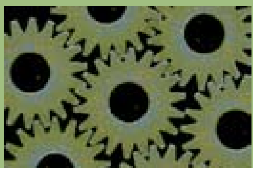


Brain Ticklers

RESULTS FROM FALL 2006

Perfect

	Baines, Elliot “Chip” A., Jr.	NY	Δ	’78
*	Kimsey, David B.	AL	A	’71
*	Pecsvaradi, Thomas	PA	Z	’64
*	Schleehauf, Martin W.	NY	N	’79

Other

*	Alderson, William S.	MI	E	’43
	Alexander, Jay A.	IL	Γ	’86
	Aron, Gert	IA	B	’58
*	Beaudet, Paul R.	Father of member		
	Bohdan, Timothy E.	IN	Γ	’85
	Chow, Jordon	NJ	Γ	’05
	Compton, Alfred	AL	B	’75
*	Couillard, J. Gregory	IL	A	’89
	DeFrank, Mason	NJ	E	’06
	Hellmann, Ilana	NY	H	’06
	Hess, Richard I.	CA	B	’62
	Jones, Donlan F.	CA	Z	’52
	Hull, Chris	6th grade		
	Kelly, Daniel	6th grade		
	Pozzi, Christopher	6th grade		
	Lalinsky, Mark A.	MI	Γ	’77
*	Madajian, David	CA	A	’82
	Madajian, Jonathon	Son of member		
	Marrone, James D.	IN	A	’87
	Meyer, Richard W.	IL	B	’62
	Midgley, James E.	MI	Γ	’56
*	Nabutovsky, Joseph	Father of member		
	Nevins, Russell T.	MA	B	’77
	Pyers, Dean	OH	Z	’84
	Quintana, Juan S.	OH	Θ	’62
	Reed, David G.	VA	Γ	’03
	Reed, Michele B.	Wife of member		
	Rentz, Peter E.	IN	A	’55
	Robillard, David J.	MD	Γ	’88
	Saikali, Jeffrey	OH	B	’96
*	Schmidt, V. Hugo	WA	B	’51
	Scholz, Gregory R.	PA	B	’00
	Siskind, Kenneth S.	RI	A	’86
	Siskind, Amanda	Daugh. of member		
*	Spong, Robert N.	UT	A	’58
	Stanley, Eileen C.	NY	Ξ	’06
*	Stribling, Jeffrey R.	CA	A	’92
	Strong, Michael D.	PA	A	’84
	Valko, Andrew G.	PA	Λ	’80
	Van Houten, Karen	ID	A	’76
	Voellinger, Edward J.	Non-member		

* Denotes correct bonus solution

FALL REVIEW

The Fall Ticklers proved to be more difficult than usual. Only about one-fourth of the entries had correct solutions to No. 4, about the plum pudding, and to No. 5 about the hanging string. Both were slightly more difficult than the bonus about dropping a needle on a grid. We were lenient in grading No. 3, as there is some ambiguity about stretching or compression as the rug is rolled.

WINTER SOLUTIONS

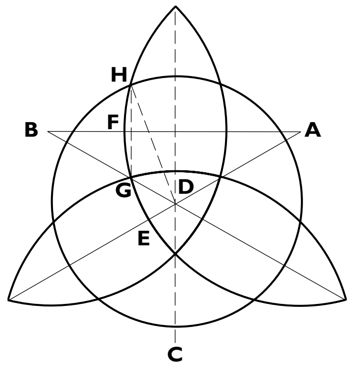
Entries for the Winter Ticklers will be acknowledged in the Summer column. Meanwhile, here are the solutions.

1 The probability that the colony of amoebae will go on forever is 2/3. Let q be the probability of a successful split and p be the probability that the colony will go on forever. With probability q, there will be two amoebae in the second generation. The probability that these two will generate an infinite chain is $1 – (1 – p)^2$, because $(1 – p)^2$ is the probability that neither will do so. Therefore, $p = q[1 – (1 – p)^2]$, since both sides of this equation equal the probability of long-term survival. Simplifying, $qp^2 + (1 – 2q)p = 0$. Solving for p and discarding the 0 root gives $p = (2q – 1)/q$. With q = 75 percent, $p = [2(0.75) - 1]/0.75 = 2/3$. This problem can also be solved using a probability tree.

