Brain Ticklers

RESULTS FROM SPRING

Perfect			
*Bohdan, Timothy E.	IN	Γ	'85
*Christianson, Kent B.	CA	A	'85
Crawford, Martin	TN	A	'54
*Gerken, Gary M.	CA	H	'11
*Griggs Jr., James L.	OH	A	'56
*Gulian, Franklin J.	DE	A	'83
Gulian, William F.	Member's son		
*Hackstock, Tobias D.	MI	H	'02
*Harpole, George M.	CA	E	'74
*Johnson, Mark C.	IL	A	'00
*Kimsey, David B.	AL	A	'71
Minnick, Michael V.	SC	A	'81
*Norris, Thomas G.	OK	A	'56
*Prince, Lawrence R.	CT	B	'91
*Richards, John R.	NJ	B	'76
Riedesel, Jeremy M.	OH	B	'96
*Schmidt, V. Hugo	WA	B	'51
Shepperd, Stanley W.	MA	B	'70
*Slegel, Timothy J.	PA	A	'80
*Spong, Robert N.	UT	A	'58
*Strong, Michael D.	PA	A	'84
*Widmer, Mark T.	OH	A	'84
Willman, Christopher R.	WA	A	'94
Other			
Alexander, Jay A.	IL	Γ	'86
Aron, Gert	IA	B	'58
Beaudet, Paul R.	Member's father		
Brule, John D.	MI	B	'49
Bunch, Christopher	10th grade		
Colbourne, Richard J.	PA	E	'78
Colbourne, Jessica	Member's daughter		
*Couillard, J. Gregory	IL	A	'89
Handley, Vernon K.	GA	A	'86
Harvey, Arthur J.	OH	A	'83
Jones, Donlan F.	CA	Z	'52
Jones, John F.	WI	A	'59
Jones, Jeffery C.	Member's son		
Jordan, R. Jeffrey	OK	Γ	'00
Katz, Robert P.	NY	P	'89
Lalinsky, Mark A.	MI	Γ	'77
Partanen, Thomas A.	MI	B	'70
Pendleton III, Winston K.	MI	Γ	'62
Quan, Richard	CA	X	'01
Rentz, Peter E.	IN	A	'55
Rentz, Mark	Member's son		
Routh, Andre G.	FL	B	'89
Schweitzer, Robert W.	NY	Z	'52
Sigillito, Vincent G.	MD	B	'58
Snodgrass, Brian W.	IN	Δ	'06
*Spring, Gary S.	MA	Z	'82
*Stribling, Jeffrey R.	CA	A	'92
Summerfield, Steven L.	MO	Γ	'85
*Voellinger, Edward J.	Non-member		
Wetterlind, Eric J.	MN	B	'93

* Denotes correct bonus solution

Don A. Dechman, *TX A '57,* is calling it quits and retiring after 20 years of faithful service as a Brain Ticklers judge. **Jeffrey R. Stribling,** *CA A '92,* a longtime solver of Brain Ticklers with a sound record of correct answers, has agreed to take his place. Jeff will be writing the winter columns from now on.

SUMMER REVIEW

The Spring Ticklers appear to have been relatively easy, as almost half the entries were perfect. Of the regular problems, No. 2, about the divisibility of an expression by 4^n, was the most difficult, but even it received 75% correct answers. All the other problems, including the Bonus, had over 90% correct solutions. The Double Bonus (3rd grade test problem) elicited some really ingenious replies.

SUMMER SOLUTIONS

1 The shortest distance between the two cities is **9511 km.** The radius at the equator of Topsis is 4000 km, so the circumference of the planet is $2\pi 4000$ km. The distance along the equator between Topsy and Turvy is 4000π km. Pythagoras tells us that the surface distance from the equator to a Topsis pole is $\sqrt{(4000^2 + 3000^2)}$ = 5000 km. Project the surface of half a "hemisphere" onto a plane as a circular sector with radius r = 5000 km and minor arc length L = 4000π km. The central angle of the arc is $\theta = L/r = 0.8\pi$ radians. The shortest distance between the two cities is a chord that connects the endpoints of the arc. The length of the chord is given by the common formula $(2r)\sin(\theta/2)$ = 2(5000 km) $\sin(0.4\pi)$ = (10000km)(0.951056) ~= 9511 km.

