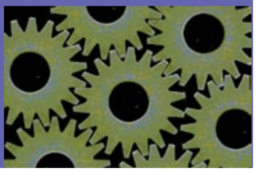


Brain Ticklers

RESULTS FROM FALL 2008

Perfect

*Gerken, Gary M.	Non-member
*Skowronski, Victor J.	NJ A '71
*Spong, Robert N.	UT A '58
*Stribling, Jeffrey R.	CA A '92

Other

Bernacki, Stephen E.	MA A '70
Bertrand, Richard M.	WI B '73
Bohdan, Timothy E.	IN Γ '85
Brule, John D.	MI B '49
*Doniger, Kenneth J.	CA A '77
Harris, Kent	Non-member
Jones, Donlan F.	CA Z '52
*Kimsey, David B.	AL A '71
Lalinsky, Mark A.	MI Γ '77
Marks, Lawrence B.	NY I '81
Marks, Benjamin	Member's son
*Marx, Kenneth D.	OR A '61
*Midgley, James E.	MI Γ '56
*Quintana, Juan S.	OH Θ '62
Rentz, Peter E.	IN A '55
Routh, Andre G.	FL B '89
*Schmidt, V. Hugo	WA B '51
Schorp, Katrina M.	TX Λ '08
*Sifferman, Scott D.	AZ B '04
Siskind, Kenneth S.	RI A '86
Siskind, Amanda	Member's daughter
Spring, Gary S.	MA Z '82
Strong, Michael D.	PA A '84
Sutor, David	Member's son
Van Houten, Karen J.	ID A '76
Voellinger, Edward J.	Non-member
*Wolff, Nicholas L.	NE A '00

* Denotes correct bonus solution

FALL REVIEW

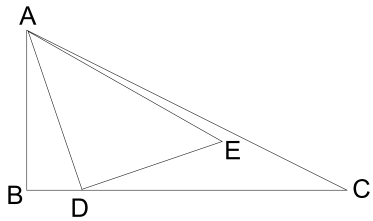
The Fall set proved to be rather difficult. We received more correct answers to the bonus (double pendulum) than we did to regular problems 2 (placing 8 queens on a chessboard), 4 (drawing all four suits from a deck of cards), and 5 (beetle on falling stick).

In looking over some old papers, I found an entry for the Winter '08 Ticklers from **Donlan F. Jones,** *CA Z '52*, that was inadvertently not acknowledged. We apologize for the oversight.

WINTER SOLUTIONS

The Winter Tickler entries will be acknowledged in the Summer column. Meanwhile, here are the answers.

1 The smallest triangle meeting the problem's requirements is a (12, 16, 20) right triangle that fits inside a (11, 60, 61) right triangle. A little thought will show that the two triangles can be arranged as shown in the accompanying figure. Let angle BAD = α, angle DAE = β, and angle BAC = θ. Then, for triangle ADE to fit in triangle ABC, we must have $\alpha + \beta < \theta$ or $\cos^{-1}AB/AD + \cos^{-1}AD/AE < \cos^{-1}AB/AC$. It seems reasonable to try to find triangles whose shortest sides differ by one unit, such as (6, 8, 10) and (5, 12, 13), which unfortunately doesn't work. Trying other multiples of (3, 4, 5), we have (9, 12, 15) and (8, 15, 17), which also doesn't work, but (12, 16, 20) and (11, 60, 61) does work, since $\cos^{-1}11/12 + \cos^{-1}12/20 = 23.56^\circ + 53.13^\circ = 76.69^\circ < \cos^{-1}11/61 = 79.61^\circ$.



2 The smallest integer that is four times as big when 4 is appended at the beginning as when 4 is appended at the end is 10,256. Let N be the number. Then, $4(10^n) + N = 4(10N + 4)$, where n is the number of digits in N. Let $N = 4M$. Then, $10^n + M = 40M + 4$, or $39M = 10^n - 4$. Divide 96, 996, 9996, … by 39 until a perfect division is found. Since 99,996/39 = 2,564, N = 4(2,564) = 10,256.

