Brain Ticklers

RESULTS FROM SPRING

Perfect

Couillard, J. Gregory	IL	Α	'89
Doniger, Kenneth J.	CA	Α	'77
*Kimsey, David B.	AL	Α	'71
Marx, Kenneth D.	OR	Α	'61
*Norris, Thomas G.	OK	Α	'56
Parks, Christopher J.	NY	Γ	'82
*Schmidt, V. Hugo	WA	Β	'51
*Slegel, Timothy J.	PA	Α	'80
*Stribling, Jeffrey R.	CA	Α	'92
Strong, Michael D.	PA	Α	'84

Other

Alexander, Jay A.	IL	Γ	'86
Aron, Gert	IA	Β	'58
Bernacki, Stephen E.	MA	Α	'70
*Bohdan, Timothy E.	IN	Γ	'85
Brule, John D.	MI	Β	'49
Crawford, Martin	TN	Α	'54
Ebersold, Dakota	Son of member		
*Gerken, Gary M.	CA	Η	'11
Gulian, Franklin J.	DE	Α	'83
Gulian, William F.	Son of member		
Handley, Vernon K.	GA	Α	'86
Hasek, William R.	PA	Γ	'49
Johnson, Roger W.	MN	Α	'79
Jones, Donlan F.	CA	Ζ	'52
Jones, John F.	WI	Α	'59
Jones, Jeffery C.	Son of member		
Kern, Peter L.	NY	Λ	'62
Lalinsky, Mark A.	MI	Γ	'77
Majewski, Sara	Wife of member		
Marrone, James I.	IN	Α	'61
McTigue, Thomas M.	NY	Ξ	'74
Penlesky, Richard J.	WI	Β	'73
Prince, Lawrence R.	CT	Β	'91
*Quan, Richard	CA	Χ	'01
Rentz, Peter E.	IN	Α	'55
Richards, John R.	NJ	Β	'76
Rose, Gerald C.	IN	Β	'57
Ross, Karen M.	IN	Α	'79
Saikali, J.	OH	Β	'96
Schweitzer, Robert W.	NY	Ζ	'52
Sigillito, Vincent G.	MD	Β	'58
*Spong, Robert N.	UT	Α	'58
Summerfield, Steven L.	MO	Γ	'85
Svetlik, J. Frank	MI	Α	'67
Vinoski, Stephen B.	TN	Δ	'85
*Voellinger, Edward J.	Non-member		

*Denotes correct bonus solution

SPRING REVIEW

Problem 3 (couples crossing river) had less than half of the submitted answers being correct. Many responders assumed it was safe for a wife to get out of the boat as another woman's husband got in while her husband was on the other river bank, but this was not allowed. While problem 4 (marbles) and the Bonus (magic square) had about two thirds of the submitted answers correct. Readers' entries for the Summer Ticklers will be acknowledged in the Winter *Bent*. Meanwhile, here are the answers.

SUMMER SOLUTIONS

1 This question was incorrectly worded in the Summer column. We should have asked for the location of the treasure, not the gallows. The gallows itself is not locatable given the instructions on the parchment. Considering the wording mistake, we will not negatively score this question for any entry, and apologize for any angst it may have caused.

The treasure is midway between the oak and pine and same distance perpendicular. In Cartesian coordinates, place the gallows at point (a,b), the oak at (1,0), and the pine at (-1,0). The vector representing the walk from the gallows to the oak is (1-a,-b). The same distance, but in the direction 90 degrees to the right is the vector (b,a-1). The first spike therefore, is placed at the point (b+1,a-1). The walk from the gallows to the pine is the vector (-a-1,-b), so turning left 90 degrees and stepping off the same number of steps is the vector (-b,-a-1), which places the second spike at (-b-1,-a-1). The point midway between the locations of the two spikes is computed by ((b+1,a-1)+(-b-1,-a-1))/2 = (0,-1). This places the treasure on the perpendicular bisector of the line from the oak to the pine, (on the left for an observer standing at the oak facing the pine) at half the distance that separates the two trees.