2 The largest prime p less than 1,000 for which $N = p^3 + 2p^2 + p$ has exactly 42 factors is 823. If $N = p_1{}^a p_2{}^b \ldots p_n{}^n$, then the number of factors of N is $(1 + a)(1 + b)\ldots(1 + n)$. Now, $42 = 2(3)(7)$, so $a = 1$, $b = 2$, and $c = 6$. Also, $N = p(p + 1)^2$. From this, it is clear that $p = p_1$ and $p + 1 = p_2 p_3{}^3$. Therefore, we need to find two primes such that $p_2 p_3{}^3 – 1$ is a prime, but, for this to occur, either p_2 or p_3 must equal 2. Assuming $p_3 = 2$, we have $8p_2 – 1 < 1{,}000$ or $p_2 < 125$. Starting with 113 (the largest prime less than 125) and working down, we find that 103 is the first prime for which $8p – 1 = 823$ is prime.

3 LYNDON x B = JOHNSON is 570140 x 6 = 3420840. By inspection, N must be zero; and the only possibilities for O and B are: (2, 6), (4, 6), (5, 3), (5, 7), (5, 9), (8, 6). From this, the only possibilities for DO × B = SO are 12 × 6 = 72, 14 × 6 = 84, or 15 × 3 = 45. Trying LY × B = JOH with each of these possibilities shows that the only solution is that given above.

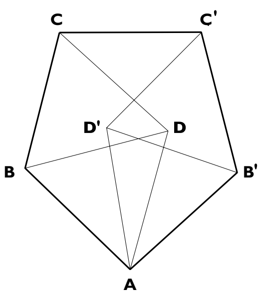
4 The radius r of the inner circle is $\sqrt{(5 - 2\sqrt{3})}/\sqrt{3} = 0.7155$. Refer to the figure below.



Label the centers of the three circles whose arcs form the propeller A, B, and C; and let D be the center of the central circle. Construct the bisectors of the three blades of the propeller. Each of these lines passes through one of the points A, B, or C, and they intersect at point D. It’s clear that the six angles at D are each 60°. Therefore, angle BAD is 30°, and since points E and F bisect arcs of length x, arc EGF $= x/2 + x/2 = x$, so each arc length of x intercepts a central angle of 30°. Let y equal the length of a chord subtending an arc of length x. Then, by the law of cosines, $y^2 = 1^2 + 1^2 - 2(1)(1)\cos 30^\circ = 2 - \sqrt{3}$. Now, DH $= r$, HG $= y$, and DG $= \sqrt{3}y/3$ (two thirds of the altitude of an equilateral triangle of side y). Applying the law of cosines to triangle DHG gives $r^2 = y^2 + y^2/3 – 2y(\sqrt{3}y/3)\cos 120^\circ = (4 + \sqrt{3})y^2/3 = [(4 + \sqrt{3})/3](2 - \sqrt{3}) = (5 - 2\sqrt{3})/3$, which gives the above answer for r.

5 The three numbers are 136, 369, and 630. The only three-digit cube with a three as one of its digits is 343, but 343 satisfies none of the other statements. Checking the twelve triangular numbers for which one of the digits is a 3 (136, 153, 231, 253, 300, 325, 351, 378, 435, 630, 703, and 903) shows that only for 136 and 630 are three of the statements true and three false. Next, checking all three-digit primes with a digit that is a 3 shows that the third number is not a prime, and we know it is not a triangular number and not a cube. Therefore, its middle digit is the average of the other two digits, the third digit differs from the second digit by 3, and it has a two-digit prime factor whose digits differ by 3 or whose sum is a cube. Only 369 satisfies these conditions.

BONUS. The solution consists of two rhombuses, ABCD and AB'C'D', with 1 cm sides and distances BD and B'D' also each 1 cm, arranged so that CC' is 1 cm, as shown in the figure. Although the answer is easy once you see it, it is very difficult to visualize at first.



