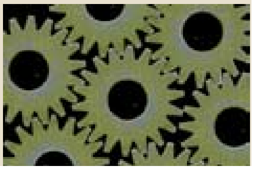


Brain Ticklers

RESULTS FROM FALL 2005

Perfect

	Carver, Robert M.	OH K '87
	Couillard, J. Gregory	IL A '89
	Hafner, Kurt	non-member
	Mitchell, Donald B.	CA Γ '59
	Pecsvaradi, Thomas	PA Z '64
*	Post, Irving G.	PA E '58
*	Schmidt, V. Hugo	WA B '51
*	Strong, Michael D.	PA A '84
	Voellinger, Edward J.	non-member

Other

*	Beaudet, Paul	Father of member
	Bernacki, Steven E.	MA A '70
	Brule, John D.	MI B '49
	Calico, Austin C.	OH B '04
*	Christenson, Ryan C.	UT B '93
	Dunne, Fiona	CA Σ '06
*	Garnett, James M.	MS A '65
	Handley, Vernon K.	GA A '86
	Hynes, David A.	MA Z '59
	James, Catherine A.	Wife of member
	Jordon, R. Jeffery	OK Γ '00
	Kassner, Rudy	non-member
*	Kimsey, David B.	AL A '71
	Klaver, David	NY A '05
	Marks, Lawrence B.	NY I '81
*	Nabutovsky, Joseph	Father of member
	Quintana, Juan S.	OH Θ '62
	Rentz, Peter E.	IN A '55
	Robillard, David J.	MD Γ '88
	Routh, Andre G.	FL B '89
*	Schleehauf, Martin W.	NY N '79
*	Snelling, William E.	GA A '79
*	Spong, Robert N.	UT A '58
*	Steiner, Ray P.	AZ A '63
	Stepanian, Shant	NJ A '06
*	Stribling, Jeffery R.	CA A '92
	Svetlik, Frank	MI A '67
	Vinoski, Stephen B.	TN Δ '85
*	Vogt, Jack C.	OH E '56
*	Yee, David G.	NJ B '04

* Indicates correct bonus solution

FALL REVIEW

The answer to Fall No. 3 can be expressed more simply as arccos(1/3). Several responders gave an answer of 87.5% for Fall No. 4, based on the contestants using delayed responses to provide information to each other. The judges believe that the silences involved constitute communication, which is prohibited by the problem statement (similar action is not allowed in contract bridge), but since everyone may not agree, credit was given for this answer.

WINTER SOLUTIONS

Winter entries will be acknowledged in the Summer column. In the meantime, here are the solutions.

1 Thirty people had both cake and ice cream. One way to quickly visualize this problem is to draw a figure, as shown. (The colored portion shows

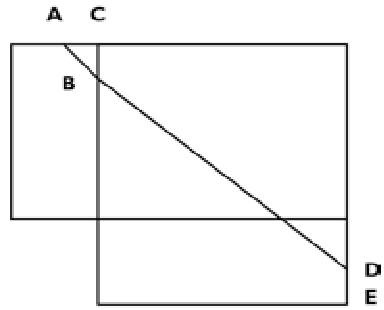
10	Hot Dog		
P.S.	20	Potato Salad	
Cake		30	Cake
Ice Cream			40

the number of people who didn't eat the indicated item.) Because the number of people who didn't eat a particular item sums to 100 (10 + 20 + 30 + 40 = 100), the only way for no one to have eaten all four items is for there to be no overlap in these values. The solution is easily obtained from the figure.

2 Al and Sam played in the 13th set. The total number of sets is 19 = (15 +14 + 9)/2. Ralph played nine sets; therefore, he sat out 10 sets. Because no one sat out two sets in a row, Ralph must have played and lost in all the even-numbered sets and sat out all the odd-numbered sets.

3 S+A+I+N+T+T+R+O+P+E+Z = 33. With 15 equations in 17 unknowns, the problem seems unsolvable. But, H, P, and R always occur with T, which does not occur otherwise. Replacing H+T, P+T, and R+T with H_T, P_T, and R_T eliminates one variable, and the rest can be solved in terms of one of them, say E, to give A = 11+E; C = -1; D = 8; F = 11; H_T = -3E; I = 8+2E; N = 1-E; O = 1+2E; P_T = 9+E; Q = -3-E; R_T = -4-2E; S = -2-2E; U = E; X = 0; and Z = 9-2E. On substituting these values into S+A+I+N+T+T+R+O+P+E+Z, the Es cancel out, leaving 33.

4 The shortest ribbon is 84.8769 cm long. The easiest way to solve this problem is to flatten the box as shown. (The figure shows half the box. Make a mirror image and line up AC with DE to get the other half.) In the figure, AB = 1, AC = BC = DE = √2/2. BD is a straight line that is the hypotenuse of a right triangle with sides of length 30 - √2 and 30; therefore, BD = 41.4385, and the total length of string is 2(41.4385 + 1) = 84.8769. A few trials show that



the length of the string is minimized when triangle ABC is isosceles with as short a hypotenuse as possible.

5 Andrew's son is married to Bernie's daughter; Bernie's son to Cliff's daughter; Cliff's son to Don's daughter; Don's son to Ed's daughter, and Ed's son to Andrew's daughter. There are five statements in the puzzle, which will be referenced as (S1), (S2), (S3), (S4), and (S5). (S1) refers to A's daughter; therefore, either A's daughter is married to B's son, or A's son is married to B's daughter. Similarly, (S2) means that either C's son is married to D's daughter, or C's daughter is married to D's son. This leaves eight possibilities. (S3), which means that B's daughter-in-law's brother is D's son-in-law, eliminates five; and (S4), which means that C's daughter-in-law's brother is E's son-in-law, eliminates two more, leaving only the solution given above. (S5) is poorly worded, but its only meaningful interpretation is that no brother/sister pair is married to another brother/sister pair; fortunately, (S5) is not needed to solve the puzzle.