3 The clock mender set the two clocks at 9:48 a.m. the previous Monday. Since both clocks strike 8 simultaneously, one clock must gain while the other loses, for a total gain plus loss of 12 hr = 720 min (or a multiple of 720). Let x = minutes gained by clock A per day and y = minutes lost by clock B per day. Therefore, in m minutes, A gains $m(1{,}440 + x)/1{,}440$ min, and B loses $m(1{,}440 - y)/1{,}440$ min. The difference between these two values is a multiple of 12 hr. Thus, $m(x + y)/1{,}440 = 720z$, or $m = 1{,}440(720z)/(x + y)$. But m is an integer < 8(1,440); thus $x + y$ divides 1,440(720). Furthermore, x and y are both < 60, so that $x + y$ = 96, 100, or 108. If $x + y = 96$, m = 10,800 min = 7 days, 12 hr, outside the clock menders open time. If $x + y = 108$, m = 9,600 min = 6 days, 16 hr, also outside the clock menders open time. Therefore, $x + y = 100$, m = 10,368 min = 7.2 days. Since both clocks have changed an exact number of minutes, both x and y must be multiples of 5, so x = 55 or 50 or 45, with corresponding gains of 6:36, 6:00, and 5:24, or 7 d, 11 hr, 24 min; 7 d, 10 hr, 48 min; and 7 d, 10 hr, 12 min. Therefore, possible original times were 8:26, 9:12, and 9:48. Only 9:48 a.m. the previous Monday is within working hours.

4 The probability that the dominos can form a single chain is 7/18. A double-six set contains 28 dominos. Therefore, a continuous chain will contain 27 joints, four each of six of the pip values (blank through 6) and three of the seventh value. This means that the two pip values at the ends of the chain must be the same. Thus, a necessary condition that a continuous chain can be formed with the chosen dominos on the ends is that the chosen dominos must share a pip value. As well as being necessary, this is also a sufficient condition. Arrange the 28 dominos in a ring (there are many ways to do this). If the chosen dominos are adjacent, break the ring at the joint between them, and you have the desired chain. If they are not adjacent, break the ring into two chains by splitting it at the two joints with the common pip value of the chosen dominos. There are now two cases: (1) the chosen dominos are in different chains or (2)

they are in the same chain. In case (1), this leaves four chain ends with the same pip value; join the two ends that are not on the chosen dominoes, and you have the desired chain.

In case (2), close the chain that does not contain the chosen dominos to form a ring. Break the chain and ring at common pip value joints, and splice them together. The chosen dominos are now at the ends of a single chain. Thus, having a common pip value is necessary and sufficient.

There are seven dominos with a given pip value; therefore, the number of ways to pick two dominos with a given value is C(7, 2) = 7(6)/2 = 21, where C(m, n) is the combinations of m things taken n at a time. Since there are seven possible pip values, there are 7(21) = 147 ways to choose two dominos with a common pip value. The total ways to pick two dominos is C(28, 2) = 28(27)/2 = 378, so the desired probability is 147/378 = 7/18.

5 Three planes are sufficient to allow one to fly safely around the world. A, B, and C take off and fly 1/8 way around the world, consuming ¼ tank of fuel each. A transfers ¼ tank to each of B and C and returns to the island on his remaining ¼ tank of fuel. B and C fly another 1/8 way at which point B transfers ¼ tank to C and returns to the island on his remaining ½ tank of fuel. C flies a further halfway around the world on his full tank. Meanwhile, A and B refuel and take off and fly toward C. At the 1/8 point, A transfers ¼ tank of fuel to B and returns to the island. B meets C at the 3/4 point and transfers ¼ tank of fuel, leaving B ½ tank to return to the island. Meantime, A has refueled and meets C at the 7/8 point and gives him ¼ tank of fuel to complete his round-the-world trip.

Bonus. The expected value of your score in trying for 6s in Yahtzee is 455/36 = 12.6389, or a little over two 6's in three tosses. Our original approach was as follows. Let p, q, and r equal the number of 6s thrown on the first, second, and third tosses, respectively, and p_1, p_2, and p_3 the corresponding probabilities. Now, p_1 = C(5, p)$(1/6)^p(5/6)^{5-p}$, p_2 = C(5-p, q)$(1/6)^q(5/6)^{5-p-q}$, and p_3 = C(5-p-q, r)$(1/6)^r(5/6)^{5-p-q-r}$, where C(m, n) is the combinations of m things taken n at a time, and $P(p, q, r) = p_1p_2p_3$. The corresponding expected value is $6(p + q + r)$P(p, q, r). To get the total expected value, it is necessary to sum over all possible combinations of p, q, and r from (0, 0, 0), (0, 1, 0), (0, 0, 1), etc., through (5, 0, 0), (0, 5, 0), (0, 0, 5). A spreadsheet is helpful for this calculation, or you can use a decision tree. The following table summarizes the results.