COMPUTER BONUS. The best answer is $659^{.4365} \approx 17$. The actual value of the left side is approximately 16.99978714, which has an error of 0.00021286. Email Don Dechman at dondechman@verizon.net if you would like a copy of the QBasic program he used to solve this problem.

NEW SPRING PROBLEMS

Here's a new batch of Ticklers for your spring enjoyment. All of these problems, except perhaps the Double Bonus, can be solved with nothing more than a hand calculator, so give the little gray cells some exercise.

1 Find a primitive Pythagorean right triangle whose area is an integer consisting of repetitions (more than one digit long) of the same digit. An example of such an integer is 77,777.

—*The Penguin Dictionary of Curious and Interesting Numbers* by David Wells

2 After grading the advanced-math test but before handing it back, the teacher asked the five students in the class to predict how well other members of the class (but not themselves) did on the test. The teacher received the following five pairs of statements, each pair being made by a different student:

i. Bill did better than Ed;
   Chuck did better than Bill.
ii. Debbie did better than Chuck;
   Art did better than Debbie.
iii. Ed did better than Art;
   Chuck did better than Ed.
iv. Bill did better than Ed;
   Debbie did better than Bill.
v. Chuck did better than Bill;
   Art did better than Chuck.

The student with the highest grade was completely correct; the boy who came in last was completely wrong; and the other students had one correct prediction and one incorrect prediction. Rank the students from highest to lowest grade. There were no ties.

—Martin Hollis in *New Scientist*

3 The game of Quod, invented by Keith Still, is played on an 11 x 11 grid of cells (like an oversized chessboard) minus the four corner squares. Players take turns placing markers on the cells, with the first player to form a square being the winner. As long as a player's four markers form the corners of a square, it is valid. Its sides do not have to be parallel to the sides of the board. How many different squares are possible?

—*How to Cut a Cake and Other Mathematical Conundrums* by Ian Stewart

4 Let S = 1(1!) + 2(2!) + 3(3!) + … + 222(222!). What is the remainder when S is divided by 2007?
—adapted from *Mathematics Teacher*

5 A fruit basket contains:
- MELONS
- PLUMS
- APPLES
- LEMONS
- and a BANANA.

If 2 MELONS are greater than 35 PLUMS but less than 3 APPLES, and 99 LEMONS are greater than 16 APPLES but less than 210 PLUMS, what is the size of a BANANA? The usual rules for cryptics apply. That is, the same letter always represents the same digit, and different letters represent different digits.

—Albert Haddad in *New Scientist*

BONUS. A bag of marbles is prepared by putting six black marbles in the bag. Then, a die is thrown, and a number of white marbles is added equal to the number shown on the die. You are given the bag. You know the procedure used to fill the bag but not the number of white balls in the bag. You draw three marbles without replacement, and they are all white. What is the probability that the next marble you draw will be black?

—from an old exam at Jesus College, Cambridge

DOUBLE BONUS. Given a consecutive run of positive integers from 1 through N, divide them into four sets such that for each set, no two numbers of that set have a sum that equals a third member of the same set. What is the maximum value of N, and what are the members of the four sets?

—*Doctor Ecco's Cyberpuzzles* by Dennis E. Shasta

Send your answers to any or all of the Brain Ticklers to: Jim Froula, Tau Beta Pi, P.O. Box 2697, Knoxville, TN 37901-2697. If your answers are plain text only (no HTML, no attachments), you can e-mail them to BrainTicklers@tbp.org. The cutoff date for receiving entries is the appearance of the Summer BENT in July. Spring entries will be acknowledged in the Fall column. The details of your calculations are not necessary, and the Double Bonus is not graded. We welcome any interesting new problems that may be suitable for the column. Jim will send your entries to the judges: **F.J. Tydeman,** *CA Δ '73;* **D.A. Dechman,** *TX A '57;* **J.L. Bradshaw,** *PA A '82;* andthe columnist for this issue

—**H.G. McIlvried,** *PA Γ '53.*