect. Now, knowing that Beth is younger than she is, Dawn is able to correctly announce Beth's age. What are the ages of Al, Beth, Carl, and Dawn, and what are Al's and Carl's incorrect guesses? It may help to know that Dawn's age is divisible by 13 and they all know that their ages are all different.

—*Brain Puzzler's Delight* by E.R. Emmet

Computer Bonus You are floating in a sea of 7's on a raft with the number 101. You discover that you can take a 7 and insert it into your raft to enlarge it (getting 7101; 1701; 1071; or 1017). Unfortunately, every time you do this, the raft divides itself by its smallest prime factor (leaving in the above case 2367; 567; 357; or 339). If the raft goes below 100, it will sink. What is the maximum number of insertions you can make before you sink? Remember, 1 is not a prime.

—*Technology Review*

Send your answers to any or all of the Spring Brain Ticklers to **Curt Gomulinski, Tau Beta Pi, P.O. Box 2697, Knoxville, TN 37901-2697** or email to *BrainTicklers@tbp.org* as plain text only. The cutoff date for entries to the Spring column is the appearance of the Summer Bent in early July. The method of solution is not necessary, unless you think it will be of interest to the judges. We welcome any interesting problems that might be suitable for the column. The Computer Bonus is not graded. Curt will forward your entries to the judges who are **F.J. Tydeman,** *CA Δ '73*; **D.A. Dechman,** *TX A '57*; **J. C. Rasbold**, *OH A '83;* and the columnist for this issue,

—H.G. McIlvried III *PA Γ '53*

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