Brain Ticklers

RESULTS FROM FALL 2007

Perfect

* Fenstermacher, T.Edward	MD	B	'80
Mayer, Michael A.	IL	A	'89
* Schmidt, V. Hugo	WA	B	'51
* Stribling, Jeffrey R.	CA	A	'92
Strong, Michael D.	PA	A	'84

Other

Adamo, Paul M.	TX	A	'85
Adamo, Megan	Daughter of member		
Alexander, Jay A.	IL	Γ	'86
Aron, Gert	IA	B	'58
Beaudet, Paul R.	Father of member		
* Brule, John D.	MI	B	'49
Conway, David B.	TX	I	'79
Couillard, J. Gregory	IL	A	'89
Eckley, Paul L.	NV	A	'75
James, Catherine A.	Wife of member		
Jones, Donlan F.	CA	Z	'52
Kern, Peter L.	NY	Λ	'62
* Kimsey, David B.	AL	A	'71
Lau, Kee-Wai	Non-member		
Linnemann, Pete	Non-member		
Meyer, Aaron S.	CA	E	'09
Quintana, Juan S.	OH	Θ	'62
Rasbold, J. Charles	OH	A	'83
Rentz, Peter E.	IN	A	'55
Scholz, Gregory R.	PA	B	'00
Spong, Robert N.	UT	A	'58
Venema, Todd M.	OH	H	'08
Vinoski, Stephen B.	TN	Δ	'85
* Voellinger, Edward J.	Non-member		

* Denotes correct bonus solution

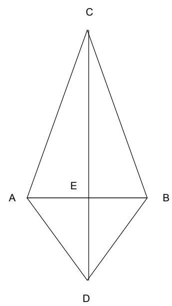
FALL REVIEW

Of the regular Fall problems, the one about packing widgets in boxes proved to be most difficult, with only half the entries having correct answers. The bonus about hitting a golf ball was the most difficult overall, with less than one fourth of the entries having correct solutions.

WINTER SOLUTIONS

1 Clara's solution is 1,024, 1,597, and 358. The secret to this puzzle is that there are only two four-digit Fibonacci numbers with all different digits; therefore, FIVE = 1597 or 2584; and FOUR is a four-digit square, starting with the digit 1 and not containing the digits 5, 7, or 9, or starting with the digit 2 and not containing a 4, 5, or 8. The only possibilities for FOUR are 1024, 2601, and 2916. For, 1024, S and X are two of 3, 6, and 8, giving six possibilities for SIX. Of these, only 358 is a semiprime. Similar logic with the other possibilities gives two other solutions: 2601, 2584, 753 and 2916, 2584, 753. Since these repeat some integers, Clara's solution is that given above.

2 The volume of the pyramid is 0.071463927. Consider the accompanying figure, where the top triangle (ΔABC) represents one of the star's points and the bottom triangle (ΔABD) represents one-fifth of the pentagonal base of the pyramid. The base angles of ΔABC are 72° and the apex is 36°.



ΔABD has base angles of 54° and an apex of 72°. Let CE = x, DE = y, and AE = z. Then, $x + y = z(1/\tan 18° + 1/\tan 36°) = 1$. Solving for z using the values $\tan 18° = \sqrt{5}(5 - 2\sqrt{5})/5$ and $\tan 36° = \sqrt{(5 - 2\sqrt{5})}$ gives $z = \sqrt{(5 - 2\sqrt{5})}/(1 + \sqrt{5})$, from which $x = \sqrt{5}/(1 + \sqrt{5})$, and $y = 1/(1 + \sqrt{5})$. Let h be the altitude of the pyramid. Then, $h^2 = x^2 - y^2 = 2/(1 + \sqrt{5})$. Let A = area of base. Then, $A = 5yz = 5\sqrt{(5 - 2\sqrt{5})}/(1 + \sqrt{5})^2$, and $V = hA/3 = (10/3)\sqrt{(5 - 2\sqrt{5})}/(1 + \sqrt{5})^3 = 0.071463927$.

3 $x = 4\sqrt{5}$. Given $(x + 9)^{1/3} - (x - 9)^{1/3} = 3$, cube both sides to give:
$(x + 9) - 3(x + 9)^{2/3}(x - 9)^{1/3} + 3(x + 9)^{1/3}(x - 9)^{2/3} - (x - 9) = 27$.

