Brain Ticklers

RESULTS FROM SPRING

Perfect

*Beaudet, Paul R. Member's father
*Bohdan, Timothy E. IN Γ '85
Ciaravino, Vito A. MI Γ '01
*Couillard, J. Gregory IL A '89
*De Vincentis, Joseph W. TX Γ '93
*Ehrgott Jr., M. Charles FL E '92
*Gerken, Gary M. CA H '11
*Gibbs, Kenneth P. MO Γ '76
*Golembiewski Jr., S. L., PA B '90
*Keenan, Timothy P. PA Z '62
*Kimsey, David B. AL A '71
Parks, Christopher J. NY Γ '82
*Schmidt, V. Hugo WA B '51
Skorina, Frank K. NY M '83
*Slegel, Timothy J. PA A '80
*Snyder, Thomas M. GA A '70
*Treseder, Robert C. UT B '81
*Voellinger, Edward J. Non-member

Other

*Alexander, Jay A. IL Γ '86
Aron, Gert IA B '58
Bernacki, Stephen E. MA A '70
Brule, John D. MI B '49
*Celani, Paul E. MD Γ '97
Corey, Patrick J. WA A '58
*Crawford, Martin TN A '54
*Griggs Jr., James L. OH A '56
Handley, Vernon K. GA A '86
*Johnson, Roger W. MN A '79
Jones, Donlan F. CA Z '52
Kelley, Eugene W. MI A '41
Lalinsky, Mark A. MI Γ '77
Lloyd, Margaret M. MA B '12
Loeb, Daniel E. CA B '86
Martin, James A. WV A '66
*Norris, Thomas G. OK A '56
Pinkerton, Audrey S. TX A '90
Pinkerton, Kate Member's daughter
Quan, Richard CA X '01
*Quintana, Juan S. OH Θ '62
*Spong, Robert N. UT A '58
Stetson II, Scott B. CA T '12
*Strong, Michael D. PA A '84
Summerfield, Steven L. MO Γ '85
Sutor, David C. Member's son
Vinoski, Stephen B. TN Δ '85
*Weinstein, Stephen A. NY Γ '96

* Denotes correct bonus solution

SPRING REVIEW

The Spring Ticklers appear to have been easier than usual, with more than 40% of the entries being perfect. The most difficult regular problems were No. 1 (domino probability) and No. 3 (transit time through the moon) with about half of the entries having correct solutions, about the same as for the Bonus (point in a square).

We were pleased to receive an entry from **Eugene W. Kelley,** *MI A '41*, who has set a new record for the length of time between year of graduation and year of submitting a Brain Tickler entry, 73 years in Gene's case.

SUMMER SOLUTIONS

Readers' entries for the Summer problems will be acknowledged in the Winter *Bent.* Meanwhile, here are the answers:

1 **13,990** square feet will be removed from the field. To compute the original area of the field, calculate the radius r of the outfield fence. The center of the arc lies along the line from home plate through second base to center field. We know the distance down the foul line is 350 feet, and the distance to center field $260 + 90\sqrt{2} = 387.28$ feet. Home plate, the right field foul pole, and the center of the circle of which the outfield arc is a part, form a triangle with side lengths 350, r and $387.28\text{-}r$. The angle at home plate between the foul line and the line to center is 45°, so use the law of cosines to determine r to be 288.97 feet. Heron's formula gives the area of the triangle as 12,165 ft². Using the law of sines, the triangle's angle at the circle's center is found to be 121.1°, so the outfield arc is 180° – 121.1° = 58.9°, and half the area swept with radius r is 42,935 ft². The total area of the field is twice the area of the triangle plus the circle's sector, or 110,200 ft². The reconfigured field is more easily computed as a quarter of a circle with a radius of 350 feet, that is, $\pi 350^2/4 = 96{,}211$ ft². We found the difference to be 13,989.5003 ft². While 13,990 is the more accurate answer, we also accepted 13,989 ft².

