RESULTS FROM SPRING

Perfect			
*Bohdan, Timothy E.	IN	Γ	'85
*Couillard, J. Gregory	IL	A	'89
*Gee, Albert	CA	A	'79
*Gerken, Gary M.	CA	H	'11
*Griggs Jr., James L.	OH	A	'56
*Gulian, Franklin J.	DE	A	'83
Gulian, Joseph D.	Son of member		
*Handley, Vernon K.	GA	A	'86
*Harvey, Arthur J.	OH	A	'83
*Johnson, Mark C.	IL	A	'00
*Marx, Kenneth D.	OR	A	'61
*Norris, Thomas G.	OK	A	'56
*Parks, Christopher J.	NY	Γ	'82
*Richards, John R.	NJ	B	'76
*Slegel, Timothy J.	PA	Ạ	'80
*Tang, Angela L.	NY	Δ	'18
Other			
Alexander, Jay A.	IL	Γ	'86
*Anderson, Kurt E.	KS	Γ	'90
Barthel, Gerald R.	OH	B	'67
Budd, Christopher M.	AZ	B	'94
*Chapman, Thomas W.	Son of member		
*Dechman, Don A.	TX	A	'57
*Foster, David E.	VA	B	'94
Grewal, Rashi	NJ	Γ	'09
*Johnson, Roger W.	MN	A	'79
*Kimsey, David B.	AL	A	'71
Lalinsky, Mark A.	MI	Γ	'77
Niemi, Michael G.	MD	Γ	'07
Pendleton III, Winston	MI	Γ	'62
Pineault, Wayne	IL	Γ	'79
*Quan, Richard	CA	X	'01
Rentz, Mark	Son of member		
*Riedesel, Jeremy M.	OH	B	'96
*Schmidt, V. Hugo	WA	B	'51
*Shepperd, Stan W.	MA	B	'70
*Sigillito, Vincent G.	MD	B	'58
Silver, Robert E.	NY	P	'80
*Spong, Robert N.	UT	A	'58
*Strong, Michael D.	PA	A	'84
Summerfield, Steven L.	MO	Γ	'85
*Voellinger, Edward J.	Non-member		
*Zison, Stanley W.	CA	Θ	'87

*Denotes correct bonus solution

SPRING REVIEW

The spring batch of Ticklers was apparently somewhat easier than other recent sets. The hardest regular problem was No. 4, about upgrading a poker hand, with only ⅔ of the entries that answered this Tickler having gotten it correct. The rest of the regular problems had close to 100% correct answers. The Bonus (about three triangular numbers whose product is a perfect square) was correctly solved by about ¾ of the entries.

SUMMER ANSWERS

1 The priest is **52** years old. As listeners, we don't know the actual whispered hint and we might not know all the visitor's ages, but we can figure out the priest's age.

The priest's first hint indicates that the product of the three visitors' ages is 2652, and the cantor calculates the prime factorization of 2652 as $2^2 \times 3 \times 13 \times 17$. Ignoring any potential upper bound on age, the factors can be combined to produce the following set of 28 age triples: S = {(12 13 17) (6 17 26) (6 13 34) (3 26 34) (4 17 39) (2 34 39) (2 3 442) (4 13 51) (2 26 51) (3 17 52) (1 51 52) (3 13 68) (1 39 68) (2 17 78) (1 34 78) (2 13 102) (1 26 102) (1 17 156) (1 13 204) (3 4 221) (2 6 221) (1 12 221) (1 6 442) (2 2 663) (1 4 663) (1 3 884) (1 2 1326) (1 1 2652)}.

The content of the priest's second whispered hint is unknown, but it narrows the potential solution to a member of S' = $\{(y_i\ m_i\ o_i)\}$. S' is a subset of S and the values y_i, m_i, and o_i in each age triple are the ages of the youngest, middle, and oldest visitor. The cantor is still unsure which triple matches the visitors' ages, so we know the cardinality of S' is greater than 1.

Let the priest's age be denoted by p. Let o_o be the smallest o_i in S'. Let o_1 be the second smallest o_i in S'. With the priest's third statement about his birthday, we learn that the priest is older than any of the visitors, and that the cantor knows the priest's age. Combined with the previous hints, the cantor is able to use the new information to determine the visitors' ages. Knowing the priest's age, and that the priest is older than all the visitors indicates that there is only one triple in S' where all of the ages are less than p. That is, we can conclude the age relationship $o_o < p <= o_1$ exists.

Finally the cantor makes a statement that allows anyone listening to unambiguously determine the priest's age. From this, we conclude $o_o + 1 = o_1$, so therefore $p = o_1$. Only $o_o = 51$ and $p = o_1 = 52$ fits from S, so the priest is 52 (and the oldest visitor is 51).

2 The maximum probability of winning is **4 / (4 + π),** which occurs when the diameter is **4 / (4 + π).** Each tile is 1 square unit. Let d be the diameter of the disk. You lose if the disk does not touch the boundary of the tile; this happens if the center of the disk is at least $d/2$ from the edge. The area of that portion of the tile is $(1–d)^2$. You also lose if the disk covers the corner of a tile, that is, if the center of the disk is within $d/2$ of a corner. That area is $\pi(d/2)^2$.