Rearranging gives:
$(x^2 - 81)^{1/3}[(x + 9)^{1/3} - (x - 9)^{1/3}] = -3$.
But the expression in square brackets equals 3. Therefore, we have $(x^2 - 81)^{1/3} = -1$. Cubing and rearranging gives $x^2 = 80$, for which the positive root is $x = 4\sqrt{5} = 8.9443$.

4 The length of the side of each cube is 29 1/3 inches. A little thought will indicate that the first cube was not fully submerged when the second cube was added and that neither the first or second cube was fully submerged when the third cube was added, but at the end all three cubes were submerged. Let A = surface area of the pond, L = length of the side of a cube, and h = initial depth of water in the pond. Writing equations for the amount of water displaced by the three cubes, we get:

After 1st cube: $L^2(h + 3) = 3A$
After 2nd cube: $2L^2(h + 7) = 7A$
After 3rd cube: $3L^3 = 11A$.

Solving these three equations simultaneously gives L = 29 1/3 inches, h = 21 inches, and A = 47.80 ft².

5 There are several ways to line up the dominoes that vary slightly from each other. If n is the highest double, then the total of all the pip values is $T = n(n + 1)(n + 2)/2$. For $n = 6$, $T = 168$; thus, 12 is the highest square that can be accommodated, and the pip values of the five unused dominoes must total 168 – 144 = 24. Since four of the unused dominoes are doubles, a little thought will indicate that there are only three possibilities, as shown in the first column of the accompanying table. To reach the squares from 1 through 12, one must successively add 1, 3, 5, …, 23 and do this using the minimum number of dominoes, which turn out to be 1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 3, and 5, respectively. Solutions exist for all three of these cases *(See grid below)*. All three solutions have minor variations. Obviously, if the same value appears at both ends of the layout when a given sum has been reached, two options are available for the next domino. We

Unused Dominoes	*Example Layout*
11, 22, 33, 44, 40	02|21|15|53|31|16|66|62|23|30|00|01|14|43|36|65|54|42|25|55|50|06|64
11, 22, 33, 44, 31	12|24|40|02|25|54|46|61|15|56|63|30|00|01|14|43|32|26|66|60|05|55|53
11, 22, 33, 55, 20	24|40|03|31|16|66|63|32|26|65|50|00|01|12|25|54|44|43|35|51|14|46|60

gave credit for any of the possible solutions; these can be found by trial and error or with the help of a computer.

Bonus. The probability of getting a winning hand in the game of 35 is 0.0391. The first step is to determine the number of possible winning combinations for a given number of cards in the same suit. For example, with 4 cards in the same suit, there are 16 winning combinations: KQJ5; KQ105; KJ105; QJ105; KQ96; KJ96; K1096; QJ96; Q1096; J1096; KQ87; KJ87; K1087; QJ87; Q1087; and J1087. (A computer is helpful in tabulating winning combinations.) Each of these possibilities has to be multiplied by the number of ways to get the rest of the 9 cards, as given in the table at right above. From the total (to avoid double counting), we need to subtract the number of hands with winners in two suits. For 4,4 hands, we have 16 winning combos for each 4-card suit, 6 ways to pick the two suits, and 26 ways to pick the 9th card for a total of 16x16x6x26 = 39,936. Similarly, for the 4,5 case, there are 16 winning combos for the 4-card suit, 206 winning combinations for the 5-card suit, and 12 ways to pick the two suits for a total of 16x206x12 = 39,552, so 143,863,512 - 39,936 - 39,552 = 143,784,024. The total ways to get 9 cards is C(52, 9) = 3,679,075,400. Thus, P =143,784,024/3,679,075,400 = 0.039081564.

Double Bonus. The composer is Wolfgang Amadeus Mozart. Let x equal the age he would be in 2006 and y equal his age at death. Then, $x = 10(y - 10)$, which means that x ends in a 0. Let the year he died be $D = 1000 + 100a + 10b + c$. Then, $y = 2006 - 1000c - 100b - 10a - 1 = 2005 - 1000c - 100b - 10a$, which means that y ends in a 5 and x is divisible by 50. Now, $D = 2006 - x + y = 2006 - x + x/10 + 10$, or $D = 2016 - 45z$, where $z = x/50$. Trying different values for z, only 5 gives a consistent value for y ($x = 250$; $y = 250/10 + 10 = 35$) that gives a death year ($2006 - 250 + 35 = 1791$) such that $2006 - 1971 = 35$. The composer is Wolfgang Amadeus Mozart, who would have been 250 in 2006.

