**Brain Ticklers**

The most difficult regular Fall problem was No. 5 about a girl in a swing, with fewer correct answers than the Bonus about the football punter. No. 4 about four spheres was the second most difficult, with only a couple more correct answers than the Bonus.

The Winter entries will be acknowledged in the Summer column. Meanwhile, here are the answers.

1. The smallest integer leaving remainders of 1, 2, 3, 4, and 5, and 6 when divided by 2, 3, 4, 5, 6, and 7, respectively, is 419. The answer is one less than the least common multiple of 2, 3, 4, 5, 6, and 7 or $2^235(5)(7) = 420 - 1 = 419$.

2. The most senior pirate offers one gold coin to the least senior pirate and a second coin to the third most senior pirate and keeps 98 coins for himself. If there were only one pirate, there would be no problem; he would keep all 100 coins. With two pirates, the senior pirate keeps all the coins because his vote represents half the votes. With three pirates, the senior pirate gives one coin to the least senior pirate and keeps 99; the least senior pirate votes for this, as he gets nothing if the senior pirate is voted down and it comes down to a split between two pirates; the second most senior pirate gets nothing. With four pirates, the senior pirate needs one other vote, so he gives one coin to the next least senior pirate and keeps 99; the pirate getting the one coin votes for this, as he gets nothing if it comes down to a split among three pirates. With five pirates, the senior pirate needs two votes, so the senior pirate offers one coin to the least senior and one coin to the third most senior pirate and keeps 98 for himself; the pirates getting the coins vote for this settlement, as they get nothing if it comes down to a split among four pirates.

3. My wife shook four hands. When asked how many hands they had shaken, nine people gave nine different answers, and since the most hands anyone could shake is eight, their answers must have been 0 through 8. Assume the person with 8 handshakes is a woman. Her spouse must have shaken 0 hands, for otherwise no one could have 0 handshakes, because she shook hands with everybody else. Next, assume the person with 7 handshakes is also a woman. Her spouse must have shaken 1 hand; otherwise, no one could have 1 handshake, because she shook hands with everyone except the person with 0 handshakes. Similar reasoning shows that the people with 6 and 2 handshakes are a married couple, as are those with 5 and 3 handshakes. Thus, in general, the sum of the handshakes of a person and the spouse is 8. The only unmatched number is 4; therefore, my wife shook 4 hands, as did I.

4. The serial numbers of the two transfers are 98,999 and 99,000. The sum of the digits of two consecutive integers cannot be an even number unless the smaller number ends in 9. Let the two numbers be $A$ and $B$. Assume $A$ ends in 9, and let the sum of the first four digits of $A$ be $S_A$. Then, the sum of the digits of $B$ is $S_A + 1$. Therefore, $S_A + 9 + S_A + 1 = 62$, so $S_A$...
The eagle is flying at a speed of 21,645 m/s. Consider an xy-coordinate system. Let a vertical line at x represent the flight path of the sparrow, and let the hawk start at the origin. At time t, the sparrow is at \((x, v_x t)\), where \(v_x\) is the sparrow’s speed, and the hawk is at \((x, y)\). Now the slope of the tangent to the hawk’s flight path at \((x, y)\) is \(dy/dx = (v_y v_x - x v_y)/y\). Solving for t gives: \(t = (v_y v_x)/v_y - x v_y/y\). Also, \(v_y = S = f y/(1 + p^2) dx\). Solving for t and equating the two values gives: \((v_y f)/(1 + p^2) dx = (x v_y v_x + y/v_y)/x\). Differentiating and simplifying gives: \((v_y f + v_x) y = (x v_y v_x + y/v_y)/x\) dx, which upon rearranging is: \(dy/(1 + p^2) = [(v_y f + v_x)/x] dx = ndx/x\), where \(n = v_y/y\). Integrating between 0 and \(p\) and 0 and \(x\) gives: \(ln[p + \sqrt{1 + p^2}] = n ln[x/(x-x)]\) or \(p + \sqrt{1 + p^2} = x\sqrt{x-x}\). Replacing \(p\) with \(dy/dx\) and multiplying by \(dx\) gives \(dy + \sqrt{1 + p^2} dx = dy + dx = [x v_y v_x + y/v_y]/x\) dx. Integrating between 0 and \(x\), yields \(y - S = v_y t + v_x = x/(1-n)\), where the subscript \(c\) refers to capture. Solving for \(t\), gives \(t = x/(1-n) v_y + v_x = n x/(1-n)\). Since \(t\) is the same for the hawk and eagle, we have \(n x/(1-n)^2 = n x/(1-n)^2\). Therefore, \(20 n/(1-n)^2 = 0.5(40)/(1-0.5)^2 = 80/3\) and \(n/(1-n)^2 = 4/3\). This gives \(3 n = 4 - 4 n^2\) or \(4 n^2 + 3 n - 4 = 0\). Solving gives \(n = [-3 + \sqrt{9 + 64}]/8 = 0.693\), so the eagle is flying \(1/n = 1.443\) times as fast as the sparrow or 21,645 m/s. The problem can also be solved using numerical integration by computer.