The probability of winning $P(d) = 1 – (1–d)^2 – \pi(d/2)^2 = 2d – d^2(1 – \pi/4)$. Differentiating with respect to d, $P'(d) = 2 – 2(1 – \pi/4)d$. Setting $P'(d) = 0$ and solving, we find $P(d)$ is maximized at $d = 4 / (4 + \pi)$. Plugging $4 / (4 + \pi)$ into $P(d)$, we find $P(4 / (4 + \pi)) = 4 / (4 + \pi)$.

3 There are **88** ways to walk up a flight of 13 stairs. Let S_n be the number of ways to climb n steps. Clearly, there is one way to climb 1 step and one way to climb two steps (one at a time). Three steps can be climbed in two ways, that is, one step three times or three steps once. So $S_1 = 1$, $S_2 = 1$, and $S_3 = 2$. From there, observe that S_n is defined by the recurrence relation $S_n = S_{n-1} + S_{n-3}$. Therefore, S_n = 1, 1, 2, 3, 4, 6, 9, 13,19, 28, 41, 60, 88. Hence $S_{13} = 88$.

4 There are **six** cubes with 2 sides of each of three colors. Since there are 8 corners on the 3-inch cube, each color (red, white, blue) must have 8 cubes with three faces of their color, 12 cubes with two faces of their color, and 6 cubes with one face of their color.

No cube may have more than three faces of a single color. So there are three possible color distributions across the faces: 3-2-1, 3-3-0, and 2-2-2.

A cube with a single face of an individual color must have a 3-2-1 distribution. There are 3 colors and each needs 6 cubes with a single face, so 18 of the 27 cubes must have the 3-2-1 distribution.

Each color needs a total of 8 cubes with 3 faces of their color, so 3 of the 27 cubes must have a 3-3-0 distribution (each color appears on two of these cubes). The one cube that has no faces of given color will be at the center of the 3-inch cube.

Six cubes remain, each with the 2-2-2 distribution.

5 Drawing the **King of Clubs** or better gives a player a greater than 50-50 chance of winning. Find the least ranked card such that if one picks four other cards out of the remaining 51, the chance of being highest ranked is greater than 50%. Numerically, this is equivalent to finding the Nth ranked card, where N is the least integer such that $p = C(N-1,4)/C(51,4) \geq 0.5$. For $N = 45$, $p \approx 0.543$. For $N = 44$, $p \approx 0.494$. So, we pick the 45th ranked card, which is the King of Clubs.

Bonus A and B's numbers are **34** and **55,** respectively.

Observe that because the integers on each of the logician's hats must be positive, for if any two logicians have the same number, then the third will immediately know her number, since hers must be sum of the other two and not the difference.

Let [a b c] be the integral ratio of the three numbers. The only triples possible are ones where one of the three is the sum of the other two. On her first turn, A will only be able to deduce her number if the ratio of the three is [2 1 1]. When A passes, all three logicians know that [2 1 1] is not possible.

On B's first turn, as a result of A passing, B will be able to guess correctly if she observes that A is twice C, or [2 3 1]. In addition, similar to A, B can also know her number if A and C's numbers are equal. So when B passes, all know that both [2 3 1] and [1 2 1] have been eliminated.

On C's first turn, C can properly deduce her number if it is the sum of a and b in any the previously eliminated triples. And like A and B's respective first turns, also if A and B's numbers are equal. When C passes, the ratios [2 1 3], [2 3 5], [1 2 3] and [1 1 2] are eliminated.

With each additional passing logician's turn, the list of not-possible ratios grows. On her turn, a logician examines all the newly eliminated ratios from each of the two previous turns. For each eliminated ratio, she adds the values of the other two logicians and replaces the value of the guessing logician with the sum. For A's second passing turn, she eliminates [4 3 1], [3 2 1], [4 1 3], [8 3 5], [5 2 3] and [3 1 2].

On each turn, the eliminated triples grow at a Fibonacci rate. B eliminates 10 triples on her second pass, C 16 on her second, A 26 on her third, and B 42 on her third. However, on her third turn, C deduces her number to be 89. Of the 68 triples under consideration, the c in only one equals 89, that is [34 55 89]. (Had C's number been non-prime, any triple with a c that divides her number would do.)