2 The expected difference for 10000 flips is **79.79.** For n flips, there are 2^n equally possible outcomes, and sum of the absolute differences for those outcomes is $2\sum_{i=0}^{floor(n/2)} C(n,i)\,(n-2i)$, where $C(m,k)$ is the combination function $m!/[k!(m-k)!]$.

Empirically, we observe that EV(1) = 1. For n even, EV(n) = EV(n-1), and for n odd, EV(n)= EV(n - 2) [n/(n - 1)]. A simple recursive program computes that ∏(odd ints <= 10000) / ∏(even ints <= 10000) = 79.786.

Alternatively, $EV(n) = (2\sum_{i=0}^{floor(n/2)} C(n,i)(n-2i))/2^n = (\sum_{i=0}^{floor(n/2)} nC(n,i) - 2\sum_{i=0}^{floor(n/2)} iC(n,i)) / 2^{n-1} = (n\sum_{i=0}^{floor(n/2)} C(n,i) - 2n\sum_{i=0}^{floor(n/2)} C(n-1,i-1)) / 2^{n-1} = n(\sum_{i=0}^{floor(n/2)} C(n,i) - 2\sum_{i=0}^{floor(n/2)/2} C(n-1,i)) / 2^{n-1}$.

Observe that the difference of the two sums is the difference between two successive lines in Pascal's triangle, so we get the closed form for EV(n) = n C[n-1,floor((n-1)/2)]/2^{n-1}. The closed form, which can be rewritten as n!/2^{n-1}(floor((n-1)/2)!)(ceiling((n-1)/2)!), is still laborious to compute for large n. Using Stirling's approximation n! ~ $\sqrt{(2\pi n)}(n/e)n$, we can see that as n gets large, EV(n) ~ $\sqrt{(2n/\pi)}$. For n = 10000, this approximation yields 79.789.

3 111211312212313213322223233311 Start by listing the 27 possible three-digit combinations in numerical order. Then, begin with 111 and add successively the smallest number available (cross off each combination as you use it). The first few additions would be 112 (the smallest after 111) to give 1112. Next, add 121 to give 11121; then 211 to give 111211; then 113 to give 1112113; then 131 to give 11121131. Now, we would like to use 311, but we can't, because there are no more combinations 11x, so we have to use 312 to give 111211312. Proceeding in this way gives the sequence shown above. Remember that the sequence bends around to form a circle.

4 Denmark won 4, Estonia 3, Azerbaijan 2, Cyprus 1, Belarus 0. Each team played each other team once, and since each won a different number of games, there is a strict hierarchy to the teams. That is, every team beat all others that won fewer games than themselves and lost to all others that won more. Three games were won by the away team, which means two were won by Team 4, and one by Team 3. Team 0 lost twice at home, and Team 2 once. A=2 was given, and since B and C played at A, B<2 and C<2, so D>2 and E>2. D was home to E, so

D > E, and D=4, E=3. Since B played away at C, then B < C, so B=0, C=1.