**BONUS.** Ninety-six different dodecahedrons can be distinguished, with a distribution of 1, 1, 3, 5, 12, 14, 24, 14, 12, 5, 3, 1, 1 for 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 red faces, respectively. We have found no better way of determining this answer than just counting, but one needs to be very careful because of the many possible orientations. Fred Tydeman wrote a computer program which examined all 60 orientations to check for duplicates. MathWorld (mathworld.com/wolfram/polyhedron-coloring.html) says that the number of colorings for a dodecahedron is given by \(11n^15 + n^4 + n^6/60\), where \(n\) is the number of colors. For \(n = 2\), this equation gives 96; however, the derivation is too complex to present.

**C. Computer Bonus.** The unique solution is 549,386,721 \(\times 743,816,529 = 23,439^2 \times 27,273^2 = 639,251,847^2\). The equation \(A + B = C^2\), where \(A, B,\) and \(C\) are different nine-digit integers each using the digits 1 through 9 exactly once, has 620 solutions, but the solution with both \(A\) and \(B\) as perfect squares is unique. Write a computer program to look for perfect squares among the 5(8!) = 201,600 nine-digit numbers that use the digits 1 to 9 exactly once and do not end in 2, 3, 7, or \(8\), and save the square roots of the 30 numbers that are perfect squares. Then, check the 30/29(2) = 435 ways to multiply two of these square roots until you find a product that is a nine-digit number using the digits 1 through 9. A computer can accomplish this task in less than a minute! Email [dondechman_2000@yahoo.co](mailto:dondechman_2000@yahoo.co) for a copy of his QBasic program.

**NEW SPRING PROBLEMS**

1. Solve the following cryptic multiplication, where each different letter stands for a different digit, the same letter always stands for the same digit, and there are no leading zeros. An * can stand for any digit.

   **RNAGEL**

   **R** **N** **A** **G** **E** **L**

   What is the value of RNAGEL?

   —150 Puzzles in Crypt-Arithmetic by Maxey Brooke

2. A spider is chasing an ant. The spider is crawling counterclockwise at a speed of 701 cm/min on the circumference of a circle with a diameter of 100 cm. The ant is crawling at a speed of 700 cm/min, also counterclockwise, on a semicircle consisting of the upper half of the spider’s circle plus a diameter. At the start of the chase, the ant is at the left end of the diameter and about to crawl along it, and the spider is at the other end of the diameter and ready to start crawling along the circumference of the circle (to which it is restricted). They commence crawling at the same instant. How many complete circuits of the circle must the spider make before it catches the ant? Idealize the problem by treating the spider and the ant as points.

   —John H. Cook

3. Doris, the librarian, wishing to fill an empty bookshelf, asked her assistants how many books it would take to fill the shelf exactly. Al said it would take 2 catalogs, 3 dictionaries, and 3 encyclopedias; Bob said it would take 4 catalogs, 3 dictionaries, and 2 encyclopedias; and Connie said it would take 4 catalogs, 4 dictionaries, and 3 encyclopedias. As it turned out, only two of these estimates were correct. Desiring to fill the shelf with only one kind of book, Doris found that only one of the three types would exactly fill the shelf and that 15 of that type were required. Which type of book did Doris use? All the catalogs are the same width, all the dictionaries are the same width, and all the encyclopedias are the same width.