2 The distribution of gems to the children is given by the first 5 rows in the table. The total value of the gems is 9($2,500) = $22,500, and each child is to receive 45/5 = 9 total gems, including at least one of each type (Diamond, Ruby, Sapphire, Opal, and Zircon), with a value of $22,500/5 = $4,500. There are 7 ways to choose 9 gems (including at least one of each type) with a value of $4,500, as shown in the table:

	D	R	S	O	Z
a	1	2	3	2	1
b	1	3	2	1	2
c	2	1	2	3	1
d	2	2	1	2	2
e	3	1	1	1	3
f	2	1	3	1	2
g	1	3	1	3	1

Since each child is to have a unique combination, the solution must come from selecting 5 of these combinations, which can be done in C(7, 5) = 21 ways, where C(i, j) is the number of combinations of i things taken j at a time. Of the 21 possibilities, only 3 satisfy the constraint that there be exactly 9 of each type of gem. The three sets meeting all the constraints are (a, b, c, d, e), (b, c, e, f, g), and (a, d, e, f, g), where the letters refer to rows in the table. We want the distribution with the fewest threes-of-a-kind, which is the first of these, i.e., the first 5 rows of the table.

3 The expected number of rounds for all N to have dropped out is **N**. As N grows large, the probability that the prize is won by exactly two contestants is approximately **0.80189**. The slips can be viewed as a permutation of the elements of a set of size N. The number of permutations is $N!$, but no contestants are eliminated if the drawing is a derangement of N. For N=1, the number of draws is trivially 1, and the probability that the prize is split in two is 0. For N=2, the expected number of draws is 2, and the prize (of course) must be split in two. For N>2, we wrote a straightforward program to calculate the probability of m being eliminated. Using recursion, we computed the expected values based on earlier results of N-m contestants.

4 Ann ate 1 donut, Beth ate 2, Carol ate 3, and Diane ate 5. From Ann's question, we know that she ate fewer than 5 donuts, for otherwise she would already know that Beth did not eat more donuts than she did. From Beth's answer to Ann's question, we know that Beth ate at least 2 donuts; if she ate only 1, she would have known that she didn't eat more donuts than Ann. From Beth's question, by similar reasoning, we know that Beth ate fewer than 5 donuts, and from Carol's answer to Beth's question, we know that Carol ate at least 3 donuts, for if she had only eaten 1 or 2, she would know that she didn't eat more than Beth. We also know that Carol ate less than 5 donuts; otherwise, she would have known that Beth did not eat more donuts than she did. Thus, Ann ate 1, 2, 3, or 4 donuts; Beth ate 2, 3, or 4; and Carol ate 3 or 4. Of the 24 combinations in the Ann-Beth-Carol sample space, 4 are impossible, as they sum to 11 or 12. Of the rest, only one has a unique value for the number of donuts Diane ate, so Diane could deduce the distribution only if she had eaten 5 donuts. This means that Ann ate 1 donut, Beth ate 2, and Carol ate 3.

5 Six 1-ohm resistors that are connected to form a tetrahedron have a resistance of 0.5 ohm between any two vertices. Label the four vertices of the tetrahedron A, B, C, and D, and suppose we are measuring the resistance between A and B. A and B are connected by three paths: AB, which contains a single 1-ohm resister, and ACB and ADB which each contain two 1-ohm resisters. There is a 1-ohm resister between C and D, but because of symmetry, no current flows through this resister. Thus, we have in effect, three resistances in parallel, which means that the reciprocal of the resistance between A and B is 1/R = 1/1 + 1/2 + 1/2 = 2, so R = 0.5 ohm.