2 Envision the circuit as a Triangular Dipyramid, that is, a polyhedra with 5 vertices and 6 triangular faces, with a resistor along each edge. Points A and E are on opposite ends of the polyhedra. By symmetry, resistors along edges **B-C**, **C-D** and **B-D** carry no current and can be eliminated. This leaves a circuit with three identical parallel resistances A-B-E, A-C-E and A-D-E, each of which must be 3 ohms, in order combine for an effective resistance of 1 ohm. The value of each individual resistor is half of 3 ohms, or **1.5 ohms.**

3 The probability that the last person sits in their assigned seat is **0.5** (50%), no matter how many seats the theatre holds. Call P_n the theatre patron who enters the theatre with *n* seats remaining. The acoustic engineer is P_{200}, the last person is P_1. Except for the engineer, every patron takes their own seat if it is available. This means that if P_n has to choose a seat (as opposed to taking their own), they have a choice of the engineer's seat, or the *n-1* seats of the patrons that follow them. Once the engineer's seat is taken, it is guaranteed that every following patron, including P_1, will sit in their own seat. P_1 will sit in either the engineer's seat or their own. Call success the result that P_1 chooses their own seat. Define S(n) to be the probability of success if P_m does not sit in their own seat. Clearly, S(1) = 0 since the definition of failure is P_1 not sitting in their seat. P_2 has a choice between the engineer's seat or P_1's, so S(2) = .5. P_3 can choose between the engineer's seat, P_2 or P_1, then $S(3) = ⅓(1 + S(2) + 0) = ⅓(1.5) = 0.5$. Generalizing, $S(n) = (1/n)(1 + \Sigma_{i=1}^{n-1} S(i)) = (1/n)(1 + (.5)(n-2)) = (1/n)(n/2) = 0.5$.

4 The number of ounces of Apples, Bananas, Cranberries and Dates respectively in the mixes are:

Mix	A	B	C	D
FrootMix	1	4	5	6
MixedFroot	7	5	3	1
TootyFrootie	25	31	29	27

The boxes of *FrootMix* (FM) and MixedFroot (MF) each weigh 16 ounces total and contain integral amounts of each of the four ingredients. The respective weights of each fruit must be different, so there are only nine ways to allocate the portions that sum to 16: 10-3-2-1, 9-4-2-1, 8-5-2-1, 8-4-3-1, 7-6-2-1, 7-5-3-1, 7-4-3-2, 6-5-4-1 and 6-5-3-2. The box of *TootyFrooties* (TF) has 112 ounces of product, which combines 4 boxes of FM and 3 boxes of MF. That is, $FM_{tot}=MF_{tot}=16$ and $4FM_{tot}+3MF_{tot}=TF_{tot}=112$. Of the ingredients in TF, Apples are the least by weight, so maximum $TF_a = (112-6)/4 = 26$. Also, Apples are the least ingredient in FM and the greatest in MF, so the 26 or fewer ounces in TF can be created by three combinations: $FM_a=1$ and $MF_a=6$ to make $TF_a=22$, $FM_a=1$ and $MF_a=7$ to make $TF_a=25$, or $FM_a=2$ and $MF_a=6$ to make $TF_a=26$. Dates are the greatest ingredient in FM and the least in MF, so the minimum $TF_d=4(6)+3(1)=27$. Observe that, algebraically speaking, the next least TF_d possible is 4(6)+3(2)=30. However, even if $TF_a=22$, then $TF_d=30$ makes $TF_b+TF_c=112-30-22=60$ and one of TF_b or TF_c would be less than TF_d, which breaks the TF ingredient weight ordering. So $TF_d=27$, which implies $FM_d=6$ and $MF_d=1$. Then $FM_c=5$ because Cranberries are the second largest ingredient in FM. Given $TF_d<TF_c$ and $MF_c=3$, therefore $TF_c=29$ to meet the requirement $TF_c<TF_b$. Only the 7-5-3-1 combination fits MF for $TF_a<27$, $MF_c=3$ and $MF_d=1$, so $MF_a=7$ and $MF_b=5$. Therefore, $FM_a=1$. We already know $FM_c=5$ and $FM_d=6$, so $FM_b=4$.