Computer Bonus P_{170} = **42208400.** Completing the eight integer sequence, $P_{171} = 47627751$, $P_{172} = 53047102$, $P_{173} = 58466453$, $P_{174} = 63885804$, $P_{175} = 69305155$, $P_{176} = 74724506$, $P_{177} = 80143857$. The corresponding denominators are $Q_{170} = 13435351$, $Q_{171} = 15160384$, $Q_{172} = 16885417$, $Q_{173} = 18610450$, $Q_{174} = 20335483$, $Q_{175} = 22060516$, $Q_{176} = 23785549$ and $Q_{177} = 25510582$. The sequences P_i and Q_i are an interesting study. Large jumps in the sequences occur at irregular intervals. At other places, P_i and Q_i increment monotonically for intervals of consecutive elements. Observe that between P_{170} / Q_{170} and P_{177} / Q_{177} above, $P_{i+1} = P_i + 5419351$ and $Q_{i+1} = Q_i + 1725033$. Not coincidentally, the increments correspond to $P_{169} = 5419351$ and $Q_{169} = 1725033$. In addition to including a P_i with units digit 0, and being of minimum length eight, only runs where the increment ends with the digit 1 can generate a desired count-up units digit sequence. The alternate example $P_{14} / Q_{14} = 355 / 133$ provides a long-ish run of fixed increments from $i = 15$ through $i = 162$, but in that interval, the units digit in P_i strictly alternates between 3 and 8. After P_{170}, the five next sequences of at least length 8 start at P_{189} (length 42), P_{781} (9), P_{986} (10), P_{1169} (8) and finally P_{1866} starting an incredibly long length of 10388 consecutive P_i with unit-digit unit increments.

NEW FALL PROBLEMS

1 The following cryptic additions are to be solved simultaneously, that is, letters have the same value in both cryptics; different letters are different values; * can represent any value. There are no zeros anywhere. The two sums are mirror images (i.e., reverses) of each other.

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 ******      MIRROR
+MIRROR     +IMAGE*
 ------      ------
 IMAGE*      ******
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—an Enigma by Albert Haddad in *New Scientist*

2 An antimagic square is an $N \times N$ grid containing the integers from 1 through N^2, such that the sum of each row, column, and main diagonal is a different number, and those sums form a consecutive sequence of integers. There are no 3×3 antimagic squares, but there are some that are close in that the sequence of sums is consecutive except for one missing value. Find a 3×3 near antimagic square in which the three digit number represented by the top left (hundreds position) to bottom right (units position) main diagonal is a minimum. Present your answer as a 3×3 grid containing the digits 1 through 9.

—H.G. McIlvried III, *PA Γ '53*

3 On average, how often (once every N years) does a blue moon (2nd full moon in the same month) fall on New Year's Eve in Los Angeles?

—F.J. Tydeman, *CA Δ '73*

4 As a promotion, a pizza shop advertises a medium plus a large pizza, with up to five toppings each, for just the price of a large pizza. The advertisement further states that a patron has a choice of 1,048,576 different combinations of two pizzas. Assuming this statement is correct, if a topping can be used only once on a given pizza and the two pizzas can each have a different combination of toppings, how many different toppings does the pizza shop have?

—Allan Gottlieb's Puzzle Corner in *Technology Review*

5 Another order of merit (no ties) for A, B, C, D, E—this time for Deception. In their usual chatty way, they are making remarks about their places. The remarks of those who were first and second are false, the rest are true.

A: D was third.
B: E was not first.
C: I was not last.
D: C was lower than B.
E: B was second.

What were their places (first to last)?

—*Brain Puzzler's Delight* by E.R. Emmet

Bonus Consider a unit radius circle whose center is at the origin of a set of Cartesian coordinates. Construct a 2Θ sector of this circle lying along the x-axis, with half the sector lying above the x-axis and half lying below the x-axis. Inscribe in this sector a series of circles of radii R_1, R_2, R_3, ..., whose centers are on the x-axis. The first circle is tangent to the arc of the sector and its two sides. The second circle is tangent to the first circle and the sides of the sector; the third circle is tangent to the second circle and the sides of the sector; ad infinitum. Obviously, the ratio of the sum of the circles' areas to the sector's area is less than 1. For what value of Θ is this ratio the largest, and what is this largest ratio?

—Allan Gottlieb's Puzzle Corner in *Technology Review*

Computer Bonus The prime 5 exactly divides the sum of all the primes smaller than itself, (2+3)/5 = 1. The next prime with this property is 71. What is the next (after 71) such prime?

—*Prime Numbers, the Most Mysterious Figures in Math* by David Wells

Postal mail your answers to any or all of the Brain Ticklers to **Dylan Lane, Tau Beta Pi, P.O. Box 2697, Knoxville, TN 37901-2697** or email to *BrainTicklers@tbp.org* as plain text only. The cutoff date for entries to the fall column is the appearance of the Winter *Bent* which typically arrives in early January (the digital distribution is several days earlier). The method of solution is not necessary. We welcome any interesting problems that might be suitable for the column. The Computer Bonus is not graded. Dylan will forward your entries to the judges who are **H.G. McIlvried III,** *PA Γ '53*; **J.C. Rasbold,** *OH A '83*, **J.R. Stribling,** *CA A '92*; and the columnist for this issue,

—F.J. Tydeman, *CA Δ '73*

The Editor of *The Bent* is frequently searching for well-written, general interest, feature articles for the magazine. Many of you have the appropriate experience, are qualified, and capable of preparing such a feature. If you have a topic that you believe would be suitable, email proposals to tbp@tbp.org. Members working on interesting research or design projects are encouraged to submit an article. Manuscripts should be 1,000-3,000 words, double-spaced, and submitted as a text or MS Word document. Publishing decisions cannot be guaranteed.