Brain Ticklers

RESULTS FROM SPRING

Perfect

Alverson, Robert L.	OH	B	'85
Beaudet, Paul R.	Father of member		
*Couillard, J. Gregory	IL	A	'89
Eanes, Robert S.	TX	Γ	'67
*Gee, Albert	CA	A	'79
Gee, Nora W.	CA	A	'79
Gee, Aaron J.	CA	Ψ	'16
*Griggs Jr., James L.	OH	A	'56
*Hurd, Jonathan A.	MA	B	'79
*Johnson, Mark C.	IL	A	'00
LaCroix, Daniel J.	MI	B	'18
Mangis, J. Kevin	VA	A	'86
*Norris, Thomas G.	OK	A	'56
Schmidt, V. Hugo	WA	B	'51
*Sheiman, Arthur E.	CA	B	'81
Slegel, Timothy J.	PA	A	'80
Spong, Robert N.	UT	A	'58
Stein, Gary M.	FL	Δ	'04
Strong, Michael D.	PA	A	'84
*Verkuilen, William W.	WI	B	'92
Willard, Duane J.	NY	Θ	'71

Other

Aron, Gert	IA	B	'58
Benedict, Daniel H.	PA	H	'09
Bertrand, Richard M.	WI	B	'73
Beutner, Thomas J.	IN	A	'87
Christiansen, Reed L.	MN	A	'83
*Conway, David B.	TX	I	'79
Dechman, Don A.	TX	A	'57
Edge, Billy L.	GA	A	'71
Ehrgott Jr., Charles	FL	E	'92
Fogel, Arlene Beck	MA	Δ	'77
*Gerken, Gary M.	CA	H	'11
Grewal, Rashi	NJ	Γ	'09
Handley, Vernon K.	GA	A	'86
Harvey, Arthur J.	OH	A	'83
Hasek, William R.	PA	Γ	'49
Heierman, William E.	GA	A	'64
Heutchy, David A.	PA	Γ	'69
*Johnson, Roger W.	MN	A	'79
*Jones, John F.	WI	A	'59
Jones, Jeffrey C.	Son of member		
Kovalick, Albert W.	CA	H	'72
Lalinsky, Mark A.	MI	Γ	'77
Lowe, Michael	MT	A	'15
Marks, Lawrence B.	NY	I	'81
Marks, Benjamin	Son of member		
Marrone, James I.	IN	A	'61
McNulty, Whitney P.	LA	A	'85
Medvecz, David J.	IN	A	'83
Minnick, Michael V.	SC	A	'81
Pendleton III, Winston	MI	Γ	'62
Prince, Lawrence R.	CT	B	'91
Prine, John G.	WA	B	'67
Quan, Richard	CA	X	'01
Rentz, Peter E.	IN	A	'55
Richards, John R.	NJ	B	'76
Rizzo, Robert J.	NY	Γ	'72
Rubin, James D.	MI	Γ	'82
Schweitzer, Robert W.	NY	Z	'52
Sigillito, Vincent G.	MD	B	'58
Snelling, William E.	GA	A	'79
Summerfield, Steven L.	MO	Γ	'85
Vinoski, Stephen B.	TN	Δ	'85
*Voellinger, Edward J.	Non-member		
Yuan, Feibi	OH	Γ	'15

* Denotes correct bonus solution

SPRING REVIEW

The response to the Spring Ticklers was surprising, with more than 60 entries. We haven't had that many for a long time. Perhaps the column was a little easier than usual and encouraged more readers to submit their answers. The easiest regular problem was No. 4 about the four digits forming 24 different integers, with only one wrong answer. The hardest was No. 3 about the paving stones. The Bonus proved to be rather difficult, with only half the responses containing answers and only a third of those being correct.

SUMMER ANSWERS

Readers' entries for the Summer Ticklers will be acknowledged in the Winter *Bent*. Meanwhile, here are the answers.

1 Tatiana's age is **14.** This tickler can be solved without knowing the polynomial. Observe that when two numbers are substituted into a polynomial $f(x)$ with integral coefficients, the difference between the two resulting values $f(i) - f(j)$ is divisible by the difference of the two numbers $i - j$. We deduce this because any constant drops from the difference, and only like powers of i and j remain, which are always divisible by $i - j$.

Let t denote the age of the child. We know from the conversation that $t > n > 7$. Since $n - 7$ divides $85 - 77 = 8$, n must be one of 8, 9, 11, or 15. Because $t - 7$ divides 77, t must be 14, 18, or 84. To have $t - n$ divide 85, the difference must be 5, 17, or 85. Of the possible values, $14 - 9 = 5$ fits, so $n = 9$ and $\mathbf{t = 14}$.

