BRAIN TICKLERS

Results From Spring

Perfect Scores

*Bohdan, Timothy E.	IN	G '85
*Clift, D. Wayne	UT	B '91
*Couillard, J. Gregory	IL	A '89
*Gerken, Gary M.	CA	H '11
Griggs Jr., James L.	OH	A '56
*Johnson, Mark C.	IL	A '00
*Kimsey, David B.	AL	A '71
*Norris, Thomas G.	OK	A '56
Richards, John R.	NJ	B '76
*Roche, James R.	IN	G '85
Roche, Kevin M.	Son of member	
*Slegel, Timothy J.	PA	A '80
*Voellinger, Edward J.	Non-member	

Other

Alexander, Jay A.	IL	G '86
Allen, Stephen L.	MO	B '78
Aron, Gert	IA	B '58
Ausburn, Chad	OK	A '00
*Bannister, Kenneth	PA	B '82
Barr, Robert A.	IL	E '85
Benedict, Daniel H.	PA	H '09
Bernacki, Stephen E.	MA	A '70
Bertrand, Richard M.	WI	B '73
Catanich, Steven P.	CA	R '83
*Celani, Paul E.	MD	G '97
Chatcavage, Edward	PA	B '80
*Dechman, Don A.	TX	A '57
*DeFillipo, Lawrence E.	NY	O '79
*Gaston, Chuck A.	PA	B '61
*Gulian, Franklin J.	DE	A '83
Gulian, Joseph D.	Son of member	
Handley, Vernon K.	GA	A '86
*Harvey, Arthur J.	OH	A '83
Johnson, Roger W.	MN	A '79
Jordan, R. Jeffrey	OK	G '00
Kneip, Paul M.	IA	A '89
Kovalick, Albert W.	CA	H '72
Lalinsky, Mark A.	MI	G '77
*McHenry, S. Dale	MO	B '81
Miller, David J.	MA	E '71
Parks, Christopher J.	NY	G '82
Partanen, Thomas A.	MI	B '70
Pendleton III, Winston K.	MI	G '62
Penlesky, Richard J.	WI	B '73
Preble, Harry L.	MA	D '61
*Quan, Richard	CA	C '01
Riedesel, Jeremy M.	OH	B '96
Roggli, Victor L.	TX	G '73
Routh, Andre G.	FL	B '89
*Schmidt, V. Hugo	WA	B '51
Schweitzer, Robert W.	NY	Z '52
Sigillito, Vincent G.	MD	B '58
Sklarin, Wyatt	Non-member	
Spong, Robert N.	UT	A '58
Spring, Gary S.	MA	Z '82
Spring, Mitchell G.	Son of member	
*Strong, Michael D.	PA	A '84
Summerfield, Steven L.	MO	G '85
Sutor, David C.	Son of member	
*Sylvester, Noah	Son of member	
*Thompson, Ryan L.	PA	B '20
Ulman, Dave A.	MI	H '83
Zison, Stanley W.	CA	Q '83

*Denotes correct bonus solution

Spring Review

Spring #3, the cubic cryptarithm, was the easiest Tickler, with nearly every respondent recording a correct solution.

Each of the other six problems resulted in similar numbers of correct responses, roughly 40 percent of the total entries.

Subjectively though, Tickler #2 was the most difficult. As mentioned in the last issue, the judges' intent was for readers to find a method to determine the least upper bound on the number of weighings needed to find an odd penny out of 365. Given that the trivial solution 2 was accepted as a correct response in several entries, this Tickler was particularly confounding.

Summer Answers

1: **$i3\sqrt{11}$**. Remember that trigonometric functions are also defined on the complex plane. Call $\alpha = \arcsin(10)$, and $\sin(\alpha) = 10$. Since $\sin^2(\alpha) + \cos^2(\alpha) = 1$, $\cos(\alpha) = \sqrt{(1-\sin^2(\alpha))} = \sqrt{(1-\sin^2(\alpha))} = \sqrt{(1-100)} = \sqrt{(-99)} = i3\sqrt{11}$.

2: The perimeter of the octagon is **10m**, and the area of the square is **45m²**. Assume that the pond is bound by the equilateral convex octagon formed by one segment from each of the eight lines drawn by Samantha. (*Numerous other concave octagons, both symmetrical and asymmetrical, may be formed using the drawn lines. Unfortunately, the wording for this Tickler did not clearly exclude those options.*) Call the area of the square A, the length of a side of the square s, so $s^2 = A$. Overlay a cartesian coordinate system on the square with the lower left corner at (0,0). The line from the corner at origin to the center of the opposite vertical side is given by $y = x/2$. The line from the lower right corner to the center of the opposite vertical side is given by $y = -x/2 + s/2$. The intersection of the two lines meet at one corner of the octagon, the point $(s/2, s/4)$. An adjacent corner of the octagon is found at the intersection of $y = x/2$, and a line from upper right corner to the opposite horizontal side, which has the equation $y = 2x - s$. The intersection is at point $(2s/3, s/3)$. The distance between the two adjacent points on the octagon is $d = \sqrt{((2s/3 - s/2)^2 + (s/3 - s/4)^2)} = \sqrt{5}s/12$. The perimeter $p = 8d = 2\sqrt{5}s/3$. Since p must be integral, s must be an integral multiple of $3\sqrt{5}$, or $s = 3k\sqrt{5}$. Since $A = s^2 = 45k^2 < 100$, then $k = 1$, $s = 3\sqrt{5}$, and $p = 10$, $A = 45$.