5 TBP = √(PUZZLE + 1) has the solution **908 = √(824463 + 1).** Rewrite the equation as TBP^2-1 = PUZZLE. Looking at the least significant digits, recognize that P defines E by the relation $E = (P^2-1)$ modulo 10. From the most significant digits, observe that P limits T to certain values such that $\sqrt{(10P)} \le T \le \sqrt{(10(P+1))}$. These two relations give us only 12 (P,E,T) triples to consider: (8,3,9), (6,5,8), (6,5,7), (5,4,7), (4,5,7), (4,5,6), (3,8,6), (3,8,5), (2,3,5), (2,3,4), (1,0,4), (1,0,3). Use trial and error on B together with each triple to find TBP candidates that generate a PUZZLE such that the 100 and 1000 columns are equal. Only TBP candidates 908 and 928 fit, which generate PUZZLE values 824463 and 861184 respectively. The latter PUZZLE value does not work as P and L are the same, leaving the unique solution TBP = 908 and PUZZLE = 824463.

Bonus **Six** prisoners can escape in 12 hours, starting at 155, 275, 395, 515, 635, 683 minutes past 6:00pm, respectively. That is, **8:35pm, 10:35pm, 12:35am, 2:35am, 4:35am and 5:23am.**

Robinson starts 300 meters west of the window, marches to the window and returns. Her cyclic route is 600 meters long and takes 6 minutes to walk. She is far enough from the prisoners' window to be out of the line of sight for only 2 of the 6 minutes, starting t=-1 minutes. Smith, starting outside the prisoners' window, has an 8 minute cycle and is out of range of the window for 4 minutes starting at t=2. Philips, starting 500 meters west of the window with a 1100 meter route, has an 11 minute cycle and is clear of the window for 6 minutes at t=-3. Finally, Quinn has a 10 minute cycle, and is out of range for 7 minutes starting at t=1.5.

A guard must be out of range for a minimum of 2 consecutive minutes for a prisoner to escape, so the window of opportunity for a prisoner to start is actually 2 fewer minutes than each guard is out of range. Robinson is the most limiting factor, and a prisoner must start their escape at the precise moment that she steps out of sight. Therefore, successful attempts can only begin at t=6r-1, where r is a positive integer. The other three guards impose the constraints that t must fall in the inclusive ranges [11p-3, 11p+1], [10q+1.5, 10q+6.5] and [8s+2, 8s+4] where p, q, and s are also positive integers. The values r=26, p=14, q=15, s=19 give t=155. Similar reasoning (by hand or spreadsheet) finds the other five answers less tha the 720 minutes that marks 6:00am.

Computer Bonus. The most significant digits of the two 1,001-digit, base-10, automorphic numbers are **3** and **6**, respectively. Observe that for an N-digit automorphic number, the last (N-1) digits must form a (N-1)-digit automorphic number. It is straightforward to determine that 1, 5, and 6 are 1-digit automorphic and that 25 and 76 are 2-digit automorphic. One need only attempt 10 (the base) multiplications to grow each candidate by a digit. Clearly, it is helpful if the programming environment used supports arbitrary precision integers with multiply and division operations. If one recognizes that automorphic numbers come in pairs, then the work is cut in half. Alternately, using the formula from wikipedia and starting with N=76, larger automorphic numbers can be found from $(3N^2 - 2N^3) \bmod 10^{2k}$ until one has at least 1,001 digits.

NEW FALL PROBLEMS

1 Arthur, Basil, Clarence, Dudley and Earl are married, but not respectively, to Alice, Barbara, Clarissa, Dorothy and Eve. The hometowns of the five men are, not respectively, Andover, Bristol, Chippenham, Delhi and Ealing; and, their youthful ambitions were, again not respectively, to have become an Architect, a Barber, a Chemist, a Dentist and an Engineer.

For each man his name, the name of his wife, his hometown and his youthful ambition all begin with different letters.

The five ladies make remarks (which are unfortunately not all true) as follows:

Alice: The would-be Dentist is married to Barbara.