2 Each floor is an annulus, that is, the shape of an ordinary washer. The area of the hexagon floor is $\mathbf{900\pi}$ **m²**, its outside perimeter is $\mathbf{120\pi}$ **m,** and its height above the ground is $\mathbf{(5\sqrt{241})/2}$ **m.** Let floor i be defined by the regular polygon with i sides. The circumradius of the polygon defines the radius R_i of the outer circle of the annulus. The apothem of the polygon defines the radius r_i of the inner circle. For a polygon with i sides, the ratio of the two radii $r_i / R_i = \cos(\pi / i)$, for $i > 2$. The area of the annulus is the difference of the areas of the two circles, or $A_i = \pi(R_i^2 - r_i^2)$.

Viewing the cross section of the building, let R_s be the radius of the outer hemisphere, and r_s be the radius of the inner hemisphere. Define h_i as the distance from the ground to floor i. Observe that $h_i^2 + r_i^2 = r_s^2$ and $h_i^2 + R_i^2 = R_s^2$. Combining to eliminate h_i and rearranging, $R_i^2 - r_i^2 = R_s^2 - r_s^2$. The right hand side is fixed by the radius of the two hemispheres, so we can conclude the area of each floor is identical.

We know from the problem that floor 4 is defined by a 60m square. Then $r_4=30$, $R_4=30\sqrt{2}$ and $A_4 = 900\pi$ m². So $A_4 = \mathbf{A_6 = 900\pi}$ **m².** Combining $r_i / R_i = \cos(\pi / i)$ with $R_i^2 - r_i^2 = 900$, $R_i^2(1-\cos^2(\pi / i)) = 900$, or $R_i = 30/\sin(\pi / i)$. It follows that $r_i = 30/\tan(\pi / i)$. For $i = 6$, $R_6 =$ 60m and the outer perimeter on floor 6 is $(2\pi)60 = \mathbf{120\pi}$ **m.**

Finally, use the fact that $h_4 + 5 = h_3$ to compute R_s. We can compute $R_3^2 = (30/\sin(\pi / 3))^2 = 1200$ and $R_4^2 = (30/\sin(\pi / 2))^2 = 1800$. Substituting into $h_4^2 + R_4^2 = R_s^2$, we get $h_4^2 + 1800 = R_s^2$. Substituting into $h_3^2 + R_3^2 = R_s^2$, we get $h_4^2 + 10h_4 + 1225 = R_s^2$. Combining the two and simplifying gives us $h_4 = 57.5$. Substituting back into $h_4^2 + 1800 = R_s^2$ gives $R_s^2 = 5106.25$. Using $h_6^2 + R_6^2 = R_s^2$, $h_6 = \sqrt{(5106.25 - (30/\sin(\pi / 6))^2)} = \sqrt{1506.25} = \mathbf{(5\sqrt{241})/2 \sim 38.81}$ **m.**

3 The number is **24.** The number must be two digits, so that adding 18 reverses the digits. Let x be the first digit, and y the second. The first condition is equivalent to the equation $(10x + y) = 3xy$. The second condition is $(10x + y) + 18 = (10y + x)$ or simply, $x + 2 = y$. Substituting the second into the first, $11x + 2 = 3x(x + 2)$ which simplifies to $3x^2 - 5x$

- 2 = 0. That is, $x = 2$ or $x = -\frac{1}{3}$. The digit x must be an integer, so $x = 2$, $y = 4$, and the number is **24.**

4 In the order from ***SUN*** through ***SAT,*** the code numbers for the days of the week are **10, 5, 6, 9, 4, 7, 8.** Observe that, of the integers between 1 and 10, the numbers 2, 4, 6, 8, 10 share the common factor 2, the numbers 3, 6, 9 share the common factor 3, and the numbers 5, 10 share the common factor 5. (The factor 4 is shared by 4 and 8, but that is a subset of the numbers that share the factor 2.) Of the seven days of the week, *N* is found in *SUN* and *MON*, *U* is found in *SUN*, *TUE*, and *THU*, *E* is found in *TUE* and *WED*, *T* is found in *TUE*, *THU*, and *SAT*, and finally, *S* is found in *SUN* and *SAT*. The *U*, *T*, and *S* days overlap, forming a larger group of 4 days, so those days must be coded from the multiples of 2. The *N* and *E* groups already each have one member that is a multiple of 2, so *SUN* and *TUE* each must be one of 6 or 10, and *MON* and *WED* must come from 3 or 9, and 5. Suppose *TUE* is 10, then *WED* is 5. Since maximum *THU* is 8, there is no *MON* small enough such that *MON*+*TUE* < *WED*+*THU*, and conclude that *TUE* is not 10. Therefore ***TUE* = 6, *SUN* = 10, *MON* = 5.** So *MON*+*TUE*=11, and since maximum *THU* = 8, *WED* > 3, so ***WED* = 9.** *FRI* has no common letters with other days, therefore must be either 1 or 7. ***FRI* = 7** because if *FRI* = 1 then *FRI* + *SAT* <= 9.

