Brain Ticklers

RESULTS FROM FALL 2010

Perfect

*Aron, Gert	IA Β '58
*Bachmann, David E.	MO Β '72
Beaudet, Paul R.	Father of member
*Brana-Mulero, Francisco J.	PR Α '74
*Christenson, Ryan C.	UT Β '93
*Hess, Richard I.	CA Β '62
*Mayer, Michael A.	IL Α '89
Nabutovsky, Joseph	Father of member
*Schmidt, V. Hugo	WA Β '51
*Spong, Robert N.	UT Α '58
*Thaller, David B.	MA Β '93

Other

Achterberg, Karl J.	WI Α '84
Alexander, Jay A.	IL Γ '86
Bertrand, Richard M.	WI Β '73
*Couillard, J. Gregory	IL Α '89
Dohner, John W.	CA Γ '72
Edge, Billy L.	GA Α '71
Egenriether, Brian J.	SC Γ '10
Filippova, Olga T.	PA Ζ '09
Handley, Vernon K.	GA Α '86
Harvey, Joseph M.	OH Ι '04
Hasler II, H. Victor, II	IN Β '84
Jones, Donlan F.	CA Ζ '52
*Jones, John F.	WI Α '59
Jones, McKray	Non-member
Kern, Peter L.	NY Λ '62
*Kimsey, David B.	AL Α '71
Lew, Thomas M.	TX Δ '84
*Lott, Steven R.	MD Δ '09
Marks, Lawrence B.	NY Ι '81
Marks, Benjamin	Son of member
Marrone, James D.	IN Α '87
Marrone, James I.	IN Α '61
Mastrocola, Naison E.	CT Β '08
Aiudi, Michael E.	Non-member
Muksian, Robert	RI Β '59
Nishimura, Katsuyoshi	IN Α '51
*Norris, Thomas G.	OK Α '56
*Novak, Lawrence C.	IL Α '85
*Prince, Lawrence R.	CT Β '91
*Pritchard, Leroy J.	MI Θ '69
Pyers, Dean	OH Ζ '84
Quintana, Juan S.	OH Θ '62
Rasbold, J. Charles	OH Α '83
Schleehauf, Martin W.	NY Ν '79
Sigillito, Vincent G.	MD Β '58
Solt, Matthew	Son of member
*Stribling, Jeffrey R.	CA Α '92
*Strong, Michael D.	PA Α '84
Summerfield, Steven L.	MO Γ '85
Sutor, David C.	Son of member
*Voellinger, Edward J.	Non-member
White, Warren N.	LA Β '74

* Denotes correct bonus solution

BRAIN TICKLERS 60TH ANNIVERSARY!

This spring marks the 60th anniversary of the appearance of the first Brain Tickler (which concerned cutting a rug) in the April 1951 issue of The Bent. Since then, we have presented 1,535 Ticklers and received 11,098 entries from 4,529 different readers (we are indebted to Fred Tydeman and his database for these statistics). Brain Ticklers was initiated by R.H. Nagel, Editor of The Bent from 1942-82 and Secretary-Treasurer of Tau Beta Pi during 1947-82. Bob recruited a judging panel to write the column, and it's been going strong ever since. The column has been a labor of love for the judges with the pleasure expressed by our readers being our only reward. New Spring No. 1 is in honor of Bob, who died in 1997 at the age of 79.

FALL REVIEW

The most difficult regular Fall problem was No. 5 about a girl in a swing, with fewer correct answers than the Bonus about the football punter. No. 4 about four spheres was the second most difficult, with only a couple more correct answers than the Bonus.

WINTER SOLUTIONS

The Winter entries will be acknowledged in the Summer column. Meanwhile, here are the answers.

1 The smallest integer leaving remainders of 1, 2, 3, 4, 5, and 6 when divided by 2, 3, 4, 5, 6, and 7, respectively, is 419. The answer is one less than the least common multiple of 2, 3, 4, 5, 6, and 7 or $2^2 3(5)(7) - 1 = 420 - 1 = 419$.