Barbara: The man who wanted to become an Engineer is not Dorothy's husband. The man who was born in Bristol has always wanted to be a Dentist.

Clarissa: The man who was born in Andover is married to Barbara.

Dorothy: The would-be Barber is not my husband.

Eve: The man who was born in Andover is not Basil. The man who was born in Chippenham wanted to become an Engineer.

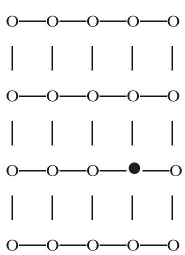
It is interesting to notice that, in these remarks, when the subject of the sentence is a man whose name begins with a letter which comes alphabetically before the initial letter of the speaker, the sentence is true; if the initial letter of the subject's name comes after the initial letter of the speaker's name, the sentence is false. In no case is the initial letter of the subject's name the same as the initial letter of the speaker's name. Find, for each man, the name of his wife, his hometown and his youthful ambition. That is, fill in the following matrix, using

Husband	A	B	C	D	E
Wife					
Hometown					
Ambition					

initial letters.

—*Brain Puzzler's Delight* by E.R. Emmet

2 Peter Pickle has drawn up this handy map of the 20 pubs in his town. On crawling nights he starts with a pint at the Swan and then moves off along the lines stopping at each pub he passes. He follows a formula on stepping out of the Swan: A, B, C, B, A, B, A, D, D, A, D, A, B, C, and finally B. A, B, C, and D stand for north, south, east, and west, but not necessarily in that order. The final B brings him to the Bull, the black dot on the map, for the first and only time. (He may visit each of the other pubs more than once.) Where is the Swan (relative to the Bull)?



—A Tantalizer by Martin Hollis in *New Scientist*

3 Find the greatest integer that can be formed in exactly 9 different ways and no more, by adding together a positive integral multiple of 7 and a positive integral multiple of 5.

—George Chrystal

4 A "near-square" is a rectangle of N x (N+1) units, and the problem is to fill it completely with as few "squarelets"—that is, smaller integral-sided squares—as possible. For a 4 x 5 near-square, you cannot do better than 5 squarelets (one 4x4 + four 1x1). With how few squarelets can you fill: a) an 18 x 19 near-square? b) a 22 x 23 near-square? (Please enclose a rough sketch of each or a clear written depiction).

—An Enigma by Stephen Ainley in *New Scientist*

5 Please solve this cryptic addition with no leading zeros, different letters are different digits, and same letter is same digit.

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  TWELVE
  TWELVE
    NINE
    NINE
    NINE
    NINE
    FIVE
+   FIVE
 SEVENTY
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—*Journal of Recreational Mathematics*

Bonus In how many ways can N people be seated at a round table so that no person sits next (unordered) to the same pair of neighbors twice? Also, show us how to do it for seven (numbered 1 to 7) people. For each seating, place the lower number of the pair next to 1 on 1's left, start and end with person 1 and go clockwise around the table (as an eight digit number). Order the resulting numbers smallest to largest. So, the first two seatings would be 12345671 and 12435761.

—Henry E. Dudeney

Double Bonus. Assuming the sudden onset of a steady vertical rainfall, will a very overweight person (in a swimsuit) remain drier (get less water on them) by walking or running a given distance to shelter? You may treat the person as a sphere. No trick answers please.

—*The Chicken from Minsk* by Y. B. Chernyak and Robert M. Rose

Postal mail your answers to any or all of the Brain Ticklers to Curt Gomulinski, Tau Beta Pi, P. O. Box 2697, Knoxville, TN 37901-2697 or email to *BrainTicklers@tbp.org* as plain text only. The cutoff date for entries to the Fall column is the appearance of the Winter *Bent* in early January (the digital distribution is several days earlier). The method of solution is not necessary. We welcome any interesting problems that might be suitable for the column. The Double Bonus is not graded. Curt will forward your entries to the judges who are **H.G. McIlvried III**, *PA Γ '53*; **D.A. Dechman,** *TX A '57*; **J.C. Rasbold,** *OH A '83*; and the columnist for this issue, **F.J. Tydeman,** *CA Δ '73*