5 The thirteen digits are **1153721781101.** The first digit is, by definition, 1, so 10^0 —> 1. Single digit integers fill the first 9 places of the string. There are 90 two digit integers that extend the string to 9 + 2(90) = 189 places. Three digit integers fill the string to 189 + 3(900) = 2,889 places. And so on, until we see that all integers ten digits or smaller cover 98,888,888,889 places, and with eleven digit integers, 1,088,888,888,889 places are covered. Therefore, the 10^{12} place is filled by the (1,000,000,000,000 - 98,888,888,889) = 901,111,111,111st place of eleven digit integers. 901,111,111,111 / 11 = 81,919,191,919 remainder 2, so the required digit is at place 2 of 91,919,191,919, or 1. Similar reasoning for places of smaller magnitudes: for 10^{11}, place 11 of 10,101,010,100 is 0; 10^{10}, place 1 of 1,111,111,111; 10^9, place 1 of 123,456,790; 10^8, place 7 of 788,888,889; 10^7, place 4 of 1,587,301; 10^6, place 1 of 185,185; 10^5, place 1 of 22,222; 10^4, place 3 of 2,777; 10^3, place 1 of 370; 10^2, place 1 of 55. The first and tenth digits are clearly 1, so the sequence of 13 digits is 1153721781101.

Bonus The travelers lifted off at **9:35pm GMT.** Let t_{GMT} and t_{space} be the time the conversations start. Call s the start/launch time. The clocks are synchronized at start, so s_{GMT} and s_{space} are identical. Interpreting the four space travelers' statements, here are the relevant facts:

- At time t_{GMT} (t_{space}), D says that it is after 11:30pm.
- At $t_{GMT}+5$ ($t_{space}+10$), A says that it is *10 minutes past the hour*. A also states that in *23 minutes* it will be 150-34=116 minutes since they set off.
- At $t_{GMT}+10$ ($t_{space}+20$), B says that it is before *10:00 pm* and before 10:30pm.
- C says that C and A are honest, and B and D are liars.

The statements made by B and D are inconsistent because it can't be both after 11:30 pm and then 20 minutes later, before 10:30 pm. At least one of the two is a liar.

C states that C and A are honest and that B and D are liars. If C is a liar, then both B and D must be honest. But we've shown that one of B and D must be a liar, therefore C and A are honest and B and D are both liars.

From A, known to be honest, we can infer that t_{GMT} is *5 minutes past the hour*. Also, since 116 minutes is *58 minutes GMT*, s = $t_{GMT}+5+23-58$ = $t_{GMT}-30$ or s = $t_{space}-60$. We conclude that s and t_{space} are both 35 minutes past the hour.

B is known to be a liar. Therefore at $t_{space}+20$, it must be after 10:30 pm. So t_{space} is after 10:10pm.

D is also a liar, so t_{space} is 11:30 or before.

Combining that t_{space} is 35 minutes past the hour, before 11:30pm, and after 10:10pm, t_{space}= 10:35pm and s = 9:35pm GMT.

Computer Bonus. For the year **1998,** the 24 digit number **556,111,667,222,778,333,889,445** is the shortest number that can be multiplied by the year to get a product which has only zeros and ones. The judges' programs cycled through potential legal products, searching for the smallest which was evenly divisible by the year. More than 268 million products were examined to find the shortest in 1998. Of course, support for arbitrary precision integers is key to a programmed solution. One judge used REXX, another Java. Running times ranged from a few minutes to a few hours, depending upon the language and hardware used. The second longest minimum multiplier is for 1980 which is only 16 digits long and required the examination of little more than 1 million products.

NEW FALL PROBLEMS

1 Solve: ONE / FIVE = .TWO with TWO being even. Different letters are different digits and there are no leading zeros.

—*Journal of Recreational Mathematics*

2 John is shopping for a used car and has found one being offered by Alice for $12,000, based on $2,000 down and a $10,000 balance to be paid to Alice over 36 months at 6% (annual) simple interest paid monthly on the unpaid balance (John's monthly payments would be $10,000/36 + unpaid balance x 0.06/12). However, if he pays cash, he can get a reduction of *D* dollars so the cash payment would be $12,000 - *D*. Although Alice will finance the car at 6%, she believes she can better invest her money at 10% elsewhere. If the 36 monthly payments are discounted to the present at a rate of 10%, how big a reduction in the cash price (to the nearest dollar) can Alice afford to give John so that buying the car for a single cash payment and the net present value of financing over 36 months (plus $2,000 added to account for the down payment), are equivalent propositions? Assume all months have 30 days, and a year is 360 days.