2 The most senior pirate offers one gold coin to the least senior pirate and a second coin to the third most senior pirate and keeps 98 coins for himself. If there were only one pirate, there would be no problem; he would keep all 100 coins. With two pirates, the senior pirate keeps all the coins because his vote represents half the votes. With three pirates, the senior pirate gives one coin to the least senior pirate and keeps 99; the least senior pirate votes for this, as he gets nothing if the senior pirate is voted down and it comes down to a split between two pirates; the second most senior pirate gets nothing. With four pirates, the senior pirate needs one other vote, so he gives one coin to the next to least senior pirate and keeps 99; the pirate getting the one coin votes for this, as he gets nothing if it comes down to a split among three pirates. With five pirates, the senior pirate needs two votes, so the senior pirate offers one coin to the least senior and one coin to the third most senior pirate and keeps 98 for himself; the pirates getting the coins vote for this settlement, as they get nothing if it comes down to a split among four pirates.

3 My wife shook four hands. When asked how many hands they had shaken, nine people gave nine different answers, and since the most hands anyone could shake is eight, their answers must have been 0 through 8. Assume the person with 8 handshakes is a woman. Her spouse must have shaken 0 hands, for otherwise no one could have 0 handshakes, because she shook hands with everybody else. Next, assume the person with 7 handshakes is also a woman. Her spouse must have shaken 1 hand; otherwise, no one could have 1 handshake, because she shook hands with everyone except the person with 0 handshakes. Similar reasoning shows that the people with 6 and 2 handshakes are a married couple, as are those with 5 and 3 handshakes. Thus, in general, the sum of the handshakes of a person and the spouse is 8. The only unmatched number is 4; therefore, my wife shook 4 hands, as did I.

4 The serial numbers of the two transfers are 98,999 and 99,000. The sum of the digits of two consecutive integers cannot be an even number unless the smaller number ends in 9. Let the two numbers be A and B. Assume A ends in one 9, and let the sum of the first four digits of A be S_4. Then, the sum of the digits of B is $S_4 + 1$. Therefore, $S_4 + 9 + S_4 + 1 = 62$, so S_4

= 26, but this allows for several possibilities, so the answer to the question about the sum of the digits of one of the transfers being between 29 and 39 must have been no, and A does not end in only one 9. If A ended in two 9s, then the sum of the digits could not be even. Therefore, A must end in three 9s; let the sum of the first two digits of A be S_2. Then, $S_2 + 27 + S_2 + 1 = 62$, so $S_2 = 17$, which means the first two digits must be 98, and the two numbers must be 98,999 and 99,000.

5 The eagle is flying at a speed of 21.645 m/s. Consider an xy-coordinate system. Let a vertical line at x_0 represent the flight path of the sparrow, and let the hawk start at the origin. At time t, the sparrow is at $(x_0, v_s t)$, where v_s is the sparrow's speed, and the hawk is at (x, y). Now the slope of the tangent to the hawk's flight path at (x, y) is $dy/dx = (v_s t - y)/(x_0 - x) = p$. Solving for t gives: $t = p(x_0 - x)/v_s + y/v_s$. Also, $v_h t = S = \int^x_o \sqrt{(1+p^2)}dx$. Solving for t and equating the two values gives: $(1/v_h) \int^x_o \sqrt{(1+p^2)}dx = p(x_0 - x)/v_s + y/v_s$. Differentiating and simplifying gives: $\sqrt{(1+p^2)}/ v_h = [(x_0 - x)/v_s](dp/dx)$, which upon rearranging is: $dp/\sqrt{(1+p^2)} = [(v_s/v_h)/(x_0 - x)]dx = n dx/(x_0 - x)$, where $n = v_s/v_h$. Integrating between 0 and p and 0 and x gives: $\ln[p+\sqrt{(1+p^2)}] = n\ln[x_0/(x_0 - x)]$ or $p + \sqrt{(1+p^2)} = [x_0/(x_0 - x)]^n$. Replacing p with dy/dx and multiplying by dx gives $dy + \sqrt{(1+p^2)}dx = dy + dS = [x_0/(x_0 - x)]^n dx$. Integrating between 0 and x_0 yields $y_c + S_c = v_s t_c + v_h t_c = x_0/(1-n)$, where the subscript c refers to capture. Solving for t_c gives $t_c = x_0/[(1-n)(v_s+v_h)] = n x_0/[v_s(1-n^2)]$. Since t_c is the same for the hawk and eagle, we have $n_e x_{0e}/(1-n_e^2) = n_h x_{0h}/(1-n_h^2)$. Therefore, $20n_e/(1-n_e^2) = 0.5(40)/(1-0.5^2) = 80/3$ and $n_e/(1-n_e^2) = 4/3$. This gives $3n_e = 4 - 4n_e^2$ or $4n_e^2 + 3n_e - 4 = 0$. Solving gives $n_e = [-3 + \sqrt{(9+64)}]/8 = 0.693$, so the eagle is flying $1/n_e = 1.443$ times as fast as the sparrow or 21.645 m/s. The problem can also be solved using numerical integration by computer.

