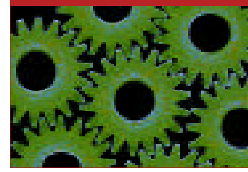


BRAIN TICKLERS

RESULTS FROM FALL 2002

Perfect

*Garnett, James M.	MS A	’65
*Hammond, Terry	OH Δ	’88
*Kimsey, David B.	AL A	’71
VanShaar, Steven R.	UT Γ	’00

Other

Aron, Gert	IA B	’59
*Biggadike, Robert H.	AR A	’58
*Bradshaw, John L.	PA A	’82
*Brzezinski, Mark A.	OH B	’00
Brown, Jefferson R.	Non-member	
*Brule, John D.	MI B	’00
*Burks, Randall	WV A	’84
*Capelli, Ronald B.	MI Γ	’73
Clift, D. Wayne	UT B	’91
*Cole, Todmund E.	CA Σ	’93
*Creutz, Michael J.	CA B	’66
Creutz, Edward C.	PA Γ	’36
Dubis, F. Michael	SC Γ	’92
Duker, Rachayl Novoseller	NY A	’97
Gardner, Orville T.	Brother	
*Garside, Jeffrey J.	WI E	’90
*Ginat, Daniel T.	NY A	’01
*Griggs, James L., Jr.	OH A	’56
Handley, Vernon K.	GA A	’86
*Harpole, George M.	CA E	’74
*Hohmann, Lawrence A.	PA A	’51
Jamgotchian, Daryl C.	CA Λ	’87
*King, Joseph A.	GA A	’04
Kronenfeld, Scott W.	WI A	’03
Langhaar, John W.	PA A	’33
*Lew, Thomas M.	TX Δ	’84
*Marrone, James D.	IN A	’87
Marrone, James I.	IN A	’61
*Mayer, Michael A.	IL A	’89
*Philbrick, John E.	NH A	’66
*Quintana, Juan S.	OH Θ	’62
Rasbold, J. Charles	OH A	’83
Rentz, Peter E.	IN A	’55
*Rosenbauer, Thomas J.	PA E	’84
Schmidt, V. Hugo	WA B	’51
Sherd, Kevin M.	OH N	’66
Skorina, Frank K.	NY M	’83
Skorina, Erik	Son	
*Spong, Robert N.	UT A	’58
*Stribling, Jeffrey R.	CA A	’92
Strong, Michael D.	PA A	’84
Surrey, Robert I.	NY Δ	’72
Tam, Helena L.	NY I	’01
Wakeley, Thomas A., Jr.	PA Z	’71

*Denotes correct bonus solution.

FALL REVIEW

Fall problem No. 3, about the probability of dropping the Q of spades in a bridge game, was the most difficult regular problem in recent memory with only five correct answers. After the A wins the first spade trick, it is tempting to argue that, since the opponents have Qxy of spades, there are four possible distributions, (Qxy, -), (Qx, y), (Qy, x), and (xy, Q), and since only the last of these leads to dropping the Q when the K is played, the odds are 1 in 4 or 0.25. However, this is incorrect because not all these distributions are equally probable. This problem involves a posteriori probability and requires the use of Bayes Theorem, which is discussed in any good book on statistics (see, for example, *The Theory of Gambling and Statistical Logic* by Richard A. Epstein). It is necessary to consider the distribution of spades before the A is played (see the Winter column). Based on these possibilities, we see that, after the first spade trick, the relative probabilities of the four distributions listed above are 0.25, 0.225, 0.225, and 0.3, respectively. Since only the fourth of these distributions results in dropping the Q on the K, the probability of this is 0.3.

Fall problem No. 4 contained a typographical error that made it impossible to find a unique solution. Therefore, it was not counted in the scoring, and a perfect score consisted of correct answers to Ticklers Nos. 1, 2, 3, and 5.

WINTER ANSWERS

Although entries for the Winter column will not be acknowledged until the Summer issue, here are the answers to the Winter Ticklers.

1. This problem concerned a game played on a circle divided into 13 sectors. The second player can force a win by, on his first play, playing to leave two symmetrically located groups of five sectors each and, thereafter, imitating the play of the first player but on the opposite side of the circle.