Bonus. The expected number of matches in the other box is 5.015. This problem is easier to understand as a coin-flipping problem. If a coin is flipped until 20 of the same side (either H or T) appear, what is the expected number of the other side in the sequence of flips? Start by considering the case where 20 H appear first. The probability of 20 H in a row is 0.5^{20}. Next, the probability of getting 20 H and 1 T, realizing that the last flip must be a H, is C(20, 1)0.5^{21}, where C(m, n) is the number of combinations of m things taken n at a time. Similarly, the probability of getting 20 H and 2 T is C(21, 2)0.5^{22}. Or, in general, the probability of getting 20 H and n T is C(19+n, n)0.5^{20+n}. Now, the expected number of tails is obtained by multiplying each of these probabilities by the number of tails and summing; thus, E_v is the sum from 0 to 19 of nC(19+n, n)0.5^{20+n}, which is easily calculated using a spreadsheet to give 7.49258. But this needs to be multiplied by 2 (getting 14.9852), because there is an equal probability of getting 20 T first. In this analogy, each H represents taking a match from box A, and each T represents taking a match from box B. Because the original problem asked for the number of matches left, not the number removed, we need to subtract from 20 to get 5.0148, the expected number of matches left in the other box. This problem can also be solved using a probability tree. Send an email to *dondechman@verizon.net* for a complete explanation of the probability-tree solution.

Computer Bonus. GEORGE = HW x BUSH is solved by 764076 = 82 x 9318. Email *dondechman@verizon.net* for a copy of his QBasic computer program that solves this problem.

NEW SPRING PROBLEMS

Here are the new Spring Ticklers. They were chosen because they can all be solved without the use of a computer, so give the little gray cells a workout.

1 A 10-digit number consists of two 4s, two 3s, two 2s, two 1s, and two 0s. The 4s are separated by four digits, the 3s by three digits, the 2s by two digits, the 1s by one digit, and the 0s by no digits. What are the smallest and largest such numbers? Leading zeros are not allowed.

—Adapted from *The Mathematics Teacher*

2 Lay out a line of length 2 along the x-axis of a coordinate system, with the center of the line at the origin. Above this line, construct an isosceles triangle with the line as a base and base angles of 75°. Below the line construct another isosceles triangle (with its vertex pointing down) with the line as a base and base angles of 45°. What is the radius of the inscribed circle of the lanceolate figure formed by the sides of the two triangles, and where is its center (distance from the origin along the y-axis)? Express your answer exactly in terms of surds.

—*Technology Review*

3 A and B each put 10 coins in the pot. A then takes a coin from the pot and tosses it, while B calls heads or tails. If B's call is correct, he takes the coin and keeps it; otherwise, A keeps the coin. B then takes a coin from the remaining pot and tosses it while A calls heads or tails. The play continues in this way until one player has accumulated 10 coins, whereupon he wins the game and takes all the coins remaining in the pot. At one point in the play, A has six coins, while B has only four. What is A's probability of winning? When the situation occurs in a game such that one player has won six coins while his opponent has won four coins, what is the expected value of the winnings (net number of coins) of the player with six coins?

—Attributed to the Chevalier de Mere, a 17th century gambler

4 You have two two-liter bottles, one (marked W) containing one liter of pure water and one (marked A) containing one liter of pure alcohol. The two things you can do are to pour liquid from either bottle down the drain or pour liquid from one bottle into the other, but you must always keep at least one liter of liquid in bottle A. What is the minimum alcohol concentration that can be achieved in bottle A? Assume that there is no volume change on mixing water and alcohol.

—Daryl Cooper

5 A modern aluminum sculpture consists of a hollow cylinder that is capped on one end by a solid hemisphere. The cylinder has an outer diameter of 100 cm and thickness of 1 cm, and the hemisphere has the same diameter as the outside of the cylinder. If, on a level surface, the sculpture balances in stable equilibrium at any point on its hemispherical surface, how long is the cylinder, and what is the minimum ceiling height in the museum to permit the sculpture to assume any stable position?

—**R. Wilson Rowland,** *MD B '51*

Bonus. Select three points at random from within a right circular cone of altitude h. What is the expected value of the distance of the closest point from the base?

—**John W. Langhaar,** *PA A '33*

Double Bonus. For many years, the Roman Catholic church used the Julian calendar, which has a leap year every year that is divisible by four, making the average calendar year longer than the sidereal year and causing the date of the first day of spring to change gradually. To correct this, Pope Gregory decreed that Thursday, October 4, 1582, would be followed by Friday, October 15. He also declared that years divisible by 100 would be leap years only if divisible by 400. For any year since 1582, if one printed two 12-month calendars, one Julian and the other Gregorian, with dates for the days of the month, at least some of the dates would not fall on the same day of the week. What is the first year for which each day of each month will fall on the same day of the week for both calendars?

—Colin Singleton in *New Scientist*

Send your answers to any or all of the Brain Ticklers to: Jim Froula, Tau Beta Pi, P.O. Box 2697, Knoxville, TN 37901. Answers in plain text (no HTML or attachments) can be emailed to *BrainTicklers@tbp.org* . The cutoff date for receiving entries is the appearance of the Summer BENT in July. Spring entries will be acknowledged in the Fall column. The details of your calculations are not necessary, and the Double Bonus is not graded. Jim will forward your entries to the judges: **F.J. Tydeman,** *CA Δ '73,* **D.A. Dechman,** *TX A '57,* **J.L. Bradshaw,** *PA A '82*, and the columnist for this issue, **H.G. McIlvried III,** *PA Γ '53.*