Bonus. Ninety-six different dodecahedrons can be distinguished, with a distribution of 1, 1, 3, 5, 12, 14, 24, 14, 12, 5, 3, 1, 1 for 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 red faces, respectively. We have found no better way of determining this answer than just counting, but one needs to be very careful because of the many possible orientations. Fred Tydeman wrote a computer program which examined all 60 orientations to check for duplicates. MathWorld (*mathworld.com/wolfram.polyhedron-coloring.html*) says that the number of colorings for a dodecahedron is given by $11n^4/15 + n^6/4 + n^{12}/60$, where n is the number of colors. For $n = 2$, this equation gives 96; however, the derivation is too complex to present.

Computer Bonus. The unique solution is 549,386,721 × 743,816,529 = $23{,}439^2 \times 27{,}273^2 = 639{,}251{,}847^2$. The equation $A \times B = C^2$, where A, B, and C are different nine-digit integers each using the digits 1 through 9 exactly once, has 620 solutions, but the solution with both A and B as perfect squares is unique. Write a computer program to look for perfect squares among the 5(8!) = 201,600 nine digit numbers that use the digits 1 to 9 exactly once and do not end in 2, 3, 7, or 8, and save the square roots of the 30 numbers that are perfect squares. Then, check the 30(29)/2 = 435 ways to multiply two of these square roots until you find a product that is a nine-digit number using the digits 1 through 9. A computer can accomplish this task in less than a minute! Email *dondechman_2000@yahoo.com* for a copy of his QBasic program.

NEW SPRING PROBLEMS

1 Solve the following cryptic multiplication, where each different letter stands for a different digit, the same letter always stands for the same digit, and there are no leading zeros. An * can stand for any digit.

```
        RNAGEL
       * * * * * * *
       -------------
        RNAGEL
       LRNAGE
      ELRNAG
     GELRNA
    AGELRN
   NAGELR
  RNAGEL
  ---------------------
  R * *N *AG * *E*L
```

What is the value of RNAGEL?

—150 Puzzles in Crypt-Arithmetic by Maxey Brooke

2 A spider is chasing an ant. The spider is crawling counterclockwise at a speed of 701 cm/min on the circumference of a circle with a diameter of 100 cm. The ant is crawling at a speed of 700 cm/min, also counterclockwise, on a semicircle consisting of the upper half of the spider's circle plus a diameter. At the start of the chase, the ant is at the left end of the diameter and about to crawl along it, and the spider is at the other end of the diameter and ready to start crawling along the circumference of the circle (to which it is restricted). They commence crawling at the same instant. How many complete circuits of the circle must the spider make before it catches the ant? Idealize the problem by treating the spider and the ant as points.

—John H. Cook

3 Doris, the librarian, wishing to fill an empty bookshelf, asked her assistants how many books it would take to fill the shelf exactly. Al said it would take 2 catalogs, 3 dictionaries, and 3 encyclopedias; Bob said it would take 4 catalogs, 3 dictionaries, and 2 encyclopedias; and Connie said it would take 4 catalogs, 4 dictionaries, and 3 encyclopedias. As it turned out, only two of these estimates were correct. Desiring to fill the shelf with only one kind of book, Doris found that only one of the three types would exactly fill the shelf and that 15 of that type were required. Which type of book did Doris use? All the catalogs are the same width, all the dictionaries are the same width, and all the encyclopedias are the same width.

—Logical Deduction Puzzles by George J. Summers

4 Take the aces, kings, queens, and jacks from a deck of cards and arrange the 16 cards in a four-by-four square array such that no two cards of the same rank or same suit are in the same row, the same column, or the same major diagonal. How many such arrangements are possible, if rotations and reflections are considered to be the same arrangement?

—A Gardner's Workout by Martin Gardner

(Continued on page 42.)