Bonus The player crossing off the first number can force a win by picking either 58 or 62. Both players make the fundamental observation that crossing off the number 1 leads to a loss. If one does that, the other player can then simply choose a prime greater than 50 for the win. Then, recognize that one player can control the game if they can restrict the moves made by the other. The dominating player can drive the game to a win by selecting certain odd, semi-prime numbers greater than 50. The losing player is forced to choose a prime less than 100/4. For example, if Bill crosses off 5, and Ann can choose 55, Ann wins the game via this sequence: B5, A55, B11, A77, B7, A91, B13, A65, B1. Bill has no latitude in his choices after he crosses off 5. Note that 5-55-11-77-7-91-13-65-5 form a cycle. If player X selects any of the primes in the cycle, player Y can win by choosing the adjacent number in either direction (assuming all numbers are available). Player X eventually loses when the cycle returns to his original number, and is forced to choose 1. Observe that 3-51-17-85-5-95-19-57-3 is another such cycle. That is, a player choosing 3 can be driven to a loss. Finally, Ann can force Bill to quickly choose 3. There are two primes, 29 and 31, whose only available multiples are 2x and 3x, as well as the trivial neighbor 1. That is, the two primes must be greater than 100/4 and less then 100/3. Ann's initial choice should be 58 or 62, which are the 2x multiples of the respective primes. If Bill responds with 2, then Ann should choose the other of 58 or 62. At that point, Bill must choose the prime 29 or 31. Ann responds with the number 3x Bill's choice and Bill is forced to cross off 3 on his next turn. Now that Bill has chosen 3, Ann simply follows the cycle above, forcing Bill's hand each time until he must choose 1 after Ann exhausts the cycle. The above can be done by hand and is a quick forced win for the first player. Using a computer, the first player can win by starting with any even number except for: 2, 52, 68, 70, 74, 76, 78, 82, 86, 92, 94, and 98. However, some of those games can be up to 64 moves long.

Computer Bonus
(a) F(1000000,1000000) = 2,202
F(1000000,1000) = 808,226
G(1000000,1000000) = 549,551
G(1000000,1000) = 549,552
(b) F(n,n) = G(n,n) = n occurs when the largest number is the last alphabetically. The ten smallest n values are: 1; 2; 200; 201; 202; 2000; 2001; 2002; 2200; 2201.

NEW FALL PROBLEMS

1 With the usual rules: Base ten, no leading zeros, different letters are different digits, same letter is same digit throughout, solve the following cryptic addition problem (which can be solved without computer help).

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  PIERRE
+ELLIOTT
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 TRUDEAU
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—Richard I. Hess, *CA B '62*

2 Gerald, Harold, Ian, John and Karl have long been members of the Christmas Compensation Club —open only to those less than 100 who suffer the severe loss which results from having one's birthday on Christmas day. Last Christmas Karl was older than Ian by three times as much as he was older than Harold, and John was 10% younger than Harold and 20% older than Ian. Gerald is older than Karl by the same amount that John is older than Ian. Find their ages.

—Brain Puzzler's Delight by E.R. Emmet

3 Our next contestant is Thomas Chalk, a retired schoolmaster from Oxbridge. Good evening, Mr. Chalk, your subject is Ruritanian royalty. The house of Kohlenbuttel gave Ruritania seven kings, the crown descending directly from father to son on each occasion. Your task, Mr. Chalk, is to say how many of these seven statements are true:

1. King Dachs was not the grandson of King Fruhling.
2. King Adolf was not the great-grandson of King Gunther.
3. King Eiche was a descendant of King Dachs.
4. King Gunther was not the great-great-grandson of King Carolus.
5. King Bohnen was not the great-great-great-great-grandson of King Eiche.
6. King Gunther was the son or

grandson of the first Kohlenbuttel king.
7. King Dachs was not the first Kohlenbuttel king.

Well, Mr. Chalk? You said two? That is correct. What is the order of the kings?

—A Tantalizer by Martin Hollis
in *New Scientist*

4 A federal marshal arrived at the jail of a lonely outpost to pick up a prisoner, only to discover five naked men locked in the cell. They all claimed to be either the sheriff or one of his deputies. The prisoners had lured in the guards and overpowered them, but the sheriff managed to throw the keys through the window so that no one could escape. The prisoners then threw out all of their clothes, so now the five naked men all stood there claiming to be officers. Fortunately, the marshal realized that the officers only told the truth and that the prisoners always lied. "Which of you are officers?" he asked.

"I am, for one," said the first man, who was the biggest.

"He'd say that in any case," said the second.

"Three of us are," said the third, who was the smallest. His right arm was in a sling.

"That's a lie," said the fourth.

"The biggest fellow says he's an officer," said the fifth, "but it's hard for you to tell if he's lying or not."