We must choose *THU* and *SAT* from 2, 4, 8: *SAT* must be more than 2 greater than *THU* to ensure that 9 + *THU* > 7 + *SAT*, so ***SAT* = 8** and ***THU* = 4**.

5 The five Tau Bates can order a total of 15 dishes in their three visits. Ordering a dish twice in the same meal will uniquely identify it. Also ordering a dish just once will uniquely identify it. They can identify three items the former way, and three more in the latter way. The remaining three can be determined on a unique pair of days. For example:

A	B	C	D	E	F	G	H	I
2			1			1	1	
	2			1		1		1
		2			1		1	1

On the first day: **AADGH.** Dish A is identified by the one that arrives twice. Second day: **BBEGI.** Dish G is known as the one that came also on day 1 and dish B is the one appearing twice. Third day: **CCFHI.** Dish H is the one served on days 1 and 3. Dish I served on days 2 and 3. Dish C twice on day 3, dish D only on day 1, dish E only on day 2, and dish F only on day 3.

Bonus The best solution that we found is **2-12-60, 2-14-35, 2-15-30, 3-5-15,** and **3-6-10;** where the numbers indicate the denominators of unit fractions. The baker slices her loaf in 2-3-12-15-60, and the two apprentices 2-3-14-15-35 and 2-5-6-10-30 respectively. If the skill constraint is removed on the baker or the apprentices, five other solutions are possible, involving the alternative slicings 2-3-11-15-110 and 2-4-5-30-60 by one of the bakers.

Computer Bonus $\mathbf{BDA8779}_{14}$ = $90{,}224{,}199_{10} = 39^5$, $\mathbf{CEE2744}_{15}$ = $148{,}035{,}889_{10} = 23^6$, $\mathbf{ITI0801}_{30}$ = $13{,}841{,}287{,}201_{10} = 343^4 = 49^6 = 7^{12}$. A naive, brute force approach is to enumerate all legal license plates, and then test to see which ones meet the x^y such that $y > 3$ criteria. A more efficient algorithm might recognize that the largest x such that $x^4 < \mathbf{ZZZ9999}_{36}$ is 529, then test pairs in the range for x from 2 to 529 and y from 4 until $x^y > \mathbf{ZZZ9999}_{36}$. With those limits, a program needs only examine about 23,000 x,y pairs, which is considerably more tractable than the brute force method. Other optimizations to reduce the search space are possible. The judges produced solutions on multiple programming platforms that generated the solutions in less than a second.

NEW FALL PROBLEMS

1 Aunt Alice is a bit hard of hearing, so that when her nephew Ned asked various neighbors some questions about their house numbers, she failed to hear the answers, though she heard the questions fine. Ned has lived on Christmas Crescent for some time, and his aunt knows Ned's number, but Alice has only just bought a vacant house there, but Ned doesn't know that. The Crescent has houses numbered rather curiously from 5 to 105, inclusive.

Ned asked the same three questions to three people, first to X, then to Y, then to Z, who live in separate houses on the Crescent. The questions were:

(i) Is the number of your house a multiple of 4?
(ii) Is it a perfect square?
(iii) Is it a multiple of 9?

No two sets of three answers were exactly the same. After hearing X's answers, Ned says to him: "If I knew whether your house number is greater than 83, I would know what it is." (Alice heard her nephew say this and was able to write down X's number correctly.)

After hearing Y's answers, Ned says to him: "If I knew whether your number is greater than 50, I could tell you what it is." (Alice heard her nephew say this, and as she happened to know that Y's number is greater than her own, she was able to write it down correctly.

After hearing Z's answers, Ned says to him: "If I knew whether your house number is greater than 30, I could tell you what it is." (Alice heard this and notes with interest that her own number is certainly less than Z's. But she too has no way of deciding whether Z's number is greater than 30. However, being anxious to continue building her reputation for logical deduction and intuition, guessed that it was greater than 30 and wrote down the number, which, fortunately was quite correct.)