'49 **Schneider, Gene W.;** August 21, 2009.
'52 **Schulz, Edward R.;** November 22, 2010.
'58 **Raney, Charles N.;** December 15, 1987.
'59 **Reynolds, James D.;** April 11, 2010.
'65 **Darsey, David M.;** November 29, 2003.
'67 **Lennington, Richard K.;** August 22, 2000.
TX Β '37 **Holcomb, Dysart E.;** February 26, 2010.
'56 **Sullivent Jr., Ernest E.;** May 30, 1995.
'56 **Young, Terence O.;** November 17, 2010.
'58 **Bruton Jr., John D.;** January 14, 2011.
'58 **Dixon, Floyd A.;** no details.
'59 **Comiskey, Eugene A.;** August 15, 2002.
TX Γ '36 **Bentz, Irvin C.;** March 7, 2004.
'38 **Sinclair, James A.;** March 2, 1994.
'57 **Bull, John S.;** August 11, 2008.
TX Δ '36 **Sherwood, Robert S.;** December 8, 2008.
'49 **McCord, William C.;** January 11, 1998.
'57 **Wende, Harvey O.;** no details.
'78 **Groves, Alvin B.;** November 22, 2010.
TX Ε '47 **Graff, William J.;** August 19, 2009.
'75 **Schrader, Monroe A.;** November 18, 2010.
TX Λ '56 **Rai, Charanjit;** March 26, 2003.
UT Α '49 **Noorda, Raymond J.;** October 9, 2006.
UT Γ '96 **Taylor, Troy L.;** July 11, 2010.
VT Α '56 **Weber, Robert;** January 2, 2008.
'68 **Beliveau, Jean-Guy L.;** July 17, 2009.
VA Α '54 **Barton, Furman W.;** June 1, 2010.
VA Β '36 **Bailey, Ralph R.;** April 19, 2007.
'36 **Maher, Francis J.;** August 10, 1995.
'39 **Austin, William E.;** March 15, 2001.
'48 **Harrison, Edwin D.;** October 23, 2001.
'57 **Bryson, Bobby L.;** no details.
'58 **Carter, Everett C.;** August 10, 2005.
'60 **Moses, Hal L.;** May 15, 1994.
VA Δ '40 **Hardy Jr., Marshall B.;** November 2, 2010.
WA Α '43 **Huey, Donald R.;** November 21, 1998.
'49 **Crossfield, Albert S.;** April 19, 2006.
'50 **Kumasaka, Kazuo;** November 18, 2010.
'57 **Schaak, John C.;** June 10, 2000.
'61 **Birkeland, Christian J.;** no details.
'62 **Brandon, Robert L.;** June 12, 2006.
'70 **Henshaw, Boyd J.;** January 1, 2003.
WA Β '35 **Garrett, John C.;** no details.
'36 **Lauckhart, Donald W.;** January 29, 1996.
'36 **Loomis, Francis J.;** January 11, 1999.
'44 **Schurman, Glenn A.;** December 30, 2010.
'49 **Bills, Daniel G.;** September 30, 2010.
'54 **Muir, Earl L.;** November 28, 2009.
'56 **Pettibone, Robert E.;** March 29, 2003.
WV Α '47 **Elkins, Rush E.;** February 27, 2006.
'59 **Szczyrbak, Jackson;** no details.
'65 **Rotruck, James L.;** January 24, 2011.
WI Α '36 **Cole, Allan W.;** March 13, 1999.
'36 **Gother, William F.;** February 9, 2002.
'36 **Wagner, Eldon C.;** May 16, 2001.
'44 **Wollering, Walter R.;** February 9, 2009.
'58 **Stafford, Thomas S.;** December 15, 2000.
WI Β '35 **Wellauer, Edward J.;** January 22, 1998.
'36 **Saveland, Walter T.;** November 29, 2005.
'36 **Storatz, Gottfried J.;** September 21, 2003.
'49 **Karas, George P.;** March 27, 2010.
'52 **Sackett, Robert W.;** October 31, 2010.
'57 **Schliesmann, Raymond G.;** May 22, 2006.
'66 **Hause, Lawrence L.;** no details.
WI Δ '97 **Nelson, Thomas C.;** January 10, 2011
ΣΤ Α '49 **Cuckler, Virgil A.;** March 21, 2007.
ΣΤ Α '50 **Blume, Myron M.;** April 24, 2010.
ΣΤ Α '50 **Hunt, Wilmer A.;** June 12, 1995.
ΣΤ Ε '53 **Kaul, Kenneth E.;** August 31, 1999.
ΣΤ Ε '56 **Gabrielson, Harold W.;** April 18, 1990.
ΣΤ Η '52 **Slehofer, Otto J.;** no details.
ΣΤ Η '53 **Desposato, Richard D.;** April 27, 2009.
ΣΤ Ξ '48 **Rubin, Sherwin;** no details.
ΣΤ Ξ '56 **Oelke, Harlan;** August 30, 2009.
ΣΤ Ρ '50 **Abbott, Charles T.;** October 17, 2010.
ΣΤ Ρ '53 **Coombs, Wendell P.;** January 1, 2007.
ΣΤ Ρ '55 **Mortensen, Glen A.;** September 26, 2003.
ΣΤ Σ '51 **Grigsby, Robert A.;** October 14, 2002.
ΣΤ Σ '52 **Day, Winthrop J.;** April 29, 2003.
ΣΤ Τ '49 **Socolowski, Norbert J.;** February 15, 2008.
ΣΤ Τ '55 **Pavlat, John R.;** August 11, 2008.
ΣΤ Τ '50 **Burge Jr., Furman H.;** February 7, 2008.
ΣΤ Ψ '48 **Monito, Gregory A.;** May 19, 2006.
ΣΤ Ω '50 **Carlson, Eugene E.;** May 19, 1995.
ΣΤ ΑΒ '49 **Gerrity, John W.;** March 14, 2005.
ΣΤ ΑΔ '57 **Hansen, Keith A.;** August 16, 1990.