2. This problem concerned choosing a four-person committee from a club containing 10 men and 12 women. There are $C(12, 4) = 495$ ways for the committee to consist of four women, where $C(n, r)$ is the number of combinations of n things taken r at a time. There are $C(12, 3)C(10,1) =$ 2,200 ways to have three women and one man, but we must eliminate $C(11, 2) = 55$ of these because Mr. and Mrs. Bickerson will not serve together. Finally, there are $C(12, 2)C(10, 2) =$ 2,970 ways to have two men and two women, but again we must eliminate $C(11, 1)C(9, 1) =$ 99 of these ways to accommodate Mr. and Mrs. B. Therefore, the total number of different committees is 495 + 2,200 - 55 + 2,970 - 99 = 5,511.

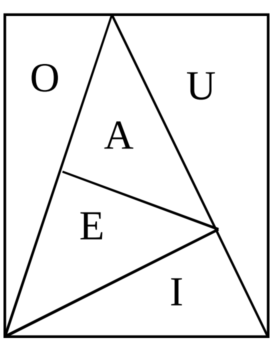
3. This problem concerned cashing a check in which the dollars and cents values were inadvertently transposed. Let d be the original number of dollars and c be the original number of cents. Then, the correct value of the check (in cents) was $100d + c$, but Mary actually received $100c + d$. After spending 350 cents, she had $200d + 2c$ left. Therefore, $100c + d - 350 = 200d + 2c$. Solving for c gives $c = (199d + 350)/98$. Thus, $199d + 350$ must be divisible by 98, or $199d \equiv 3d \equiv -350 \equiv 42 \pmod{98}$, giving $d = 14$, $c = 32$, and a value for the check of $14.32. Upon discovering the error, Mary returned the overpayment.

4. This problem concerned five schoolgirls who unethically weighed themselves on a single nickel. The total of the ten weighings is 183 + 186 + 187 + 190 + 191 + 192 + 193 + 194 + 196 + 200 = 1,912 lb. Since each girl is weighed four times (once with each of the other girls), their total weight is 1,912/4 = 478 lb. Since all the weighings are different, each girl must have a different weight. Let w_1, w_2, w_3, w_4, and w_5 be the weights of the girls from lightest to heaviest. Then,

$w_1 + w_2 = 183$ and $w_4 + w_5 = 200$. Therefore, $w_3 = 478 - 183 - 200 = 95$. If we first subtract w_4 from all the weighings to obtain a set of weights of the other girls and then repeat this step with w_5 to obtain a second set of weights, three results from the first set must match three results from the second set, and one of these values must be 95. Trying 99 and 101, we find that matching values are 91, 92, 93, and 95. Since 91 + 92 = 183, we get the girls' weights as 91, 92, 95, 99, and 101, which are consistent with all the weighings.

5. This problem concerned mailing a tightly wrapped cone having to meet certain constraints. Let the radius of the base of the cone be r and its height h. Then, its volume is $\pi r^2 h/3$. If the girth is taken as the circumference of the base and the length as the height, then $2\pi r + h = 108$. Therefore, $V = \pi r^2(108 - 2\pi r)/3$. Differentiating and setting $dV/dr = 0$ gives $r = 36/\pi = 11.459$ inches and $h = 108 - 2\pi r = 36$ inches as the dimensions of the maximum-volume cone. Then, $V = 11.459^2(36)\pi = 4{,}950.4 \text{ in}^3 = 2.865 \text{ ft}^3$. Therefore, the density is $70/2.865 = 24.435 \text{ lb/ft}^3$.

BONUS. This problem involved the island of Bongo, which was divided into five counties in the shape of right triangles. The key to this problem is visualizing all the possible ways to construct a rectangle composed of three right triangles having equal areas and two unequal larger right triangles. The correct arrangement is shown in the accompanying figure. Areas A and E are isosceles triangles; I is a right triangle with the same area as A and E; and O and U are larger right triangles. If we let the length of a side of A or E equal 1, then the lengths of the sides of I, O, and U are (√2/2, √2), (√10/5, 3√10/5), and (3√10/10, 3√10/5), respectively. From these values, we see that the length of Bongo is 1.2 times its width. Therefore, the length is 1.2(45) = 54 km, and the area is 45(54) = 2,430 km^2. Note that the only requirement for the union of A, E, and I was that it be a triangle, but not necessarily a right triangle.



