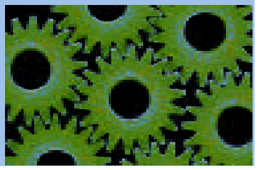

Brain Ticklers Celebrate 50th Year!

FIFTIETH ANNIVERSARY

In this issue we continue our review of our first 50 years of existence by providing data on the decade from 1991 to 2000. During this time we received 2,190 entries, compared to an average of 1,781 for the previous four decades. The most perfect solutions were submitted by James L. Griggs Jr., *OH* A *'56;* Michael D. Strong, *PA* A *'84;* and Francis W. Pasterczyk, *MA* Z *'58;* and the most correct bonus solutions were submitted by the same three Tau Bates. To summarize our first 50 years, we received a total of 9,314 entries from approximately 4,150 unique people. Of these entries, about a third (3,296) were perfect, and about a quarter (2,406) included correct Bonus solutions. We used 1,263 different Ticklers; but don't worry, we have enough new problems in our files to keep the Brain Ticklers going at least another 20 years.

FALL REVIEW

The Fall problem set proved to be harder than usual, with only a few perfect solutions being submitted. The most difficult problems were No. 1 about the snow plow and No. 4 about Uncle George's birthday, which required some pretty subtle logic. Both these Ticklers proved to be considerably more difficult than the Bonus about the value of the geometric series.

WINTER ANSWERS

The Winter entries will be acknowledged in the Summer column. In the meantime, here are the answers.

1. There are 6 x 6 = 36 possible red die/white die fractions. The value of the denominator can vary from 1 through 6. For each of these, the numerator can also vary from 1 to 6. Since the sum of the integers from 1 through 6 is 21, the sum of all possible fractions is $21(1 + 1/2 + 1/3 + 1/4 + 1/5 + 1/6) = 1{,}029/20$. The expected value is then $1{,}029/720 = 343/240 = 1.429167$.

2. It will take the painters 108 minutes to paint the room. Let x and y be the fractions of the flat work and trim, respectively, done by Al. Then, $t_A = 2x + y$, and $t_B = 3(1 - x) + 2(1 - y) = 5 - 3x - 2y$, where t_A and t_B are the times Al and Ben spend painting. For minimum total time, $t_A = t_B$. Setting these terms equal and simplifying gives $x = 1 - 3y/5$. Time to complete the job is given by $T = (t_A + t_B)/2 = (5 - x - y)/2 = 2 - y/5$. This is obviously a minimum when $y = 1$. Therefore, $T = 2 - 1/5 = 1.8$ hr. The same result can be reached by pure logic by realizing that Al does the trim twice as fast as Ben but the flat work only 1.5 times as fast. Thus, the best strategy is to let Al do all of the trim work while Ben starts on the flat work. Then, at the time Al finishes the trim in one hour, Ben will have completed 1/3 of the flat work. Then, working together, they can complete the remaining flat work in $(2/3)/(1/2 + 1/3) = 0.8$ hr.

3. For the shell to float, the weight of the water displaced must equal the weight of the shell plus the weight of the boy. The volume of the spherical segment below the water line is $\pi h^2(3R - h)/3$, where h is the height of the segment and R is the outer radius of the shell. Since the freeboard is 10 cm, $h = R - 10$. With the density of water being 1 g/cm^3, the weight of water displaced is $\pi(2R^3 - 30R^2 + 1000)/3$. The volume of the shell is $2\pi(R^3 - r^3)/3$, where r is the inner radius. The thickness of the shell is $R - r = 2.5$ cm.

Inserting these values and simplifying gives the volume of the shell as $5\pi(3R^2 - 7.5R + 6.25)/3$.

Combining these results, using 2.5 as the specific gravity of concrete and 45 kg as the weight of the boy, gives $\pi(2R^3 - 30R^2 + 1{,}000)/3 = 12.5\pi(3R^2 - 7.5R + 6.25)/3 + 45{,}000$.

Combining terms gives $R^3 - 33.75R^2 + 46.875R - 21{,}024.98 = 0$.

Solving this cubic by trial and error or by using the cubic formula gives R = 43.69 cm for a diameter of 87.38 cm.

4. Ignoring rotations and reflections, there are two possible arrangements of the eight queens on a chessboard. Number the columns on the chessboard A through H and the rows 1 through 8. Each row and column must have one, and only one, queen, with no queens on the major diagonals and no queens threatening each other. Therefore, one queen must be on B1, C1, or D1. (The other squares on the first row are either on a main diagonal or reflections of the listed squares.) For each of these possibilities, there are three or four ways to place a queen on the second row, and then two or three ways for a queen on the third row, and so forth for the rest of the rows. Starting with a supply of quadrille paper, it is not too difficult to try all the possibilities to arrive at the two arrangements shown in the accompanying figures.

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	Q						
						Q	
Q							
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		Q					
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5. The poker hand contained the 2 of spades, 5 and 9 of hearts, 4 of diamonds, and 8 of clubs. Because all four suits are present, there must be at least two red and two black cards and only one suit with two cards. Since hearts total 14 and there are no aces or face cards and all cards have a different value, the hearts must be (4, 10), (5, 9), or (6, 8). Because the sum of the odd cards equals the sum of the even cards, there must be two odd and three even cards. Now the maximum of the odd cards is 7 + 9 = 16, and the minimum of the even cards is 2 + 4 + 6 = 12. Therefore, these sums must be 12, 14, or 16. With this background, it is relatively easy to deduce that the hand consisted of the cards listed above.