No. of cards in suit	*No. of winning combinations*	*No. of ways to pick rest of hand*	*No. of suits*	*No. of winning hands*
4	16	C(39, 5) = 575,757	4	36,848,448
5	206	C(39, 4) = 82,251	4	67,774,824
6	938	C(39, 3) = 9,139	4	34,289,528
7	1,602	C(39, 2) = 741	4	4,748,328
8	1,279	C(39, 1) = 39	4	199,524
9	715	C(39, 0) = 1	4	2,860
		Total		143,863,512

NEW SPRING PROBLEMS

Now for some new Ticklers to keep your brain active during Spring break.

1 A photographer has been hired to take a group picture of a high-school reunion that is attended by 100 graduates. How many pictures should he take to have at least a 95 percent probability that he has at least one picture in which no one has his or her eyes closed? Assume that the average person blinks six times a minute and that their eyes are closed for one tenth of a second on each blink. Assume the shutter clicks instantaneously.

—**H.G. McIlvried III,** *PA Γ '53*

2 Six unit squares can be joined along their edges to form 35 different hexominos, the simplest being a one by six rectangle. How many of these hexominos can be folded along edges joining the squares to form a unit cube? A rotation or reflection is not considered to be a different hexomino.

—*The Colossal Book of Mathematics* by Martin Gardner

3 Many of you puzzle fans are familiar with the famous cryptic, SEND + MORE = MONEY. Here is the response: ALAS + LASS + NO + MORE = CASH. We want the solution to this cryptic addition with the biggest NO. Each different letter represents a different digit, but the same letter always represents the same digit. There are no leading zeros.

—*Madachy's Mathematical Recreations* by Joseph S. Madachy

4 The game of snooker is played with 22 balls (15 red balls, six colored balls, and a cue ball) on a six pocket table that resembles a standard pool table. The red balls are not numbered, but the colored balls are numbered 2, 3, 4, 5, 6, and 7.

A player scores one point for each red ball pocketed and the indicated number of points for pocketing a colored ball. A player must pocket a red ball, then any colored ball, then another red ball, then another colored ball, etc. The red balls are not respotted, but as long as any red balls are on the table, colored balls are respotted.

After all 15 red balls are pocketed, the six colored balls are sunk in numerical order without being respotted.

Al recently ran the table, sinking one ball at a time. His cumulative score after pocketing each of the first 14 colored balls and the 15th red ball was always a semiprime (the product of two primes), but his score after pocketing each of the final six colored balls was never a semiprime.

What was his final score?

—Richard England in *New Scientist*

5 Triangular numbers are numbers of the form $T_n = n(n + 1)/2$. They are called triangular numbers because T_n points can be arranged, bowling pin style, to form a triangle.

The first few triangular numbers are 1, 3, 6, 10, and 15. What is the sum of the reciprocals of the triangular numbers from 1 to infinity?

—*Mathematical Journeys* by Peter D. Schumer

Bonus. Fred has five dice and decides to see how long it will take him to throw five 6s using the following procedure. He tosses all five dice. He then picks up and re-tosses those that are not 6s. He repeats this process until he has all 6s. What is the expected number of tosses to get five 6s? Give the answer exactly in terms of a fraction.

—**F.J. Tydeman,** *CA Δ '73*

Computer Bonus. A weak prime is a prime such that, when any of its digits is replaced (one at a time) by any different digit, the resulting integer is no longer prime. The two smallest weak primes are 294,001 and 505,447. What is the smallest weak prime such that, when its digits are reversed, the result is also a weak prime? This one may take some time. As an exception to our usual rule, we will acknowledge correct solutions in the Fall column.

—**R. Dudley White,** *VA A '76*

Send your answers to any or all of the Spring Brain Ticklers to **Jim Froula, Tau Beta Pi, P. O. Box 2697, Knoxville, TN 37901-2697**, or email to *BrainTicklers@tbp.org* only as plain text. The cutoff date for entries to the Spring column is the appearance of the Summer BENT in June. The method of solution is not necessary. We welcome any interesting problems that might be suitable for the column. Jim will forward your entries to the judges who are **F.J. Tydeman,** *CA Δ '73;* **J.L. Bradshaw,** *PA A '82;* **D.A. Dechman,** *TX A '57;* and the columnist for this issue,

—**H.G. McIlvried III,** *PA Γ '53.*

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