BRAIN TICKLERS

Results From Spring

Perfect Scores

*Berthold, Kristopher D.	TX	Β	'04
Coleman, Benjamin J.	NH	Β	'20
Couillard, J. Gregory	IL	Α	'89
Fleischer, Bruce	Father of member		
Johnson, Mark C.	IL	Α	'00
Rowe, Steven A.	ME	Α	'81
Barr, David A.	Non-member		
*Slegel, Timothy J.	PA	Α	'80
*Spong, Robert N.	UT	Α	'58
Sylvester, Noah	Son of member		

Other

Aron, Gert	IA	Β	'58
Bannister, Kenneth A.	PA	Β	'82
Bohdan, Timothy E.	IN	Γ	'85
Dechman, Don A.	TX	Α	'57
*Griggs Jr., James L.	OH	Α	'56
*Holcomb, J. Eric	OH	Α	'82
Johnson, Roger W.	MN	Α	'79
Kimsey, David B.	AL	Α	'71
Kovalick, Albert W.	CA	Η	'72
Kraska, Don H.	MI	Γ	'63
Lalinsky, Mark A.	MI	Γ	'77
Munsil, Wesley E.	CA	Β	'71
*Norris, Thomas G.	OK	Α	'56
Norris Jr., Thomas G.	PA	Γ	'79
Parks, Christopher J.	NY	Γ	'82
Scott, Darrell J.	NC	Δ	'82
Silver, Robert E.	NY	Ρ	'80
Spring, Gary S.	MA	Ζ	'82
Summerfield, Steven L.	MO	Γ	'85
*Voellinger, Edward J.	Non-member		
Wallace, Jean E.	IA	Α	'81
Zison, Stanley W.	CA	Θ	'83

*Denotes correct bonus solution

Spring Review

Readers did very well on the Spring puzzles. One hundred percent of the submitted answers to Question 1 (the base 9 cryptarithm) were correct. Between 60 and 90 percent of the submitted answers to the remaining questions were correct.

The Double Bonus was something of an experiment. When this question was used in the Winter 1979 issue, 0 / 154 respondents submitted a correct answer. The judges were curious to see if results would be different in this age of ready access to computers. For the Spring 2021 issue, 3 / 30 respondents submitted a correct solution.

Summer Answers

1: $w = 6$. Six wrongs do not make a right. There are 21 answers for $w = 2$, for example WRONG = 12,734 and RIGHT = 25,468. Four answers for $w = 3$, including WRONG = 13,082 and RIGHT = 39,246. Four answers $w = 4$, such as WRONG = 16,093 and RIGHT = 64,372. Five answers for $w = 5$ is one possibility, WRONG = 19,452 and RIGHT = 97,260. But, there are zero answers for $w = 6$.

2: The pairings are: **N1-D2, N2-D3, N3-D1,** for a winning probability **P = 31 / 60 ≅ 0.5167.** Each team has exactly three players, so the Nets captain has 3! = 6 options for pairing sets. Since the two teams are equally strong, any strategy that matches two equally ranked individuals will result in P = .5 as the Nets advantage in one unequal pair is precisely offset by a disadvantage in the other. Therefore, one only needs to consider the two pairing sets that have three unequal pairs.

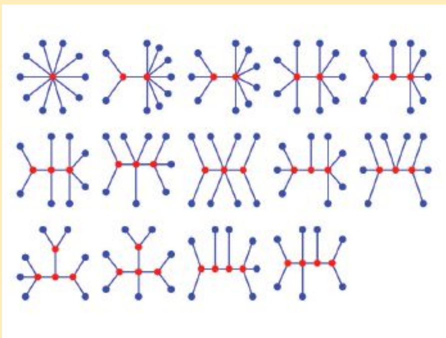
With the N1-D2, N2-D3, N3-D1 pairing set, the probability that (N1, N2, N3) are all winners is 1/10, (N1, N2, D1) is 3/10, (N1, D3, N3) is 1/15, and (D2, N2, N3) is 1/20. The probability of winning the tournament P = 1/10 + 3/10 + 1/15 + 1/20 = 31/60.