Correction
Steven G. Jenks, *OR* Α *'73*, was incorrectly added to Chapter Eternal in the Spring 2007 issue of THE BENT. He is alive and well.

BRAIN TICKLERS

(Continued from page 37.)

5 In the town of Isobar, every rainy day is followed by a sunny day. Every sunny day is followed by either a rainy day or a sunny day with equal probability. To the nearest day, what is the expected number of sunny days in a 365-day year? On a rainy day, it rains all day; on a sunny day, it is sunny all day.
—*The Surprise Attack in Mathematical Problems* by L.A. Graham

Bonus. Suppose that, instead of being a sphere, the Earth were a right circular cylinder (with a diameter equal to its height) of the same total volume and mass. What would be the gravitational acceleration on a person standing in the center of one of the circular faces? Assume the earth is a perfect sphere with a radius of 6,370 km and a uniform specific gravity of 5.518. Use a value of 6.674×10^{-11} m^3/s^2 kg for G, the universal gravitational constant.
—Howard G. McIlvried III, *PA Γ '53*

Computer Bonus. Within the set of prime numbers is a subset of primes, the sums of whose digits are also primes; we will call such numbers double primes. For example, 23 is a double prime because it is a prime number whose digits sum to 5, which is also prime; while 13 is not a double prime since it is a prime whose digits sum to 4, which is not prime. Let P_N be the Nth smallest prime, and let D_N equal the number of double primes less than or equal to P_N. For $N = 1, 2, 3, 4,$ and 5, $D_N/N = 1$. However, as more primes are considered, D_N/N generally (although not continuously) decreases. For example, when $N = 9$, $D_N/N = 2/3$. Determine the value of N such that $D_N/N < 1/e$ for the first time, where e is the base for the natural logarithms.
—Samuel L. SanGregory, *OH M '88*

Send your answers to any or all of the Spring Brain Ticklers to Jim Froula, **Tau Beta Pi, P.O. Box 2697, Knoxville, TN 37901-2697** or only as plain text by email to *BrainTicklers@tbp.org*. The cutoff date for entries to the Spring column is the appearance of the Summer BENT in late June. It is not necessary to include the method of solution. The Computer Bonus is not graded. The judges welcome any interesting problems that might be suitable for the column. Jim will forward your entries to the judges: **F.J. Tydeman,** *CA Δ '73;* **D.A. Dechman,** *TX A '57;* **J.L. Bradshaw,** *PA A '82;* and the columnist for this issue, **H.G. McIlvried,** III, *PA Γ '53*.