No. of 6s	Score	Prob.	E_v
0	0	0.0649	0
1	6	0.2363	1.4175
2	12	0.3440	4.1279
3	18	0.2504	4.5076
4	24	0.0911	2.1877
5	30	0.0133	0.3982
Total		1.0000	12.6389

However, **Toby Berger,** *CT A '62*, provided a much more insightful and beautiful solution. Consider the first die. The probability that it is tossed all three times without yielding a 6 is $(5/6)^3$ = 125/216. Thus, the probability that it does yield a 6 is 1 - 125/216 = 91/216, while its expected contribution to the overall score is 6(91/216) = 91/36. The same is true for each of the other four dice, so the expected score is 5 times 91/36 = 455/36.

Computer Bonus. The only solution in integers to the equation $y^3 = x^2 - 15$, besides $x = 4$, $y = 1$, is x = 1,138, y = 109. It is easy to solve this problem using a spreadsheet or a QBasic computer program.

NEW SPRING PROBLEMS

For a break from Spring finals, give these Ticklers a try.

1 Scott, the astronaut, has landed on Planet X, which has the same density as Earth but only half the mass. Finding no atmosphere, he decides to leave; he fires his hyperdrive (that has the ability to generate thrust while consuming very little fuel) and lifts off. Unfortunately, just as he reaches half the escape velocity, his engine shuts off prematurely. How much time does Scott have to eject in his escape pod before his ship is pulled back to the planet's surface by gravity and crashes? Assume that the space ship has a constant mass of 1,000 kg and that its engine generates a constant thrust of 35,000 N as long as it is firing. Ignore the effect of the planet's rotation and the change in gravity with distance from the surface of Planet X.

—**Scott L. Bilker,** *NJ Γ '90*

2 Students Al, Bill, Carl, Don, and Ed live in rooms A, B, C, D, and E in the dorm, although not necessarily in that order. They have just taken a test in which they each got a different score. When questioned, they made the following remarks:

Al a1: Carl scored lower than Bill.
a2: Don is in Room D.
Bill b1: Ed scored lower than Carl.
b2: Don is in Room C.
Carl c1: Al is in Room C.
c2: The student in Room B did not have the second-best score.
Don d1: Carl is in Room E.
d2: Ed is not in Room D.
Ed e1: I scored lower than Bill.
e2: Al got the highest score.

Remarks in which the first person mentioned scored lower than the speaker are true, while remarks in which the first person mentioned scored higher than the speaker are false. Also, when making a remark about himself, only those who scored highest or next highest tell the truth. How did the students rank in the test, and what rooms are they in?

—*Brain Puzzler's Delight* by E.R. Emmet

3 Solve the following cryptic division, which results in a repeating decimal.

TBPI/AAAAA = .ONANDONANDON…

We want the solution with the biggest PI. The usual rules for cryptics apply.

—**H.G. McIlvried III,** *PA Γ '53*

4 Ignoring colons, a 24-hour digital watch shows many palindromes during the day, e.g., 1:01:01 and 23:55:32.

BRAIN TICKLERS

(Continued from page 39.)

How many palindromes show in 24 hours, what are the longest and shortest times between palindromes, and what two palindromes have a difference that is closest to 12 hours? A 24-hour clock goes from 23:59:59 to 0:00:00.

—*Technology Review*

5 What is the probability of getting a royal flush (A-K-Q-J-10 in the same suit in any order) in seven-card stud poker played with a 53-card deck (standard 52-card deck plus a joker), if the joker is wild and can be used to represent any card?