Conversely, with the N1-D3, N2-D1, N3-D2 pairing set, the probability of winning the tournament P = 1/10 + 1/5 + 3/20 + 1/30 = 29/60.

If the teams were to carry more players, one could use a similar strategy to compute the probabilities, although a program or spreadsheet would make the task more manageable. If there were five on each team: N1-D2, N2-D3, N3-D4, N4-D5, N5-D1 is the best strategy for P = 601 / 1,134 ≅ 0.5300.

Surprisingly, seven players does not follow the pattern: N1-D3, N2-D4, N3-D5, N4-D6, N5-D7, N6-D2, N7-D1. P = 16,601 / 30,720 ≅ .5404.

3: There are **14** trees with 11 vertices. Of the 14 trees, there are a maximum of **4** non-leaf vertices. In the movie, Will Hunting drew only 8 of 10 graphs with exactly 10 vertices. Despite the drama, this problem is not nearly as hard as cinema presents. We asked for unique (aka non-isomorphic) trees (that is, a graph with no cycles) that are irreducible (that is, no vertex has exactly two edges). Below is a diagram of the 14 trees. Of the 14 trees, 4 had exactly 4 non-leaf vertices (the bottom row).



4: For x=20 and y=43, the largest order that cannot be filled is **797;** the general form is $xy - (x + y)$. Traditionally known as the *Frobenius Coin* problem, and now sometimes referred to as the *Chicken McNugget* problem, this Tickler concerns the case where there are exactly two box sizes. Using trial and error, it is relatively easy to figure out the specific answer for x=20 and y=43. A rigorous proof of the general form is too long for this column.

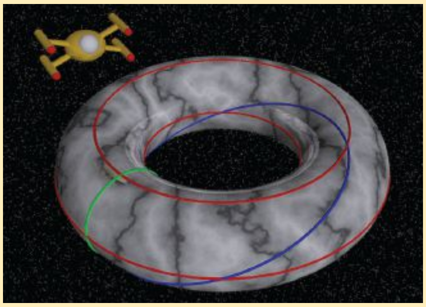
5: **110 km.** Let R be the major radius of the torus, that is, the distance from the center of the torus to the center of the encircling tube. Let r be the minor radius of the torus, that is, the radius of the tube. Of course, $R > r$. A circular channel can be formed in one of three ways:

1. By the intersection of the torus with a plane parallel to equatorial plane of torus; such circles have a radius q such that $R - r \le q \le R + r$; for example, the red circles in the diagram

2. By the intersection of the torus with a plane perpendicular to the equatorial plane of the torus; such circles have a radius $q = r$; for example, the green circle in the diagram

3. By the intersection of the torus with a plane at an angle $\sin(r / R)$ to the equatorial plane passing through the center; these so called Villarceau circles have a radius $q = R$; for example, the blue circle in the diagram

Multiple circles of red type 1 do not intersect, nor do multiple circles of green type 2. However, every type 1 circle must intersect with every circle of types 2 and 3.



Hannah and Jack's circles are different lengths, but do not intersect, so they must be red type 1. Dana and Ron's circles intersect Jack's, so they must be types 2 and 3, but it is not immediately clear which is which. We can calculate $q_{dana} = 50/2\pi$, and $q_{ron} = 60/2\pi$. Since $R > r$, the longer $q_{ron} = 60/2\pi = R$ (blue type 3), and the shorter $q_{dana} = 50/2\pi = r$ (green type 2). Sarah's channel is the longest possible, so has the maximal radius (red type 1) $R + r = 60/2\pi + 50/2\pi = 110/2\pi$. Sarah's channel has length 110 km.

BONUS: WISE.

For purposes of this answer, define italics as being the sum of the provided digits, e.g. *ONE* = O + N + E.

Observe that given the pairs *FOUR* / *FOURTEEN*, *SIX* / *SIXTEEN*, *SEVEN* / *SEVENTEEN* and *NINE* / *NINETEEN*, ***TEEN* must be 10.** If not, at least four of the sums must be wrong, and we know that at most one is.

