Brain Ticklers

RESULTS FROM FALL 2011

Perfect

*Schmidt, V. Hugo	WA B '51
*Spong, Robert N.	UT A '58
Stribling, Jeffrey R.	CA A '92
Tellechea, Gabriel	TX A '87
*Thaller, David B.	MA B '93
Voellinger, Edward J.	Non-member

Other

Alexander, Jay A.	IL Γ '86
Aron, Gert	IA B '58
Bertrand, Richard M.	WI B '73
Black, Janine	Non-member
Bohdan, Timothy E.	IN Γ '85
Brule, John D.	MI B '49
Couillard, J. Gregory	IL A '89
Eckley, Paul L.	NV A '75
Egenriether, Brian J.	SC Γ '10
*Griggs, Jr, James L.	OH A '56
Handley, Vernon K.	GA A '86
Harris, Kent	Non-member
Hasler, II, H. Victor	IN B '84
Jones, Donlan F.	CA Z '52
Hoyt, Joseph	6th grade
Sortino, Nicole	7th grade
Weldon, Grayson	6th grade
Jones, John F.	WI A '59
Jones, Michael P.	Son of member
Kimsey, David B.	AL A '71
Lalinsky, Mark A.	MI Γ '77
Marks, Lawrence B.	NY I '81
Marrone, James D.	IN A '87
Mercer, Robert	Non-member
Musser, Marion R.	OH A '57
Quintana, Juan S.	OH Θ '62
Rasbold, J. Charles	OH A '83
Rentz, Peter E.	IN A '55
Schultz Cruz, Daniel	PR A '07
Shaffer, Daniel A.	OH N '11
Siskind, Kenneth S.	RI A '86
Siskind, Amanda	Daughter of member
Stetson, II , Scott B.	CA T '12
*Stout, Roger P.	AZ B '77
Strong, Michael D.	PA A '84
Summerfield, Steven L.	MO Γ '85

* Denotes correct bonus solution

FALL REVIEW

The most difficult regular problem was No. 4 about the quadratic equation, which was almost as difficult as the Bonus about satellite orbits. The most popular problem was No. 1 about the cost of a turkey, which had twice as many entries as any other Tickler.

WINTER SOLUTIONS

Readers' entries for the Winter Ticklers will be acknowledged in the Summer Bent. In the meantime, here are the solutions.

1 There are no primes in the sequence 9; 98; 987; ... Any member of the sequence ending in 0, 2, 4, 5, 6, or 8 is not prime; so only members ending in 1, 3, 7, or 9 need to be examined. However, in all these cases, the sum of the digits of the number is divisible by 3, which means the number is divisible by 3. Thus, there are no primes in the sequence.

2 You were to find two three-digit numbers that sum to a four-digit number such that among them the digits 0 through 9 are used exactly once, that is ABC+DEF=GHIJ. There are 96 solutions with the maximum and minimum values of GHIJ being 1602 and 1026.

3 The lock's combination is 2-5-3-6-4-1. Call the three guessed combinations C1, C2, and C3. Consider C2: there are 6(5)/2 = 15 ways for two digits to be correct. Try each of these 15 possibilities by crossing out the corresponding digits in C3. This will leave four digits in C3, for which at most one can be in the same position as one of C2's digits. If this doubled digit, when moved to the C2-C3 empty column, matches the corresponding digit in C1, then the combination has been found. Alternatively, if there is no doubled digit and one of the four C3 digits matches C1, then the combination has been found. For example, try 1 and 3 as the correct digits for C2, which leads to the situation shown in the table:

C1	4	5	6	1	3	2
C2			3			1
C3	2			6	4	5

If we move the 5 in C3 to the second (empty) column, everything matches and we get 2-5-3-6-4-1 for the combination, which is the only solution.

4 The expected number of tosses of a pair of dice to get 1 through 6 in order is 17.0414. The best way to tackle this problem is by a decision tree. The probability of getting a specific digit in one toss of two dice is 11/36, and the probability of getting both the desired digit and the next digit is 2/36, so the probability of getting just the desired digit is 9/36, with relative probabilities of 9/11 and 2/11 of getting one or two matches. The decision tree has 13 branches, such as (1-2-3-4-5-6), (1-2-3-4-56), (1-2-3-45-6), etc., where a double digit means both values are thrown. The expected value (E_v) of the number of tosses is the sum of the expected number of tosses for each branch, multiplied by that branch's probability. For any branch, the expected number of tosses is 36/11 times the number of steps in that branch. The table provides information on the 13 branches.

