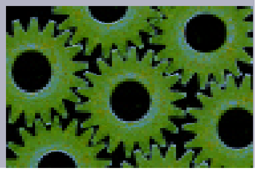


Brain Ticklers

RESULTS FROM FALL 2004

	Perfect			
*	Anderson, Paul MacLean	Son of member		
	Baines, Elliot "Chip" A., Jr.	NY	Δ	'78
	Couillard, J. Gregory	IL	A	'89
	Doniger, Kenneth J.	CA	A	'77
	Galer, Craig K.	MI	A	'77
	Levitch, Roy N.	NY	Γ	'61
	Pickelmann, Paul R.	Son of member		
	Rasbold, J. Charles	OH	A	'83
*	Schmidt, V. Hugo	WA	B	'51
*	Strong, Michael D.	PA	A	'84
	Wegener, Stephen P.	LA	A	'75
	Yee, David G.	NJ	B	'04
	Other			
	Aron, Gert	IA	B	'59
	Bachmann, David E.	MO	B	'72
	Bennefeld, Jon D.	MN	B	'97
	Bernacki, Stephen E.	MA	A	'70
*	Biggadike, Robert H.	AR	A	'58
	Brule, John D.	MI	B	'49
	Conway, David B.	TX	I	'79
	Creutz, Michael J.	CA	B	'66
	Creutz, Edward C.	PA	Γ	'36
	Forde, Jeffrey M.	CA	M	'97
*	Garnett, James M.	MS	A	65
	Gonzalez, William	FL	Δ	'04
	Griggs, James L., Jr.	OH	A	'56
	Jones, Donlan F.	CA	Z	'52
	Jordan, R. Jeffrey	OK	Γ	'00
	Karlsson, Rolf B.	MI	Z	'96
	Kimsey, David B.	AL	A	'71
	Londot, Keith L.	OH	Γ	'76
	Marks, Lawrence B.	NY	I	'81
	Mitchell, Gregory K.	NM	B	'87
	Nabutovsky, Joseph	Father of member		
	Parrish, Nathan V.	UT	Γ	'06
	Purdy, Herbert W., III	PA	E	'59
	Quintana, Juan S.	OH	Θ	'62
	Quitter, Roy	OK	Γ	'03
	Rentz, Peter E.	IN	A	'55
	Shah, Prateek	TX	H	'02
	Snyder, M. Duane	IA	B	'63
*	Spong, Robert N.	UT	A	'58
*	Stribling, Jeffrey R.	CA	A	'92
	Summerfield, Steven L., M.D.	MO	Γ	'85
	Valko, Andrew G.	PA	Λ	'80
	Van Houten, Karen	ID	A	'76
	VanShaar, Steven R.	UT	Γ	'00
	Voellinger, Edward J.	Non-member		
	Vogt, Jack C.	OH	E	'56
	Weinstein, Stephen A.	NY	Γ	'96
	Wolff, Nicholas L.	NE	A	'00
	Zapor, Richard A.	CA	E	'84

* *Denotes correct bonus solution*

	SUMMER ENTRY (Rectification)			
	Peralta-Maninat, Alfredo J.	MA	B	'54

FALL REVIEW

The most difficult regular Fall problem was No. 3 about the mixed-doubles tennis tournament with less than half of the answers being correct. Only slightly easier was No. 2 about the probability of a five-Sunday month. The bonus about the length of twilight was one of the most difficult problems we have ever used, with only a few correct answers.

WINTER SOLUTIONS

1 In minutes, the lengths of the tracks on the three CDs are (5, 7, 17, 19), (7, 11, 13, 23), and (7, 13, 17, 23). Since 2 cannot be one of the track lengths (any sum of three different primes including 2 would not be prime), the track lengths must be chosen from the 13 primes between 3 and 43. The approach is to find all three prime combinations that sum to a small enough prime so that another prime, larger than any of the three, can be added to give a sum of 60 or less. There are 14 such combinations: (3, 5, 11), (3, 5, 23), (3, 7, 13), (3, 7, 19), (3, 11, 17), (5, 7, 11), (5, 7, 19), (5, 11, 13), (5, 13, 19), (7, 11, 13), (7, 11, 19), (7, 13, 17), and (11, 13, 17). Trying all possible fourth primes with these combinations shows that the only possibilities that meet the requirements of the problem (sum of 60 or less and sum of any three a prime) are the combinations given above.