Knowing that *TEEN* = 10, then *TEN* = 10 if and only (iff) *E* = 0. That is, *E* ≠ 0 if *TEN* is wrong.

Considering *EIGHT* and *EIGHTEEN*, both *EIGHT* = 8 and *EIGHTEEN* = 18 iff *T* = 0. That is, if one of the two is wrong, *T* ≠ 0.

Similarly, considering *THREE* and *THIRTEEN*, both *THREE* = 3 and *THIRTEEN* = 13 iff *I* = *2E*. That is, if one of the two is wrong, *I* ≠ *2E*.

Suppose all five are correct: *TEN* = 10, *EIGHT* = 8, *EIGHTEEN* = 18, *THREE* = 3, and *THIRTEEN* = 13. Then *E* = *T* = *I* = 0. Then *TEN* = *N* = 10. And *NINE* = *2N* = 9, *N* = 4.5. *N* can't be both 10 and 4.5, so **exactly one of *TEN*, *EIGHT*, *EIGHTEEN*, *THREE*, and *THIRTEEN* is the incorrect sum and all the other sums are correct.**

Suppose *TEN* is the incorrect sum; *TEN* ≠ 10. *T* = 0, *E* ≠ 0, *I* = *2E*. From *TEEN* = 10, *EEN* = 10, and *IN* = 10. Given *NINE* = 9, *NE* = -1, so *E* = 11, *I* = 22, and *N* = -12. *ONE* = 1, so *O* = 2. *TWO* = 2, so *W* = 0. *FIFTEEN* = 15, so *F* = -8.5. *FIVE* = 5, so *V* =-19.5. *SEVEN* = 7, so *S* = 16.5. *SIX* = 6, so *X* = -32.5. *ELEVEN* = 11, so *L* = 9.5. Checking *TWELVE* = *ELVE* = 12. *TWENTY* = 20, so *Y* = 21. At this point, *G*, *H*, *R*, and *U* are undetermined. From *FOUR* = 4, we get *UR* = 10.5. *EIGHT* = 8, so *GH* = -25. *THREE* = 3, so *HR* = -19. Combining *UR* = 10.5, *GH* = -25, and *HR* = -19, there is no way to get any more than one of *G*, *H*, *R*, or *U* to be zero. Since we have no more than three letters equal to zero, we conclude both ***TEN* = 10** and ***E* = 0**.

Suppose one of *THREE* or *THIRTEEN* is an incorrect sum. *T* = 0, *E* = 0, *I* ≠ *2E*. *TEN* = 10, so *N* = 10. *NINE* = 9, so *I* = -11. *ONE* = 1, so *O* = -9. *TWO* = 2, so *W* = 11. *TWENTY* = 20, so *Y* = -1. *FIFTEEN* = 15, so *F* = 8. *FIVE* = 5, so *V* = 8. *SEVEN* = 7, so *S* = -11. *SIX* = 6, so *X* = 28. *ELEVEN* = 11, so *L* = -7. Checking *TWELVE* = *WLV* = 12. At this point, *G*, *H*, *R*, and *U* are undetermined. From *FOUR* = 4, we get *UR* = 5. *EIGHT* = 8, so *GH* = 19. If *THREE* = 3 then *HR* = 3, or if *THIRTEEN* = 13, then *HR* = 14. Combining *UR* = 5, *GH* = 19, and one of the *HR* equations, there is no way to get any more than one of *G*, *H*, *R*, or *U* to be zero. Since in this case we only have at most three letters equal to zero, we conclude both ***THREE* = 3** and ***THIRTEEN* = 13**.