# single steps (i)	# double steps (j)	# single 6's (k)	# branches (N)
5	0	1	1
4	1	0	1
3	1	1	4
2	2	0	3
1	2	1	3
0	3	0	1

The expected number of tosses for a branch is (i+j)(36/11). The relative probability is 9/11 for each single step, 2/11 for a double step, and 11/11 for a final single 6. Thus, the probability for a branch is $(9/11)^i(2/11)^j(11/11)^k$. The $E_v = \sum N(i+j)(36/11)(9/11)^i(2/11)^j$ = 30,189,958/1,771,561 = 17.0414.

5 PLANETS = 8+14+12+4+3+2+9 = 52. Since there are more variables (17 letters) than equations (11 planets), we cannot directly solve for the letter values, but since C and Y only occur together and I and J only occur together, there are only 15 variables, meaning that we can pick four letters, say A, E, O, and U, and express all the others in terms of those four. By calculations like VENUS–SUN = V+E = 39–18 = 21 or V = 21–E, URANUS–SUN = 36–18 = U+R+A = 18 or R = 18–A–U, and SATURN–SUN = 33–18 = 15 or T = 15–A–R = U–3, it is possible to express all the consonants in terms of A , E, O, and U. It is then a matter of trying different values for A, E, O, and U until a unique set of values is found (A=12, C=6 or 7, E=3, H=13, I=15 or 16, J=16 or 15, L=14, M=10, N=4, O=11, P=8, R=1, S=9, T=2, U=5, V=18, Y=7 or 6).

Bonus. There are 7 fundamentally different paths. There are several ways to solve this puzzle, the most obvious being a large decision tree. A clever solution (outlined below), provided by **William W. Verkuilen,** *WI B '92*, involves finding certain combinations of moves (strings) that recur repeatedly in successful paths. The figure shows the six different kinds of cells (A to F) superimposed on the 5x5 grid.

A-1	C-2	B-3	C-4	A-5
C-6	D-7	E-8	D-9	C-10
B-11	E-12	F-13	E-14	B-15
C-16	D-17	E-18	D-19	C-20
A-21	C-22	B-23	C-24	A-25

The secret to finding a path is realizing that each string starts and ends on a C-cell. Based on only certain moves being possible and the required numbers of these moves, we can deduce that there are two cases (depending on whether there are 2 or 3 B-B moves) to consider and that a complete path for Case I consists of the following 8 strings:

S1: C-A-F-A-C
S2a & b: C-A-C
S3a & b: C-E-C
S4a & b: C-D-D-C
S5: C-E-B-B-B-B-E-C
while a Case II path consists of:
S1: C-A-F-A-C
S2a & b: C-A-C
S4: C-D-D-C
S6a & b: C-D-C
S7a & b: C-E-B-B-E-C

Consider Case I. Since all placements of S5 are equivalent, we need to consider only one basic S5 string, so we'll use (4, 24)-12-15-23-11-3-18-(6, 10), where the numbers in parentheses mean either choice is valid. Then, the two S4's must be (2, 20)-17-9-(6, 24) and (4, 16)-19-7-(10, 22), while the two S3's must be 2-14-22 and 16-8-20. There are 6 possibilities for S1 (4 choices for adjacent corners and 2 for opposite corners). For each choice of S1, the two S2s are set. Try (10, 22)-25-13-5-(2, 20) for S1, which makes the two S2's 6-21-24 and 10-25-22. Since each of the 8 strings making up a path is bounded by Cs and since no 2 strings in a successful path can have the same start and end Cs, once the order of the Cs is determined, the other cells can be filled in uniquely. One way to find a path, is to create a table, as shown. In the top row, enter S2a, S2b, S3a, and S3b; and in the second row, enter their corresponding starting and ending Cs. In the third row enter S1, S4a, S4b, and S5, and in the fourth row, enter their corresponding 2 possible starting and ending Cs. Now, in row 4, cross out one value in each pair in such a way that each of the numbers 2, 4, 6, 10, 16, 20, 22, and 24 appears exactly twice and no 2 strings have the same start and end values. If you can do this, a solution has been found (see example table below). If not, try a new S1, S2a, S2b combination.