CHANGE OF ADDRESS THE BENT

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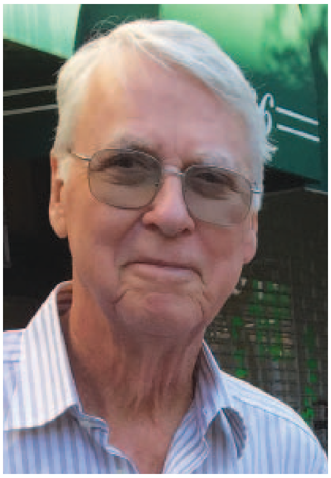
MORE THAN JUST TICKLING BRAINS

Long-time Tau Beta Pi member and volunteer, Don Dechman, *Texas Alpha '57*, knows what it means to have a positive attitude and persevere when solving math problems. He has served as one of the four Brain Ticklers judges since 1996.

In honor of Don's 80th birthday, his three sons, David, *Virginia Beta '82*, Ken, a Tufts '87 graduate, and Jim, *Texas Alpha '89*, are endowing the Don A. Dechman Scholarship. Don notes, "Scholarships are important to helping young minds get started in life."



Born in Fort Worth, TX, Don spent many years in Houston and then received his bachelor's degree in chemical engineering from the University of Texas at Austin. To this day and to his knowledge, he continues to hold the record for shortest completion of a ChE master's degree at UT—achieved in only nine months.

New Challenge

Don spent the majority of his career at Union Carbide as a process engineer designing and modifying existing chemical plants. In his time there he helped develop the low pressure oxo process to make butyl alcohol—a process which Union Carbide licensed and is used in the production of approximately 50% of the world's capacity of this chemical.

Don retired as the director of engineering for the solvents and coating materials division and decided to take on a new challenge—working for nine years for John Brown Engineers in Charleston, WV. He is currently retired full-time in Naples, FL, with Jonene, his wife since 1958.

After a decade in the workforce and with his early days in engineering behind him, he signed up for a lifetime subscription to *The Bent* and began submitting answers for the Brain Ticklers working toward getting perfect scores.

He noted that this was back when everything was done by hand with no computers or Internet for assistance. It wasn't long after that he was approached to be one of our judges.

Don finds his work as a Brain Ticklers judge worthwhile and enjoyable because of the effort they put into the column each quarter to provide instructive answers for the previous problems plus clearly stated new problems.

Don's sage advice for our students, "Don't shy away from doing math problems. If you approach them with confidence and a disciplined approach, you can work them," adding "I do the Winter issue of the Brain Ticklers and always try to start with an easier one to help everyone out!"

TELL US THE TALE...WIN A T-SHIRT

Send us your witty captions for this photo from *The Bent* archives, and if it is judged one of the best, you will win a TBΠ t-shirt.

The picture, published in the Winter 1977 issue, was taken during the 1976 Convention that was held on the campus of Texas A&M University. The camera caught delegate **Beverlee G. Steinberg,** *CA Θ '77*, checking to see if the punch really was all gone.

Email entries to *pat@tbp.org*, or mail them to HQ by Monday, November 2, 2015.

The photo for the Summer Contest, to the right, published in the Spring 1974 issue, was included in an article about Northrop University (at the time, Northrop Institute of Technology) where the California Pi Chapter was located. As part of an undergraduate senior research project investigating the effects of industrial noise on brain wave activity, the student above records the brain waves of a fellow student.

The top two most popular captions were:

"How come you can sleep during class but not here?" from **Drake R. Kijowski,** *IN A '76*, and

"No, Mr. Bond. I expect you to die!" from **Jeffery C. Gehring,** *IA A '92*.

Both will receive a t-shirt.

Two entries tied for honorable mention: "Really Professor! I'm sure there has to be a better way to take a selfie!" submitted by **Timothy L. Johnson,** *IA A '78*, and **Gregory M. Gatlin,** *MD B '83* sent us "Hang in there, just a few more full-body scans and IT Security will allow you to log in."

Thanks to all of our participants past, present, and future, for sharing your brand of engineering humor!