Brain Ticklers

Due to pressing personal business, John L. Bradshaw, PA A ’82, has found it necessary to resign his position as a Brain Ticklers Judge. We thank John for his service and wish him well. As John’s replacement, we are pleased to announce that Charles “Chuck” Rasbold, OH A ’83, has accepted our invitation to become a Brain Ticklers Judge. Chuck has been a faithful submitter of solutions for many years with a sound record of correct answers, and we look forward to working with him. His first column will appear in the Fall 2013 issue of The Bent.

WINTER SOLUTIONS

Readers’ entries to the Winter Ticklers will be acknowledged in the Summer Bent. Meanwhile, here are the answers.

1. The 00 ticket is worth $7.53, and the 88 ticket is worth $0.06. The total number of data points is twice the number of quarters or 2(46) = 368. The probability of the last digit of a score being 0 is \( p_o = 0.0681 \), and the probability of 00 is \( p_{00} = 0.00533 \). Since a ticket can win more than one quarter, the expected value of the winnings from ticket 00 is \( E_{00} = 4(0.2681)p_{00} = 0.7953 \). Similarly, \( E_{00} = 4(0.2681)p_{00} = 0.00533 \). Since a ticket can win more than one quarter, the expected value of the winnings from ticket 00 is \( E_{00} = 4(0.2681)p_{00} = 0.7953 \). Similarly, \( E_{00} = 4(0.2681)p_{00} = 0.00533 \).

2. The area of the interface is 65,369 cm². Let \( r \) be initial radius of the soap bubbles, and \( R = \) radius after they coalesce. The volume of a spherical segment = \( \pi h^2(3R - h)/3 \), where \( h \) is the height of the segment, so \( 4\pi R^3/3 - \pi h^2(3R - h)/3 = 4\pi /3 \), or \( 4\pi R^3 - 3Rh^2 + h^3 = 4\pi R^3 \). Now, \( h = R(1 - \cos \theta) \), where \( \theta \) is the angle between the radius through the center of the segment and a radius to the edge of the segment. Therefore, \( R(4 - 3(1 - \cos \theta)^2 + (1 - \cos \theta)^3) = \pi R^3(2 + 3\cos \theta - \cos^2 \theta) = 4\pi R^3 \), so \( R = 4\pi R^3(2 + 3\cos \theta - \cos^2 \theta)^{1/3} \). Now, the curved surface area of a spherical segment = \( 2\pi Rh = 2\pi R^2(1 - \cos \theta) \), and the area of the circular interface = \( \pi h^2(3R - h) + \pi R^2 \sin \theta \). Therefore, the area of the double bubble is: \( S = 2(4\pi R^3 - 2n\pi R^3(1 - \cos \theta) + 8\pi R^2(1 - \cos \theta))^2 \).

3. The three coins are 1¢, 5¢, and 22¢. This is easily solved using a spreadsheet. Let the values of the three coins be X, Y, and Z (smallest to largest). Then, program the spreadsheet’s columns as follows: Col. A—Integers from 1 through 99. Col. B=FLOOR(Col. A/Z). Col. C=FLOOR(MOD(Col. A, Z)/Y). Col. D=MOD(Col. C, Y). Col. E=Col. B + Col. C + Col. D. Cols. B, C, and D are the number of X, Y, and Z coins, respectively, needed to make change for an amount of N cents. Summing Col. E from 1 through 99 gives the total number of coins required to make change for amounts from 1¢ through 99¢. The approach is to try values for X, Y, and Z until a minimum is reached. The trial and error can be minimized based on the following observations. First, it is obvious that X must be 1. Second, it is probably good to have about the same number of each coin for the 99¢ case. Taking the cube root (since there are three coins) of 99 yields 4.6, which suggests a ratio of coin values of 5 to 1 to 2. Starting with (1, 5, 25) and trying close variations quickly shows that the minimum occurs for (1, 5, 22) and (1, 5, 23), both of which require a total of 526 coins. Since the king wants the smaller total, the answer is 1¢, 5¢, and 22¢.