—**F.J. Tydeman,** *CA Δ '73*

Bonus. Alice's doctor has prescribed that every morning she is to take one each of k different pills. The pills all look identical except that each medicine type has a unique color. In preparation for an n-day trip, Alice put n pills from each of her k bottles into one larger bottle to save space. As a result Alice, who is totally color blind but forgot that fact while packing, was forced each morning during the trip to withdraw randomly k pills from the bottle and take them. What is the overall probability that every day of the trip Alice indeed took exactly what her doctor prescribed? Present your answer as a function of n and k.

—**Toby Berger,** *CT A '62*

Double Bonus. The sequence 29, 149, 269, 389, 509 contains five consecutive primes having a common difference of 120; however, the next member of the sequence 629 is composite. In a sequence of positive integers having a common difference of 210, what is the maximum possible number of consecutive terms that are all primes? Give an example of such a sequence.

—*Technology Review*

Postal mail your answers to any or all of the Brain Ticklers to Jim Froula, Tau Beta Pi, P.O. Box 2697, Knoxville, TN 37901-2697, or email plain text (no HTML, no attachments) to *BrainTicklers@tbp.org*.

The cutoff date for entries to the Spring column is the appearance of the Summer Bent during late June.

Spring entries will be acknowledged in the Fall column. The details of your calculations are not necessary, and the Double Bonus is not graded. We welcome any interesting new problems that may be suitable for the column.

Jim will forward your entries to the judges, who are: **F. J. Tydeman,** *CA Δ '73*; **D.A. Dechman,** *TX A '57*; **J.L. Bradshaw,** *PA A '82*; and the columnist for this issue

—**H.G. McIlvried III,** *PA Γ '53*

TBP CONNECT — Tau Beta Pi's InCircle Community

THE BEST PEOPLE ENGINEERING JOB BOARD

Tau Beta Pi has renamed its career center and job board, to alert members about the improved opportunities available through the partnership with JobTarget.

During these challenging economic times, it is essential that TBΠ members have a resource where they can seek employment, support, and information—*www.tbp.org*.

JOBTARGET

2009 SPRING CONFERENCES

The District Program provides a vital link between the national leaders and individual chapters. Each spring the Directors meet with students at regional conferences to provide both retiring and new officers opportunities to improve chapter operations and to socialize. **Interested alumni are encouraged to attend,** but should email *tbp@tbp.org* for information about remaining events.

DISTRICT	LOCATION	DATE
1	Boston, MA	Feb. 20-21
2	New Brunswick, NJ	Feb. 21-22
3	Lewisburg, PA	Feb. 27-28
4	Morgantown, WV	Apr. 4
5	Atlanta, GA	Mar. 28-29
6	Auburn, AL	Feb. 6-8
7	Detroit, MI	Apr. 4
8	Angola, IN	Apr. 4
9	Wichita, KS	Mar. 6-7
10	New Orleans, LA	Feb. 28
11	Minneapolis, MN	Apr. 3-4
12	Boulder, CO	Feb. 27-28
13	Flagstaff, AZ	Apr. 18-19
14	Portland, OR	Apr. 4
15	Santa Cruz, CA	Mar. 6-7
16	San Diego, CA	April 18

TBΠ Directory

Executive Council

President Larry A. Simonson, Ph.D., P.E., *SD A '69*, SDSM&T Foundation, 501 E. St. Joseph St., Rapid City, SD 57701. *(larry@tbp.org)*

Vice President Solange C. Dao, P.E., *FL A '95*, 1110 E. Marks St., Orlando, FL 32803. *(solange@tbp.org)*

Councillor Jonathan F.K. Earle, Ph.D., P.E., *FL A '65*, 8516 SW 20th Lane, Gainesville, FL 32607. *(jonathan@tbp.org)*

Councillor Jason A. Huggins, P.E., *FL A '96*, 4701 Hickory Shores Blvd., Gulf Breeze, FL 32563. *(jason@tbp.org)*

Councillor Norman Pih, *TN A '82*, No. 10, 811 W. Cherry Ave., Flagstaff, AZ 86001. *(norman@tbp.org)*

International Headquarters

Executive Director James D. Froula, P.E., *TN A '67*, P.O. Box 2697, Knoxville, TN 37901-2697. *(tbp@tbp.org)*

Asst. Sec.-Treas. Roger E. Hawks, *NY A '75*, P.O. Box 2697, Knoxville, TN 37901-2697. *(tbp@tbp.org)*