2 The coordinates of the final point are (3, 3), which are the coordinates of the centroid of the original figure. The coordinates of the centroid of a plane figure with n sides are $(\Sigma x_i/n, \Sigma y_i/n)$, where (x_i, y_i), $i = 1, 2 ..., n$, are the vertices of the figure. Now, form a new figure by joining the midpoints of successive sides of the original figure. The x-coordinate of the centroid of this figure is given by $[(x_1 + x_2)/2 + (x_2 + x_3)/2 + ... + (x_n + x_1)/2]/n = \Sigma x_i/n$. Similarly, the centroid's y-coordinate is $\Sigma y_i/n$. Thus, the centroid is at the same point as that of the original figure. Obviously, repeating this process will not change the centroid, which, therefore, must be the final point. One could also plot the original figure and go through a few cycles to infer that the final point is (3, 3).

3 The probability of the Bulls' winning is 9/40 = 0.225. In order for the Bulls to win, they must win at least three matches. Winning cases for the Bulls generally involve their weaker players matched against the Aces' better players in some of the matches. The cases where B2, B5; B3, B5; and B4, B5 play A1, A2 are easy to analyze and lead to 2, 4, and 8 wins, respectively, for the Bulls. The cases where B1, B2; B1, B3; B1, B4; and B1, B5 play A1, A2 require a little more effort and lead to 1, 2, 4, and 6 wins for the Bulls, for a total of 27 possibilities out of a total of 5! = 120 ways the teams can be matched. Thus, the Bulls probability of winning is 27/120 = 9/40.

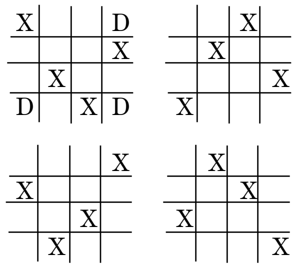
4 Joe originally displayed 9,126, and Joan saw 9,216 = 96^2. Joe multiplied by 2 to get 18,252, which Joan saw as 25,281 = 159^2. The digits that are still digits when viewed upside down are 0, 1, 2, 5, 6, 8, and 9; they all stay the same except 6 becomes 9, and 9 becomes 6. Make a list of squares that don't contain the digits 3, 4, or 7 or end in 0. Convert them to their upside-down values, double them, discard those that contain a 3, 4, or 7, and again convert the rest to their upside-down values and take square roots. When this is done, 9,216 turns out to be the only integer that works and fits on a seven-digit calculator.

5 A minimum of 19 of the 64 cells of a three-dimensional tic-tac-toe cube must be occupied to ensure that your opponent cannot win. There are two ways (not counting rotations and reflections) to block the ten possible wins on a 4 x 4 grid with only four x's, as shown below.

X			
			X
	X		
		X	

X			
		X	
			X
	X		

Use one of these arrangements for the top layer, and rotate or flip it for subsequent layers to block all vertical wins. One possible arrangement is shown at the top of the following page:



This arrangement blocks all possible wins with one and only one x, except for three of the four body diagonals (those that pass from a corner through the center of the cube). Adding the cells marked D blocks these. Thus, $4^2 + 3 = 19$ cells need to be occupied.

BONUS. The minimum number of padlocks needed to meet the requirement of the problem is 20. Label the padlocks A through T. Then, the distribution of keys is as follows:

DIR.	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
1AD	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O					
2AD	A	B	C	D	E	F	G	H	I	J						P	Q	R	S	T
1DH	A	B	C	D							K	L	M	N		P	Q	R	S	T
2DH	A				E	F	G				K		M	N	O	P		R	S	T
3DH		B			E			H	I			L		N	O	P	Q		S	T
4DH			C			F		H		J	K	L	M		O	P	Q	R		T
5DH				D			G	H	I	J		L	M	N	O		Q	R	S	T

er

can open the door, or either AD with any two DHs. Also, any four DHs can open the door. The best way to approach this problem is to try different combinations to see what works. Eventually, the light will dawn, and the solution will be clear.