3: The bicycle is traveling **left to right**, and the red dotted line represents the rear tire. On a traditional bicycle, the front wheel steers, and the rear wheel, which cannot turn, always travels in the direction of the front tire. That is, (1) a tangent drawn from the path of the rear tire must always intersect the path of the front tire and (2) the distance between a rear tangency point and the intersection with the front path will always be the same, precisely the length of the wheelbase of the bicycle. We can see from the diagram that there exist horizontal tangents from the green line that never intersect the red; therefore, the green must be the front tire and the red the rear. Furthermore, some rough measurements show that tangents from the red line drawn to the left vary in length to their intersections with the green line. However, tangents drawn to the right are of fixed length and we can conclude the bicycle is traveling left to right.

4: The probability that the first digit of 2^N is a 1 is $\log_{10} 2$ = 0.30103. Observe that for every positive *n*, there is exactly one *n*-digit number that begins with the digit one that is a power of two. There can't be two (or more), for if *k* is the smallest *n*-digit number, 2*k* must begin with a two or a three. In addition, there has to be at least one, since, given the (*n*-1)-digit power of two 2^M that starts with 1, we can show that one and only one of $(8)2^M$ or $(16)2^M$ must start with 1. This Tickler is an example of Benford's Law, which is an observation about the distribution of leading digits in sets of numerical data.

5: Initially, light both ends of the 24 minute fuse and one end of the 50 minute fuse. When the shorter fuse is extinguished, 12 minutes have passed, **light the other end of the longer fuse**. The longer fuse has 38 minutes left, and should finish in half that time, 19 more minutes. At that point, 12+19=31 minutes have passed, at which time the switch can be flipped.

BONUS: Both cases require tucking to achieve the 1-8 stacking. For the first puzzle, the steps are:

1. Fold D onto A, G onto H, E onto B, F onto C.

2. Fold 6 onto 7, 5 onto 4.

3. Tuck D/E back so that E touches F, D touches C.

4. Fold 3 onto 2.

Reading from top down, the squares are in order:

1, B, 3, D, 5, F, 7, H.

For the second puzzle, the steps are:

1. Fold D onto A, E onto H, C onto B, F onto G.

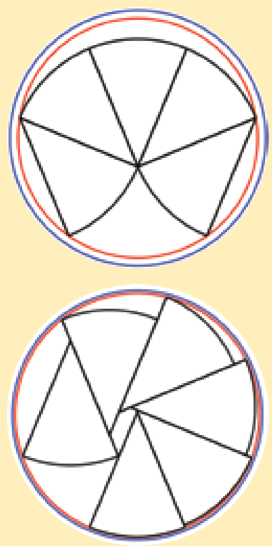
2. Fold 5 onto 4.

3. Tuck 6/7 between A/D, like a snake eating its tail, until 7 touches H, 6 touches E, 3 touches D, and 2 touches A.

Reading from top down, the squares are in order:

1, 2, C, D, 5, 6, G, H.

DOUBLE BONUS: The maximum number of slices that can fit on the marginally smaller plate is S = **6**. The smallest plate we found that can hold 6 slices is approximately **29.30 cm** in diameter. Our best 5 slice configuration is exactly D_8 x $\sqrt{((6-3\sqrt{2})/2)}$ in diameter, or approximately **28.12 cm**. This Tickler, motivated by a mealtime discussion of the family of **Timothy J. Slegel**, *PA A '80*, has no methodical solution for determining the layout of the slices, at least none was found by any of the efforts of the submitter or judges. The best configurations, as shown below, were found by trial and error with computer approximation used to initially calculate each diameter.



New Fall Problems

1: Order of Merit

Alf, Bert, Charlie, Duggie, and Ernie are arranged in an order of merit (no ties) for Honesty, Charm, and Intelligence. In the remarks which they make about the places of themselves and others, those who are 1st (highest), 2nd or 3rd for Honesty invariably tell the truth, but all the remarks made by the other two are false.

ALF: (i) Duggie was 1st for Honesty. (ii) I was not higher for Charm than I was for Intelligence.

BERT: (i) I was higher for Intelligence than I was for Honesty (ii) I was higher for Charm than I was for Intelligence.

CHARLIE: (i) I was 4th for Honesty. (ii) I was not higher for Intelligence than I was for Charm.

DUGGIE: (i) Alf was 4th for Honesty. (ii) In at least one test Ernie was lower than Charlie.

ERNIE: (i) I was 3rd for Charm. (ii) The sum of the numbers of Duggie's places is one less than the sum of the numbers of Bert's.