Finally, consider the possibility that one of *EIGHT* or *EIGHTEEN* is an incorrect sum. *T* ≠ 0, *E* = 0, ***I* = *2E* = 0**. *NINE* = 9, so ***N* = 4.5**. *TEN* = 10, so ***T* = 5.5**. *ONE* = 1, so ***O* = -3.5**. *TWO* = 2, so ***W* = 0**. *TWENTY* = 20, so ***Y* = 4.5**. *FIFTEEN* = 15, so ***F* = 2.5**. *FIVE* = 5, so ***V* = 2.5**. *SEVEN* = 7, so ***S* = 0**. *SIX* = 6, so ***X* = 6**. *ELEVEN* = 11, so ***L* = 4**. Checking *TWELVE* = *TLV* = 12. At this point, *G*, *H*, *R*, and *U* are undetermined. From *FOUR* = 4, we get ***UR* = 5**. *THREE* = 3, so ***HR* = -2.5**. If EIGHT = 8 then GH = 2.5, or if EIGHTEEN = 18, then GH = 8. Combining *UR* = 5, *HR* = -2.5, and one of the *GH* equations, we can select *G*, *H*, *R*, or *U* such that none are zero.

We have exactly four zeroes, *E*, *I*, *W*, and *S*, which can be formed to make the word *WISE*.

In summary, *E* = *I* = *W* = *S* = 0, *N* = 4.5, *T* = 5.5, *O* = -3.5, *Y* = 4.5, *F* = 2.5, *V* = 2.5, *X* = 6, and *L* = 4. *EIGHT* or *EIGHTEEN* is the incorrect sum. Either *GH* = 8 or *GH* = 2.5. *UR* = 5 and *HR* = -2.5 and we can select *G*, *H*, *R*, or *U* such that none are zero.

COMPUTER BONUS:

The elder died at age **122**. The seven stones weigh **1, 3, 7, 12, 43, 76, and 102**.

For comparison, if there were only six stones, our program found the elder's age to be 84 and the six weights are 1, 3, 17, 22, 51, 61. Five stones: age 49, weights 1, 3, 7, 12, 36. Four stones: age 26, weights 1, 2, 6, 18. Three stones: age 13, weights 1, 3, 9.

New Fall Problems

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New Fall Problems

1: Ancient Hieroglyphics

Recently, archaeologists have provided a translation of some cryptic hieroglyphic inscriptions. They are statements made by Cleopatra, Marc Antony, Julius Caesar, and Cleo's father. Not all the statements are true. In fact, no two persons made the same number of true statements. Assume that Cleopatra could love only one person at a time and that someone told more truths than Caesar.

Cleopatra: 1. I lost my asp in Memphis. 2. I love my daddy. 3. I do not love Julius.

Antony: 1. Cleo loves me. 2. Caesar is ambitious. 3. Brutus is a true friend of Caesar's.

Julius: 1. Cleo loves me. 2. Brutus is a true friend of mine. 3. Cleo did not lose her asp in Memphis.

Cleo's Dad: 1. Cleo loves either Marc Antony or Julius Caesar. 2. Cleo lost her asp in Luxor. 3. Caesar is not ambitious.

Whom does Cleopatra love?

—*The Crucible*

2: Policy Numbers

Before the days of computers, a clerk in an insurance office was given the job of assigning numbers (in ascending order) to new policies with the following restriction: each decimal digit in the number has to be different. Unfortunately, the 6 and 8 keys on his typewriter are broken. How many new numbers can he assign in the range of 5402 to 97543210, inclusive?

—*Introductory Combinatorics* by Richard A. Brualdi

3: Perfect Bridge

In the game of bridge, four 13-card hands are dealt from a standard 52-card deck. What is the exact probability of a "perfect" deal, that is, what is the probability that each of the four hands contains 13 cards of the same suit? Express your answer as either a ratio of integers, or a ratio of expressions involving factorials.

—**R.W. Rowland,** *MD Β '51*

4: Cryptic Solver

Solve this cryptic addition of two five-digit numbers in base 10 with no leading zeros and different letters are different digits: ABCDE + BCDEF = CDEFG

—**Byron R. Adams,** *TX Α '58*

5: Mercury's Orbit

On average, what percent of the time is Mercury the closest planet (of the inner eight) to Pluto? Assume all orbits are circular and coplanar. We would like two significant digits in your answer.