S2a		S2b		S3a		S3b	
6	24	10	22	16	20	2	22
S1		S4a		S4b		S5	
2 ~~20~~	~~4~~ 16	4 ~~16~~	10 ~~22~~	~~2~~ 20	~~6~~ 24	4 ~~24~~	6 ~~10~~

To extract the path, start with S1 (16, 2), move to S3b (2, 22), then to S2b (22, 10), etc. to get the order of the Cs: 16-2-22-10-4-6-24-20-16, which upon inserting the rest of the cells gives: 16-1-13-5-2-14-22-25-10-7-19-4-12-15-23-11-3-18-6-21-24-9-17-20-8-16. Two other choices in row 4 give successful paths: 2-4-6-24-20-16-10-22-2 and 20-4-6-24-2-22-10-16-20. If we choose opposite corners for S1, we get two more solutions: 6-2-22-10-4-16-20-24-6 and 20-6-24-2-22-10-4-16-20, for a total of 5 paths for Case I. For Case II, a complete circuit consists of S1, S2a, S2b, S4, S6a, S6b, S7a, and S7b. Applying the same kind of logic to Case II produces two new paths—22-4-16-2-20-24-10-22 and 20-6-24-10-22-4-10-2-20—for a total of 7. (Only the C paths are given to save space.)

Computer Bonus. The fifth smallest Keith Number that is also a prime is 1,084,051.

NEW SPRING PROBLEMS

Here are the new Spring problems. Except for the Computer Bonus, they can all be solved without a computer.

1 Solve the following cryptarithm: O/NE + T/WO + S/IX = NI/NE, where / indicates division. As expected, TWO is divisible by 2, SIX is divisible by 6, and NINE is divisible by 9, but oddly TEN is divisible by 7. Each different letter represents a different digit, the same letter represents the same digit, and there are no leading zeros.

—**H. G. McIlvried, III**, *PA Γ '53*

2 How many permutations of the integers 1 through N consist of a strictly ascending sequence followed by a strictly descending sequence. For example, for N = 9, one such permutation is (1-4-5-7-9-8-6-3-2). There must be at least two integers in a sequence, and N is considered to be a member of both sequences. Reversals are not considered to be different permutations, i.e. (2-3-6-8-9-7-5-4-1) is the same as the above example.

—Puzzle Corner in *Technology Review*

3 Find a non-trivial ($x, y, z \neq 0$) integral solution of the following cubic equation:

$$987{,}654{,}321x + 123{,}456{,}789y + z = (x + y + z)^3$$

We want the solution with the smallest positive value for $(x + y + z)$; however, this does not imply that x, y, and z are all positive.

—The Contest Center

4 In Green Town, half the farmland must be maintained in a natural state by law. To achieve this, in each field an inside strip around the four sides must be left uncultivated to provide habitat for wildlife. All fields are rectangles of the same size, 100m wide and an integral number of meters long. These border strips, which equal half the field's area, are a uniform and integral number of meters wide. It turns out that the digits in the length, when read from left to right, are in monotonically increasing order. What

(Continued on page 45.)

The Galletebeitia Family
Alvaro, *FL E '10*
Borja, *FL E '10*

The Hohmann Family
Gerald D., *PA A '51 (dec.)*
Lawrence A., *PA A '51 (dec.)*

The Johns Family
Ethan P., *MO B '11*
Owen M., *MO B '11*

The Klooth Family
Carolin M., *CA K '12*
Markus S., *CA K '12*

The Nock Family
Brian F., *NY Γ '13*
Stephen P., *NY Γ '13*

The Perkins Family
Austin W., *LA A '12*
Bradley C.B., *LA A '12*

The Willis Family
Kyle S., *IA A '12*
Ryan P., *IA A '12*

•Jack W. MacKay, *Alabama Beta '35*, still a P.E. at the age of 103, is believed to be our oldest living member. He is a published author on steel, cast iron, and ductile pipe. MacKay served as president and board member of the Alloy Casting Institute and as chairman of the Cast Iron Pipe Research Association's public relations and advertising committees. After retiring from ACIPCO, MacKay served as vice president of NTW Tire Co. until 1982. MacKay lives in Birmingham, AL, and has worked as an engineering consultant for Caldwell-MacKay, his son's company, since 1981. He was inducted last year into the State of Alabama Engineering Hall of Fame.