   —Logical Deduction Puzzles by George J. Summers

4. Take the aces, kings, queens, and jacks from a deck of cards and arrange the 16 cards in a four-by-four square array such that no two cards of the same rank or same suit are in the same row, the same column, or the same major diagonal. How many such arrangements are possible, if rotations and reflections are considered to be the same arrangement?

   —A Gardner’s Workout by Martin Gardner

(Continued on page 32.)
‘49 Schneider, Gene W.; August 21, 2009.
TX B ’37 Holcomb, Dysart E.; February 26, 2010.
‘56 Young, Terence O.; November 17, 2010.
‘58 Dixon, Floyd A.; no details.
TX Δ ‘36 Sherwood, Robert S.; December 8, 2008.
‘57 Wende, Harvey O.; no details.
TX A ‘56 Rai, Charanjit; March 26, 2003.
UT A ‘49 Noorda, Raymond J.; October 9, 2006.
VA A ‘54 Berton, Furrman W.; June 1, 2010.
‘57 Bryson, Bobby L.; no details.
‘49 Crossfield, Albert S.; April 19, 2006.
‘50 Kumasaka, Kazuo; November 18, 2010.
‘61 Birkeland, Christian J.; no details.
WA B ‘35 Garrett, John C.; no details.
‘36 Lauckhart, Donald W.; January 29, 1996.
‘44 Schurman, Glenn A.; December 30, 2010.
‘59 Szczyrbak, Jackson; no details.
‘49 Karas, George P.; March 27, 2010.
‘52 Sackett, Robert W.; October 31, 2010.
‘57 Schlesseman, Raymond G.; May 22, 2006.
‘66 Hause, Lawrence L.; no details.
ΣT A ‘50 Blume, Myron H.; April 24, 2010.
ΣT E ‘56 Gabrielson, Harold W.; April 18, 1990.
ΣT H ‘52 Siehofer, Otto J.; no details.
ΣT E ‘48 Ruben, Sherwin; no details.
ΣT E ‘56 Oelke, Harlan; August 30, 2009.
ΣT Σ ‘51 Grigsby, Robert A.; October 14, 2002.
ΣT AB ‘49 Gertty, John W.; March 14, 2005.

**Correction**

Steven G. Jenks, OR A’73, was incorrectly added to Chapter Eternal in the Spring 2007 issue of The Bent. He is alive and well.

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**BRAIN TICKLERS**

(Continued from page 37.)

In the town of Isobar, every rainy day is followed by a sunny day. Every sunny day is followed by either a rainy day or a sunny day with equal probability. To the nearest day, what is the expected number of sunny days in a 968-day year? On a rainy day, it rains all day; on a sunny day, it is sunny all day.

—The *Surprise Attack in Mathematical Problems* by L.A. Graham

**Computer Bonus.** Within the set of prime numbers is a subset of primes, the sums of whose digits are also primes; we will call such numbers double primes. For example, 23 is a double prime because it is a prime number whose digits sum to 5, which is also prime; while 13 is not a double prime since it is a prime whose digits sum to 4, which is not prime. Let $P_n$ be the $n$th smallest prime, and let $D_n$ equal the number of double primes less than or equal to $P_n$. For $N = 1, 2, 3, 4, \text{ and } 5, D_N = 1$. However, as more primes are considered, $D_N/N$ generally (although not continuously) decreases. For example, when $N = 9, D_9/N = 2/3$. Determine the value of $N$ such that $D_N/N < 1/e$ for the first time, where $e$ is the base for the natural logarithms.

—Samuel L. SanGregory, OH M’88

Send your answers to any or all of the Spring Brain Ticklers to Jim Froula, *Tau Beta Pi, P.O. Box 2697, Knoxville, TN 37901-2697* or only as plain text by email to BrainTicklers@tbpi.or. The cutoff date for entries to the Spring column is the appearance of the Summer *Bent* in late June. It is not necessary to include the method of solution. The Computer Bonus is not graded. The judges welcome any interesting problems that might be suitable for the column. Jim will forward your entries to the judges: F.J. Tydeman, CA A’73; D.A. Dechman, TX A’57; J.L. Bradshaw, PA A’82; and the columnist for this issue, H.G. Mcllvried, III, PA Γ’53.