What are the house numbers of Ned, Alice, X, Y, and Z?

—*Brain Puzzler's Delight*
by E.R. Emmet

(Continued from page 52)

2 The Rev. Obadiah Slope does not actually care about soccer but, since becoming a Rural Dean, he likes to seem as if he does. So he takes a dutiful interest in the local league (each team plays each other team once in a season). "How is it going this year, my son?" he recently asked the retired tax inspector who acts as secretary.

"Well, it's not finished yet you know," was the answer, "even though only four sides are competing this time. Parminster United have so far played two games and lost them both, with three goals for and six against in total. Quondam Athletic have also played two, winning one and drawing the other, with a total of four goals for and three against. Real Episcopi have drawn one and won two, totaling two goals for and none against. And Salem Dynamo...."

"Splendid! Splendid!" Rev. Slope broke in, conscious that the lunch hour was nigh and suddenly remembering that, "I must see a man about a misericord."

What was the score in each of Salem's matches?

—A Tantalizer by Martin Hollis in *New Scientist*

3 On a 12-hour analog clock, at what time near 8:18 are the hands symmetric about 6, that is, 6 splits the angle between the minute and hour hands? At what time near 8:18 is the clockwise angle from the hour hand to the minute hand the Golden Mean fraction of the whole circle, that is, the angle between the minute and the hour hands is 360°(√5 - 1)/2? Express your two answers (to the nearest millisecond) in the form hh:mm:ss.ttt.

—Allan Gottlieb's Puzzle Corner in *Technology Review*

4 In the game of tennis, two players hit a ball back and forth across a net into a defined court. A player loses a point by either hitting the ball outside the court or failing to return a ball before it has bounced twice. The first player to win four points, provided he is ahead by at least two points, wins the game. Thus, the final score can be 4-0, 4-1, 4-2, 5-3, 6-4, etc. The same player serves for a complete game, and on each point he has two chances to make a successful serve. Assume a player has a probability, p_1, of a successful first serve, a probability, p_2, of a successful second serve if the first serve is bad, a probability q_1 of winning a point if his first serve is good, and a probability q_2 of winning a point if his second serve is good. If he is serving, what is his probability w of winning a point (in terms of p's and q's)? What is his probability g of winning the game (in terms of w)? If p_1=0.8, p_2=0.7, q_1=0.6, and q_2=0.5, what are w and g?

—H.G. McIlvried III, *PA Γ '53*

5 Solve this cryptic addition with the usual rules: different letters are different digits, same letter is same digit, no leading zeros, base 10. FIVE + FIVE + TEN + TEN + TEN + TEN + THIRTY = EIGHTY.

—*Journal of Recreational Mathematics*

Bonus Four square blocks of sizes from 1^2 up to 4^2 can be fitted into a 7 x 5 rectangle (as shown), but not into a smaller rectangle. As you take a larger series of blocks, 1^2, 2^2, 3^2, ..., n^2, it is not always easy to choose the rectangle which will accommodate them with the least "waste". With n=4, the minimum waste is 5. What size rectangles give the minimum waste for n=11 and for n=12? Consider the rectangle as a p×q grid, and draw in the n squares. Next, fill in each of these squares with digits; 1 for the 1^2 square, 2 for the 2^2 square, 3 for the 3^2 square, … Use a for 10^2, b for 11^2, etc. Use a dash for wasted space. Present your answers as q rows of p digits, as indicated in the example below.

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3334444
3334444
3334444
2214444
22-----
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—Stephen Ainley in *New Scientist*

Computer Bonus Find the (base 10) count of and the sum of all palindromic numbers between 1 and $100{,}000{,}000_{base}$, inclusive, for *bases* 2 through 10, inclusive. A palindromic number is a number which reads the same forward and backward.

—Rolf B. Karlsson, *MI Z '96*

Postal mail your answers to any or all of the Brain Ticklers to **Curt Gomulinski, Tau Beta Pi, P. O. Box 2697, Knoxville, TN 37901-2697** or email to *BrainTicklers@tbp.org* as plain text only. The cutoff date for entries to the Fall column is the appearance of the Winter *Bent* which typically arrives in early January (the digital distribution is several days earlier). The method of solution is not necessary. We welcome any interesting problems that might be suitable for the column. The Computer Bonus is not graded. Curt will forward your entries to the judges who are **H.G. McIlvried III,** *PA Γ '53;* **J.C. Rasbold,** *OH A '83;* **J.R. Stribling,** *CA A '92*, and the columnist for this issue,

—F.J. Tydeman, *CA Δ '73*