If \( N = 2nM \), then the number of
The remainder on dividing $2^{4700063497}$ by 4700063497 is 3. Problems of this sort are most easily solved using modular arithmetic (see any book on number theory) on a spreadsheet. Let 4700063497 = N. The approach is to first express N as the sum of powers of 2 i.e. $N = 2^a + 2^b + 2^c + 2^d + 2^e + 2^f + 2^{14} + 2^{16} + 2^{18} + 2^{19} + 2^{21} + 2^{22} + 2^{25} + 2^{27}$ + 2^{29}$. Therefore, $2^{4700063497} = 2$ to the power $2^a$ to the power $2^b$ to the power $2^c$ to the power $2^d$ to the power $2^e$ to the power $2^f$ to the power $2^{14}$ to the power $2^{16}$ to the power $2^{18}$ to the power $2^{19}$ to the power $2^{21}$ to the power $2^{22}$ to the power $2^{25}$ to the power $2^{27}$ + 2^{29}. If we find the value of each of these terms (mod N) and then find their product (mod N), we will have the desired answer. This is easily accomplished on a spreadsheet set up as follows. In Col. A, list the integers from 0 to 32. In Col. B, calculate the corresponding powers of 2. Start col. C with a 2 (which equals 2 to the power $2^a$ or $2^b$); then in the 2nd row, calculate the square of the entry directly above (i.e., $2^2$ to the power $2^c$) and determine the remainder when divided by N (use the MOD function). Continuing down col. C will generate 2 to the power $2^d$ (mod N). Thus, the values in the three columns represent $n$, $2^n$, and $2$ to the power $2^n$. Next, copy to col. D values from col. C corresponding to the powers of 2 in the expression for N. Now, starting with the first value in col. D, successively multiply by the next entry and, after each multiplication, find the remainder when divided by N. (Hint: Start by multiplying 2 by 256 to get 512. Since this is smaller than N, we now multiply by 3,573,049,424 and then find the remainder upon dividing by N to get 1,076,604,755, etc.) When you get to the end, your final remainder will be 3. Incidentally, this is the smallest value of N such that $2^N = 3$ (mod N). Keep in mind that for this procedure to work, we need sufficient accuracy so that there are no rounding errors.

**NEW SPRING PROBLEMS**

Here are the new Spring Ticklers to keep the little gray cells active during Spring break. For the most part they can be solved without a computer.

1. Joe has wired 100 bulbs, labeled 1 to 100, into an electrical circuit along with a button switch. He starts with all the bulbs unlit. When he pushes the button, every light lights. When he pushes it a second time, every second light (i.e., lights 2, 4, 6, etc.) goes off. On the third push, every third light changes status, that is, if it is off, it turns on, and if it is on, it turns off. On the fourth push the same thing occurs for every fourth light, and so on for the fifth through hundredth pushes. At this point, how many lights are lit? —The Electric Toilet Virgin Death Lottery and Other Outrageous Logic Problems, by Thomas Byrne and Tom Cassidy

2. As part of an experiment to study the alertness of students, the letters A to F were permuted and briefly shown (in the form of a six letter “word”) to seven students. After an hour, the students were asked to write down the order of the letters. What the
3. A typical roll of NECCO® wafers contains 40 wafers with a random distribution of eight different flavors. Suppose NECCO® decides to make sample rolls containing only 12 wafers. What is the probability that a twelve-wafer roll will have at least one wafer of each flavor? Assume that the rolls are made up from batches of wafers that contain equal numbers of each flavor and that the wafers in each roll are selected at random.

—Howard G. McIlvried III, PA Γ ’53

4. Consider a double row of regular hexagons, ten in the top row and nine in the bottom row, arranged like the cells in a honeycomb. Starting at the upper left hexagon and zigzagging down and up, label the hexagons A through S. How many different paths are there, starting at A and ending at S, where a path consists of a series of ten to 19 letters indicating the order in which the hexagons are visited? You can only move from one hexagon to another hexagon that shares a common edge. No hexagon can be visited more than once for a given path.

—Why Do Bases Come in Threes? by Rob Eastaway and Jeremy Wyndham

5. Find three different nonzero digits such that each of the six permutations of the digits, read as six three-digit integers, is a semiprime. A semiprime is an integer that is the product of two, not necessarily different, primes. For example, 121 = 11×11 and 143 = 11×13 are semiprimes, while 153 = 3×3×17 and 105 = 3×5×7 are not.

—Richard England in New Scientist

Computer Bonus You are floating in a sea of 7’s on a raft with the number 101. You discover that you can take a 7 and insert it into your raft to enlarge it (getting 7101; 1701; 1071; or 339). Unfortunately, every time you do this, the raft divides by its smallest prime factor (leaving in the above case 2367; 567; 357; or 339). If the raft goes below 100, it will sink. What is the maximum number of insertions you can make before you sink? Remember, 1 is not a prime.

—Technology Review

Send your answers to any or all of the Spring Brain Ticklers to Curt Gomulinski, Tau Beta Pi, P.O. Box 2697, Knoxville, TN 37901-2697 or email to BrainTicklers@tbp.org as plain text only. The cutoff date for entries to the Spring column is the appearance of the Summer Bent in early July. The method of solution is not necessary, unless you think it will be of interest to the judges. We welcome any interesting problems that might be suitable for the column. The Computer Bonus is not graded. Curt will forward your entries to the judges who are F.J. Tydeman, CA Δ ’73; D.A. Dechman, TX A ’57; J. C. Rasbold, OH A ’83; and the columnist for this issue.

—H.G. McIlvried III PA Γ’53

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