After a few minutes the marshal opened the cell and told the officers to step out. Whom did he let out?

—Unknown

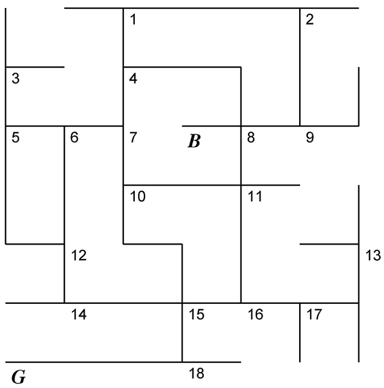
5 The State of Confusion is a 6x6 kilometer square (all roads are an integral number of km and all intersections are at right angles) that has ten towns:

A. Aberdeen	F. Fairbanks
B. Bryce	G. Gainesville
C. Cabot	H. Hawi
D. Detroit	I. Ironton
E. Eagle	J. Jackson

To assist strangers visiting Confusion, ten highway signs have been erected:

K. Cabot - 5; Aberdeen - 6
L. Fairbanks - 2; Eagle - 4
M. Fairbanks - 3; Detroit - 7
N. Detroit - 6; Bryce - 6
O. Cabot - 3; Ironton - 4
P. Bryce - 1; Hawi - 6
Q. Eagle - 2; Hawi - 7
R. Ironton - 3; Jackson - 4
S. Aberdeen - 1; Gainesville - 7
T. Jackson - 3; Gainesville - 6

The accompanying figure is a map of Confusion. There are 20 grid points on the map, indicated by numbered points, 10 of these grid points are occupied by towns and 10 by highway signs. Unfortunately, the copy of the map has gotten wet and some of the ink has rubbed off, and only two of the towns are shown (the capital, Bryce, and the university, Gainesville). Where are the other 8 towns located? Towns and highway signs are all an exact number of km from each other with distances measured along the vertical and horizontal road segments (not as the crow flies). The distance on each sign is the shortest distance to the given town. Present your answer as 8 number/letter pairs, representing the grid point and the town located there.



—*Games* magazine

Bonus Using at most fourteen 1-ohm resistors, show us a network that has a resistance of π ohms, accurate to seven significant digits. You should consider series, parallel, delta, wye, and bridge connections.

—**Hubert W. Hagadorn,** *PA E '59*

Double Bonus In Euclid's proof of the Pythagorean theorem: ABC is a right triangle; B is a right angle; ABDE, BCFG, and ACHK are the squares on the sides. Let Z be the point where EC and AF meet. Show that BZ is (or is not) parallel to AK.

—Puzzle Corner
in *Technology Review*

Postal mail your answers to any or all of the Brain Ticklers to **Curt Gomulinski, Tau Beta Pi, P. O. Box 2697, Knoxville, TN 37901-2697,** or email to *BrainTicklers@tbp.org*. The cutoff date for entries to the Fall column is the appearance of the Winter *Bent* during early January. The method of solution is not necessary, unless you think it will be of interest to the judges. We also welcome any interesting new problems that may be suitable for use in the column. The Double Bonus is not graded. Curt will forward your entries to the judges, who are: **H.G. McIlvried III,** *PA Γ '53*; **D.A. Dechman,** *TX A '57*; **J.C. Rasbold,** *OH A '83*; and the columnist for this issue,

F. J. Tydeman, *CA Δ '73*

LETTERS TO THE EDITOR

(Continued from page 9)

especially pictures that show the "Grandma" with her family. And, you don't have to be a Grandma or even of Grandma age in order to submit.

Jill S. Tietjen, P.E., VA A '76

The Right Decision

• Two excellent movies not listed in *Putting the Engineer Into Popular Entertainment* from your Summer 2014 issue of *The Bent* that have aeronautical engineers as main characters: *The Dam Busters* (1951)—B. N. Wallis was a top-notch English engineer who guided development of "bouncing bombs" that were used to destroy several German dams during World War II; *No Highway in the Sky* (1951)—James Stewart portrayed the engineer in a story from Neville Shute.

Richard T. Wylie, P.E., CA N '78