COMPUTER BONUS. The only factorian larger than 145 is 40,585 = 4! + 0! + 5! + 8! + 5!. Any seven-digit factorian must be equal to or less than 7(9!) = 7(362,880) = 2,540,000. Since 8(9!) < 10,000,000, no eight-digit or larger factorian is possible. So, all that is needed is a program to check the integers from 146 to 2,540,000. The only factorian found is 40,585. Write *dondechman@aol.com* if you want a copy of the program he used to solve this problem.

NEW SPRING PROBLEMS

All of these problems, except the computer bonus, can be solved without the use of a computer. Therefore, as Agatha Christie's Hercule Poirot used to say, exercise the little gray cells before resorting to the computer.

1 A large flower bed in front of our courthouse is laid out in the form of a regular polygon of N sides. A walk, composed of N identical stone slabs in the shape of isosceles trapezoids, surrounds the flower bed. The radius of the inscribed circle tangent to the inner edges of the walk is 19 feet, and the radius of the circumscribed circle through the outer vertices of the walk is 22 feet. The ratio of the area of the flower bed (not including the walk) to the area of the walk is almost exactly three (off by less than 0.001). What is the value of N?

—Adapted from *Technology Review*

2 At a certain gambling establishment, the game of Lucky 10 is played as follows. The house provides a pot of $10. The player then tosses two coins simultaneously. If two heads appear, the player takes $1 from the pot; if two tails, he adds $1 to the pot. If he gets a head and a tail, it is a draw, and he neither adds to nor withdraws from the pot. After 10 two-coin tosses (including draws), if there is exactly $10 in the pot, the player takes it. Otherwise, the house gets whatever is in the pot. How much should the player pay the house (to the nearest penny) to make this a fair game?

—Adapted from Joseph-Louis Lagrange (1770)

3 One morning, Ann left home and cycled north at a steady speed. At the same time, Bill left the same home and cycled south at a steady speed 1 mph faster than Ann's. An hour later, their mother realized that they had both left without their lunches, so she set out at a steady speed of 10 mph and cycled after Ann, gave her her lunch, then turned and cycled after Bill, give him his lunch, and finally reached home exactly 5 hours after she started. How long would it have taken her if she had cycled to Bill first and then to Ann?

—Susan Denham in *New Scientist*

4 Consider a four-digit integer M with all different nonzero digits. Generate two other four-digit integers, A and D, by rearranging the digits of M so that A has the digits arranged in ascending order and D has the digits arranged in descending order.
If $D - A = M$, what is the value of M?

—*nearly impossible Brain Bafflers* by Tim Sole and Rod Marshall

5 In 2004, the largest known prime was $2^{24,036,583} - 1$. If this number were written in base 10, how many digits would it contain, and what would be its leftmost and rightmost three digits?

—Adapted from *Technology Review*

BONUS. Consider an infinite plane ruled with parallel lines one meter apart. An ellipse with a major axis of 0.75 m and a minor axis of 0.50 m is thrown down at random on the plane. What is the probability that the ellipse will fall on one of the lines?

—Joseph Whitworth (1803-87)

COMPUTER BONUS. Some ways to express a power of 10 as the product of two factors without using any zeros are $10 = 2 \times 5$, $10^2 = 4 \times 25$, and $10^3 = 8 \times 125$. What is the largest power of 10 which can be so expressed?

—*The Mathematics Teacher*

Send your answers to any or all Brain Ticklers to: Jim Froula, Tau Beta Pi, P.O. Box 2697, Knoxville, TN 37901. If your answers are plain text only (no HTML, no attachment), you can email them to *BrainTicklers@tbp.org*. The cutoff date for receiving entries is the appearance of the Summer BENT in July. Spring entries will be acknowledged in the Fall column. The details of your calculations are not necessary, and the Computer Bonus is not graded. We welcome any interesting new problems that may be suitable for the column. Jim will send your entries to the judges:

F.J. Tydeman, *CA Δ '73,*
D.A. Dechman, *TX A '57,*
J.L. Bradshaw, *PA A '82,*
and the columnist for this issue,
Howard G. McIlvried III, *PA Γ '53.*