We are sorry to hear of the passing of our two oldest known members.

Charles E. McMurdo, P.E., *Virginia Alpha '29*, died December 31, 2011, at the age of 105 years, 5 months. He retired in 1971 from the Chesapeake and Potomac Telephone Company of Virginia, where he was an electrical engineer. McMurdo was a past chairman of the American Society for Quality Control, and was the oldest living alumnus of the University of Florida.

Albert E. O'Neall, *Florida Alpha '35*, passed away January 13, 2011, aged 102. He lived in Orlando, FL, for more than 60 years, and as a civil engineer, worked on water and sewage projects throughout the region. O'Neall was the oldest known living graduate of the University of Florida.

BRAIN TICKLERS

(Continued from page 37.)

is the length of a field and the width of the border strip?

—Enigma by Keith Austin in *New Scientist*

5 To increase his probability of throwing a 7, a gambler loaded a pair of dice as show in the table.

Pip Value	Prob. of Occurrence	
	Die 1	Die 2
1 & 2	1/4	1/12
3 & 4	1/6	1/6
5 & 6	1/12	1/4

One day, he accidentally grabbed one standard die and one loaded die. What was his probability of throwing a 7 with one loaded die and one standard die?

—*Mathematics Teacher*

Bonus. Arrange the integers 1 through 32 in a circle such that the sum of each pair of adjacent integers is always the square of an integer. Present your answer as a string of 33 integers starting and ending with 1.

—*Puzzles 101: A Puzzle Master's Challenge* by Nobuyuki Yoshigahara

Computer Bonus. Take any positive integer and apply the following algorithm: (a) if the number is odd, multiply by 3 and add 1; and (b) if the number is even, divide by 2. If you continue this process, you will always end up with 1, no matter what number you start with. For example, 17 requires 12 steps (17-52-26-13-40-20-10-5-16-8-4-2-1). In the range of 1 to 10,000, which integer requires the most steps and how many steps are required?

—*Mathematical Amazements and Surprises* by Alfred S. Posamentier and Ingmar Lehmann

Send your answers to any or all of the Spring Brain Ticklers to **Curt Gomulinski, Tau Beta Pi, P. O. Box 2697, Knoxville, TN 37901-2697** or e-mail to *BrainTicklers@tbp.org*, plain text only. The cutoff date for entries to the Spring column is the appearance of the Summer BENT in early July. It is not necessary to include the method of solution. The Computer Bonus is not graded. The judges welcome any interesting problems that might be suitable for the column. Curt will forward your entries to the judges who are **F. J. Tydeman,** *CA Δ '73*; **D. A. Dechman,** *TX A '57*; **J. L. Bradshaw,** *PA A '82*; and the columnist for this issue,

H. G. McIlvried, III, *PA Γ '53.*

CORRECTION

Several members noted a mistake on page seven of the Winter 2012 edition of THE BENT. The College of New Jersey was erroneously referenced by the letters NJIT. This should have been TCNJ as NJIT is the abbreviation for the New Jersey Institute of Technology, our New Jersey Gamma Chapter. We apologize for any misunderstanding this may have caused.

LOANS AVAILBLE

Since 1935, TBΠ has assisted student members with their financial needs while in school or with payment of their initiation fee through our Student Loan Fund. Eligible students should never decline membership because they cannot pay the initiation fee. Chapter officers should provide information about the program to candidates in the invitation letters or during information meetings.

We are pleased to offer loans to students in amounts up to $2,500 per member. Repayment is required after three years, and a simple interest rate of six percent is charged from the day the loan is received. Interested students can obtain additional information, promissory notes, and applications from *tbp.org*.