Bonus. Ann's probability of winning the tic-tac-toe game, ignoring draws, is 0.67. Assume Ann uses Xs. The number of possible games is 9!/(5!4!) = 126; however, when rotations and reflections are considered, there are only 22 different arrangements, although some may occur as many as 8 times in the total of 126. Of the 126 possible games, Ann clearly wins 62, Bev clearly wins 12, and 16 end in draws. This leaves 36 that can go either way. These 36 cases all have the form of three Xs in one row, three Os in another row, and two Xs and one O in the remaining row. The first three Xs can be placed in $C(5, 3) = 10$ ways, only one of which wins. The first three Os can be placed in $C(4, 3) = 4$ ways, only one of which wins for Bev. If Ann hasn't won or lost after six draws, her fourth X can be placed in 2 x 9 = 18 ways, of which 6 lead to wins. Therefore, her probability of winning in these 36 cases is
$1/10 + [1 - 1/10 - (9/10)(1/4)](6/18) = 13/40$. Then, Ann's probability of winning, PA, not ignoring draws, equals $62/126 + (13/40)(36/126) = 737/1{,}260$; and Bev's probability, PB, is
$12/126 + (27/40)(36/126) = 121/420$.
Throwing out draws, we find that Ann's probability of beating Bev is $PA/(PA + PB) = 737/1{,}100 = 67/100$.

Double Bonus. Let A and B be the two points between which the midpoint is to be found. Describe a circle with radius AB and center A and an arc slightly larger than a semicircle with radius AB, center B, and point A near the middle. Starting at B, with the pair of compasses still set at AB, strike three successive arcs on the circle to locate point C, which is the end of the diameter through AB. Draw an arc with radius BC and center C that cuts the semicircle with center B at D and E. With D and E as centers, draw arcs with radii AB. These arcs intersect at the midpoint between A and B. Thus, eight compass operations are required in this solution.

NEW SPRING PROBLEMS

We realize that computers are ubiquitous and that many of our readers will use them to solve the Ticklers, and we have no difficulty with that. However, all of the following problems can be solved without the use of a computer, and we suggest that, before resorting to a computer, you try to solve them using brain power alone.

I. Pam lives on Rhombus Road, which has houses numbered from 3 to 99, inclusive. Ron is anxious to learn her house number so he can call on her, but Pam won't tell him. His buddies, Art, Bill, Carl, and Don, have a piece of information each about the house number, but none knows the actual number: Art knows whether the house number is a perfect square; Bill knows whether it is a multiple of 3; Carl knows whether it has one or two digits; and Don knows whether it is a multiple of 5. Ron knows the type of information each has. After questioning and getting answers from Art, Bill, and Carl, Ron thinks that Don's information might confirm what Pam's number is.

After getting an answer from Don, Ron says, "Now, if I knew whether her number is odd or even, I'd be sure." After wheedling the parity of her number from Pam, Ron is sure he knows her number. He buys flowers and shows up at the address only to find an abandoned house. Later, he learns that Art and Bill had both lied. What is Pam's house number?

—adapted from *Brain Puzzlers Delight* by E.R. Emmet

2. Once when I was visiting Upper Slobovia—where the currency is denominated in oseps (each osep is worth 100 ovatnecs) and the coins in circulation have values of 5, 10, 20, and 50 ovatnecs—I was counting my change and discovered that I had exactly 10 oseps worth of coins. Also, to my surprise, I found that I had a prime number of each kind of coin. Then, I realized that I had the minimum total number of coins I could have under these conditions. How many coins did I have? Remember, 1 is not a prime number.

—The Platinic Corner

3. Solve the following cryptic addition:

YELLOW + YELLOW + RED = ORANGE

The usual rules apply: each different letter represents a different digit; the same letter always represents the same digit; and there are no leading zeros.

—*Technology Review*

4. Consider the letters from A through M. If all possible 13-letter permutations (no repeated letters) of these letters are arranged in alphabetical order, what is the 1,234,567,890th permutation in the list?

—adapted from *Mathematics Teacher*

5. At a recent party at my house, I burned two cylindrical candles. I lit the red candle, which was 2 cm longer than the white candle, at 4 p.m., and I lit the white candle 15 minutes later. At 8 p.m. I noticed that they were the same length. The red candle burned out at midnight, and the white candle burned out an hour later. If each candle burned at its own steady rate, what were their original lengths?

—*nearly impossible Brain Bafflers* by Tim Sole and Rod Marshall

Bonus. A satellite is in an equatorial circular orbit 400 km above the surface of the earth. To place the satellite in geosynchronous orbit, its engine must be fired twice, once to place it into an elliptical transfer orbit with apogee at the geosynchronous altitude, and again at this apogee to place it in circular geosynchronous orbit. By how much, in m/s, must the magnitude of the satellite's velocity be increased at each burn?

—**C.K. Galer**, *MI* Α *'77*

Double Bonus. Ann and Brandon play the following game. First, Ann puts $1 in the pot. Then, they take turns tossing a coin, with Brandon tossing first. If Brandon gets heads, he takes $1 from the pot; if he gets tails, he adds $1 to the pot. Then, Ann tosses the coin. If Ann gets heads, she takes $1 from the pot; but if she gets tails, she neither takes anything from the pot nor adds anything to the pot. They continue taking turns tossing the coin until the pot is empty, at which time the game ends. Is this a fair game? If so, why? If not, why not?

—Nikolaus Bernoulli (1687-1759)

If you have a favorite problem of your own, feel free to include it. Jim will forward it to the judging panel, consisting of:

F.J. Tydeman, *CA* Δ *'73;*
R.W. Rowland, *MD* Β *'51;*
D.A. Dechman, *TX* Α *'57;* and the columnist for this issue,
—Howard G. McIlvried III, *PA* Γ *'53.*