—John W. Langhaar†, *PA A '33*

3 Algernon, Basil and Clarence are either English or Irish, either Conservative or Liberal, and either Protestant or Catholic. In each case there is, as they all know, at least one of each.

None of them know which the others are, but Algernon and Basil have both been told that if Clarence is an Irish Catholic he cannot be a Liberal.

Basil asks Algernon whether he is a Conservative or a Liberal, and whether he is English or Irish. Algernon tells him.

After a pause for reflection, Basil is able to announce correctly the full particulars (English or Irish, Protestant or Catholic, Conservative or Liberal) of both Algernon and Clarence.

What are the particulars for the three men?

—*Brain Puzzler's Delight* by E.R. Emmet

4 Our library has several copies of Boremaster's commentary on Hegel. It is not exactly a jolly read, as you well know if you have ever waded through its 36 chapters, but is much in demand on the grounds that it is less painful than Hegel himself. Even so, I was surprised to see my friend Jones leaving the library with three copies under his arm. “Steady on, old bean,” I exclaimed, “there are other readers to think of.”

“The other copies are all on the shelf,” he replied airily, “but I had to take three to get a complete text. Some rotter has snipped whole chapters out of every copy.”

“Well, surely two copies would have done.”

“No, no two copies would yield a full text.”

“Do you mean that I shall have to check every copy if I want to be sure of a full text?”

“Oh no. Just take any three at random, as I did. You are bound to get a full text, even though no chapter is present in all copies. For each pair of chapters there is at least one copy with only one of them.”

For this to be true, what is the minimum number of copies that the library have in total? Also, provide an N row by 36 column matrix, where each row represents a copy of the book, each column represents a chapter, and each element of the matrix is either 1 if the chapter is present in the indicated copy or 0 if the chapter is absent.

—a Tantalizer by Martin Hollis in *New Scientist*

5 A hexagon inscribed in a circle has side lengths of 2, 2, 2, 11, 11, and 11. What is the exact radius of the circle?

—Puzzle Corner in *Technology Review*

Bonus Sally’s Spa has installed a security system consisting of a row of N switches wired so that, unless the following rules are followed, an alarm will go off: (1) the switch at the far right can be turned on or off at any time and (2) any other switch may be turned on or off only if the switch to its immediate right is on and all other switches (if any) to its right are off. If all N switches are initially on, what is the least number of moves required to deactivate (all switches off) the alarm system? A move consists of turning a switch on or off. Express your answer without using recursion or summation. You can use (N mod 2) or $(-1)^N$ as a way to find even versus odd.

—*Problem Solving Through Recreational Mathematics* by Bonnie Averbach and Orin Chein

Computer Bonus Let f(N) be the smallest positive integer whose prime factorization contains the N digits 1 to N each exactly once. So, f(3)=26=2x13. Find f(N) and its prime factorization, for N equal to 4 through 9, inclusive.

—Adapted from Puzzle Corner in *Technology Review*

Postal mail your answers to any or all of the Brain Ticklers to Curt Gomulinski, Tau Beta Pi, P. O. Box 2697, Knoxville, TN 37901-2697 or email to *BrainTicklers@tbp.org* as plain text only. The cutoff date for entries to the Fall column is the appearance of the Winter *Bent* in late December (the digital distribution is several days earlier). The method of solution is not necessary. We welcome any interesting problems that might be suitable for the column. The Computer Bonus is not graded. Curt will forward your entries to the judges who are **H.G. McIlvried III,** *PA Γ ’53*; **J.C. Rasbold,** *OH Α ’83*; **J. R. Stribling,** *CA Α ’92*; and the columnist for this issue,

F. J. Tydeman, *CA Δ ’73*