Find the order of merit (1st to 5th) in each of the three tests. Present your answer as a matrix with the five rankings across the top as 1, 2, ... 5.

—*Brain Puzzler's Delight* by E. R. Emmet

2: Diameter of Lily Pad

One calm day on Peaceful Pond, Ninny the Newt, weighing in at 20 g, was relaxing at the center of a floating circular lily pad weighing 300 g. Nearby, Sneaky the Snake, weighing in at 7 kg, was curled up at the far end of a floating straight log one meter long, weighing 25 kg, pointed directly at Ninny, and separated from the lily pad by only 75 cm. Spying Ninny, Sneaky began to slither along the log. Ninny scurried away to the far edge of the pad, but then, in confusion, turned around and ran back the other way, stopping at the edge closest to the log. By that time, Sneaky was at the near end of the log, poised to strike. However, Ninny was saved because the pad and log were then separated by 100 cm, just out of striking range. Assuming Sneaky and Ninny to be points, the weights of log and pad to be uniformly distributed and water resistance to be zero, what was the diameter of the lily pad?

—Adapted from Allan Gottlieb's Puzzle Corner in *Technology Review*

BTs continue on page 59.

New Fall Problems Cont.

3: Path to Goddess

The Ratselgarten in Viena is famous for its twenty goddesses, whose statues stand at the junctions of its paths. The task of keeping them spick and span belongs to Stephan Schnitzel. Once a month, he dusts and polishes them, following a route of his own design which, without leaving the paths shown, takes him to each goddess exactly twice.

Each goddess has a different letter on the plan in his office and his order of visiting is, he tells me, P A D M O I C T F K G B J R H N L Q E S P A L Q J R H N D M O I C T S F E K G B. But, as you will no doubt spot without even being told which letter to put at which junction, he has made a small error in the telling. He has inadvertently put two consecutive letters in the wrong order somewhere. What are those two letters? Which letter is at which junction with goddess A placed as close as possible to the top left corner? In the diagram, each + is a goddess/junction; - and | are paths.

—A Tantalizer by Martin Hollis in *New Scientist*.

```
+--+--+--+
|  |  |  |
+  +--+  +
|  |  |  |
+--+  +--+
|  |  |  |
+  +--+  +
|  |  |  |
+--+--+--+
```

4: Cryptarithmetic Game

Solve the following cryptic multiplication.

AB × CDE = FGHIJ

with no zeros in the 5 digit number. Standard rules apply: no leading zeros, different letters are different digits, and base 10.

—Joseph Nabutovsky,
Father of member

5: Integers Problem

Find the smallest positive integers a, b, c such that $a^3 + b^4 = c^5$.

—Allan Gottlieb's Puzzle Corner in *Technology Review*

BONUS:

The game of SETS is played with a deck of 81 cards, each card having a different combination of four properties: symbol shown (circle, triangle, or square); color of symbol (red, green, or purple); number of symbols on a card (one, two, or three of the same symbol); and type of symbol fill (outline only, solid color, or crosshatched). The deck is shuffled, and *N* cards are laid out, face up. The objective is to form sets of three cards from the laid out cards. When a set is made, it is removed and replaced with three new cards from the cards remaining in the deck. Three cards constitute a set, if, and only if, for the three cards, each of the four properties individually is either all the same or all different. Thus, three cards, one with one solid red triangle, a second with two outlined green triangles, and a third with two crosshatched red circles, do not make a set (two red cards, one green card; two cards with triangles, one with circles; and one card with one symbol, two cards with two symbols). But if we replace the third card with a card with three crosshatched purple triangles, then we have a set: three different colors (red, green, purple); all the same symbol (triangles); three different numbers of symbols (1, 2, 3); and three different fills (solid, crosshatched, outlined). What is the maximum number of cards that can be initially laid out that contains no set?

—**Howard G. McIlvried III,** *PA Γ '53*

COMPUTER BONUS:

Find a 4 by 4 magic square such that the product of the numbers in each row, column, major diagonal, the four corners, and every 2 by 2 subsquare is the same constant. The numbers in the square must all be different positive integers, but they do not need to be consecutive integers. We want the square with the smallest magic product and the top left corner is the value 1.

—**Howard G. McIlvried III,** *PA Γ '53*

Email your answers (plain text only) to any or all of the Fall Brain Ticklers to *TBP.BrainTicklers@tbp.org* or by postal mail to **Dylan Lane, Tau Beta Pi, P.O. Box 2697, Knoxville, TN 37901-2697**. The method of solution is not necessary. The Computer Bonus is not graded. Where possible, exact answers are preferable to approximations. The cutoff date for entries to the Fall column is the appearance of the Winter *Bent* which typically arrives in early January (the digital distribution is several days earlier). We welcome any interesting problems that might be suitable for the column. Dylan will forward your entries to the judges who are **J.C. Rasbold,** *OH A '83;* **J.R. Stribling,** *CA A '92;* **G.M. Gerken,** *CA H '11;* and the columnist for this issue,

— **F.J. Tydeman,** *CA Δ '73*