COMPUTER BONUS. You were asked to divide the integers from 2 through 25 into two groups such that the products of each group had a minimum difference. Although this problem can be solved by computer, with a little luck you can solve it without one. Our target factor is $(25!)^{0.5} = 3.938427 \times 10^{12}$. As a first attempt at matching this value, try 25•24•23•22•21•20•19•18•15•6 = 3.92482×10^{12}, which makes the other factor 3.938427×10^{12}. The ratio of these factors is 1.0035. Now, 1/0.0035 = 285.7; therefore, if we can replace a factor of 286 in the larger number by a factor of 285 from the smaller, we can get closer to parity. But, 285 = 15•19 and 286 = 2•11•13. Making this switch, we get for the two factors as 3,938,264,064,000 and 3,938,590,656,000, which have a difference of 326,592,000 and a ratio of 1.000083, pretty close to equal. For a computer solution, start with the fact that 25! = $2^{22}3^{10}5^{6}7^{3}11^{2}13•17•19•23$. Then, a simple program will show that the two factors with the smallest difference are $2^{13}3^{5}5^{3}7^{2}17•19$ and $2^{9}3^{5}5^{3}7•11^{2}13•23$, which agrees with the answer given above.

NEW SPRING PROBLEMS

Now, for something to keep you occupied during Spring break. The judges realize that computers are ubiquitous in today's world, but the name of this column is Brain Ticklers. With the exception of the Computer Bonus, all of the following problems can be solved without the aid of a computer. Therefore, we recommend you tickle your brain cells before resorting to your computer.

1. The ratio of the area of a right triangle with integral sides (each less than 100 units) to its perimeter is a single-digit integer. The value of this integer is written on a folded piece of paper. If you opened the paper and saw the integer, you could determine the dimensions of the triangle. Given that hint, what are the lengths of its sides?

—*The Platinic Corner*

2. Assume that Venus, Earth, and Mars are in circular, coplanar orbits around the Sun, with orbital times of 225, 365, and 687 Earth days, respectively. If all three planets are aligned along a diameter of the orbit of Mars, what is the minimum time (in Earth years) before they are again aligned on a diameter of Mars' orbit?

—**D.A. Dechman,** *TX A '57*

3. Find a 10-digit integer containing the digits 0 through 9 each once such that, for n equal to 1 through 10, the integer formed by the first n digits is divisible by n.

—*Nearly Impossible Brain Bafflers* by Tim Sole and Rod Marshall

4. This cryptic will be more interesting if you have a knowledge of German. VIER and NEUN are both perfect squares. If you were told the number represented by VIER, you could deduce the number represented by NEUN. Alternatively, if you were told the number represented by NEUN, you could deduce the number represented by VIER. What are the values of VIER and NEUN? (The same letter has the same value in both words, and there are no leading zeros.)

—Richard England in *New Scientist*

5. From a large supply of black and white marbles, 100 black and 100 white marbles are placed in an urn. Three marbles are drawn at random from the urn and, depending on the marbles drawn, certain marbles are put back in the urn:

Marbles Drawn	Marbles Put Back
3 black	1 black
2 black, 1 white	1 black, 1 white
1 black, 2 white	2 white
3 white	1 black, 1 white

If this procedure is followed until fewer than three marbles remain in the urn, what is the probability that they are all white?

—Mathematics Teacher

Bonus. Consider the parabola $y = x^2$, and let P be a point (other than the origin) on this parabola. Construct a perpendicular to the parabola at P (i.e., a perpendicular to the tangent at P). This perpendicular, when extended, will intersect the parabola at Q. Find the coordinates of P (with x positive) such that the area bounded by the perpendicular and the parabola is a minimum.

—Southwest Missouri State University Advanced Problem Archive

Computer Bonus. A palindrome is a number that reads the same forwards and backwards. What is the smallest, and reported to be the only, nonpalindromic integer whose cube is a palindrome?

—The Ambidextrous Universe by Martin Gardner

Send your answers to any or all of the Spring Ticklers to:

Jim Froula
Tau Beta Pi
P.O. Box 2697
Knoxville TN 37901-2697.

The cutoff date for submitting answers is the appearance of the Summer issue in July. If your answers involve text only, they may be e-mailed to Brainticklers@tbp.org. The details of your answers are not needed unless you think they will be of interest to the judges. The Computer Bonus is not graded.

If you have a favorite problem of your own, feel free to include it. Jim will forward your entries to the judging panel, consisting of:

F.J. Tydeman, *CA* Δ *'73;*
R.W. Rowland, *MD* Β *'51;*
D.A. Dechman, *TX* Α *'57; and*
the columnist for this issue,
—**Howard G. McIlvried III**, *PA* Γ *'53.*