—**Fred J. Tydeman,** *CA Δ '73*

BONUS: A space traveler, marooned due to the failure of her rocket engine, observes on her video screen that a small asteroid is rapidly approaching on a collision course. In a desperate effort to nudge her rocket ship out of the path of the asteroid, she decides to vent the contents of her water tank to space. The water tank has a volume of two cubic meters and is half full with water and half full with nitrogen at a pressure of 6 MPa. The initial mass of the ship and its contents is 2000 kg. What change in velocity can be achieved if the venting is stopped just as the last drop of water exits the ship (and no nitrogen escapes)? Assume the vent on the water tank is aligned with the center of gravity of the ship and that the tank is fitted with a heater so that the expansion of the nitrogen is isothermal. Ignore friction losses.

—**Greg A. Qualls,** *KS Β '80*

COMPUTER BONUS: One example of a six-digit decimal integer with all different digits, such that its square contains none of the digits in the number, is 203,879 ($203{,}879^2 = 41{,}566{,}646{,}641$). Find a second example.

—*A Passion for Mathematics* by Clifford A. Pickover

Email your answers (plain text only) to any or all of the Fall Brain Ticklers to *BrainTicklers@tbp.org* or by postal mail to **Dylan Lane, Tau Beta Pi, P.O. Box 2697, Knoxville, TN 37901-2697**. The method of solution is not necessary. The Computer Bonus is not graded. Where possible, exact answers are preferable to approximations. The cutoff date for entries to the Fall column is the appearance of the Winter *Bent* which typically arrives in early January (the digital distribution is several days earlier). We welcome any interesting problems that might be suitable for the column. Dylan will forward your entries to the judges who are **J.C. Rasbold,** *OH Α '83;* **J.R. Stribling,** *CA Α '92;* **G.M. Gerken,** *CA Η '11;* and the columnist for this issue,

— **F.J. Tydeman,** *CA Δ '73*

CHAPTER ETERNAL

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BETA WA Β
Henager, Charles Henry, *'50*, May 29, 2008.
Johnson, George W.S., *'50*, no details.
Flechsig Jr., Alfred Julius, *'57*, Aug. 28, 2015.
Scheibner, James Elwood, *'57*, July 28, 2020.
Tharp, Stanley R., *'58*, September 7, 2001.
Curtis, David G., *'65*, August 13, 2011.
GAMMA WA Γ
Decker, Delmar Lee, *'71*, March 11, 2004.

WEST VIRGINIA

ALPHA WV Α
Pettit, Gene Lee, *'52*, May 23, 2017.
Konrad, Charles E., *'61*, January 10, 2014.
Meikle, Philip Gene, *'61*, October 21, 2010.
Simon, Jerome Callahan, *'66*, Jan. 18, 2005.
BETA WV Β
Barnhart, David Bruce, *'72*, May 7, 2021.

WISCONSIN

ALPHA WI Α
Ille, William B., *'44*, May 13, 2015.
Bastian, Lehyman John, *'50*, Jan. 31, 2012.
Bartelt, Charles Henry, *'52*, March 10, 2000.
Szeremeta, Peter John, *'52*, no details.
Foss, Bryon Edward, *'64*, June 1, 2015.
BETA WI Β
Kemnitz, Robert Harold, *'42*, no details.
Jenny, Daniel P., *'43*, August 3, 2017.
Frederickson, Charles A., *'46*, Nov. 23, 2004.
Garvey, Thomas Michael, *'47*, Aug. 27, 2005.
Wilson, Arthur Daniel, *'48*, Dec. 2, 2006.
Bruemmer, William M., *'50*, Feb. 19, 2013.
Blaschke, Richard F., *'51*, October 13, 1998.
Springer, Karl Eugene, *'56*, no details.
Cronin, Patrick Lawrence, *'63*, Jan. 29, 2000.
Hauber, Janet E., *'65*, March 29, 2021.
GAMMA WI Γ
McPherson, Joseph D., *'64*, no details.
Youngwith, Richard A., *'80*, March 1, 2021.