Brain Ticklers

RESULTS FROM SPRING

Perfect

*Anderson, Paul M.	Son of member		
*Gerken, Gary M.	CA	H	'11
*Gibbs, Kenneth P.	MO	G	'76
Norris, Thomas G.	OK	A	'56
Prince, Lawrence R.	CT	B	'91
*Schmidt, V. Hugo	WA	B	'51
*Thaller, David B.	MA	B	'93

Other

Aron, Gert	IA	B	'58
Beaudet, Paul R.	Father of member		
Bohdan, Timothy E.	IN	G	'85
Buckley, Robert C.	TN	Z	'12
*Couillard, J. Gregory	IL	A	'89
Grant, Chuck	Non-member		
Chance, Sophie	Non-member		
Grewal, Rashi	NJ	G	'09
Handley, Vernon K.	GA	A	'86
Jones, Donlan F.	CA	Z	'52
Kimmel, Peter G.	Husband of member		
McCormick, Reynard	Non-member		
Rentz, Peter E.	IN	A	'55
Richards, John R.	NJ	B	'76
Riedesel, Jeremy M.	OH	B	'96
Smith, Evan	Son of member		
Smith, Josh	Son of member		
*Spong, Robert N.	UT	A	'58
*Stribling, Jeffrey R.	CA	A	'92
*Strong, Michael D.	PA	A	'84
Summerfield, Steven L.	MO	G	'85
Sutor, David C.	Son of member		
Teale, John L.	NM	B	'76
Voellinger, Edward J.	Non-member		
Young, Daniel R.	OH	A	'12

*Denotes correct bonus solution

SPRING REVIEW

Problems 3 (wafers) and 4 (paths) were the hardest regular problems, with only about 50% correct answers. The Bonus (ages) had a six part answer. Many responders got some of the answer correct, but only 40% got all six parts correct.

SUMMER SOLUTIONS

1 From premises 5 and 6, we can conclude that Christopf L. Biggleswade is **fluent in Klingon**, because he is a hippy. Using premise 2, we learn that Christopf is **unhappy**, because all hippies are unhappy. From premise 8, Christopf is **not a Ph.D. candidate** because he is unhappy. Premise 7 allows us to conclude that Christopf's **Mom is, or has been, a shaman** because he is fluent in Klingon but is not a Ph.D. candidate. Premises 4 and 1 tell us that Christopf has **shingles** because he works at the Reliable Data Dump. Finally, using premise 3, Christopf **does not suffer from mixed dominance** because his mother is, or has been, a shaman and he suffers from shingles.

2 It takes **seven** pours to divide the eight pints in the jug into two equal parts. The state space is small enough to explore all unique alternatives in the decision tree.

	8-pint	5-pint	3-pint
initial state	8	0	0
pour #1, 8 into 5 →	3	5	0
pour #2, 5 into 3 →	3	2	3
pour #3, 3 into 8 →	6	2	0
pour #4, 5 into 3 →	6	0	2
pour #5, 8 into 5 →	1	5	2
pour #6, 5 into 3 →	1	4	3
pour #7, 3 into 8 →	4	4	0

3 The final league table is:

	P	*W*	*L*	*D*	*GF*	*GA*
A	3	1	1	1	6	5
B	3	3	0	0	4	1
C	3	0	1	2	3	4
D	3	0	2	1	0	3

B defeats A 2-1, defeats C 1-0 and defeats D 1-0. A defeats D 2-0, draws with C 3-3. C draws with D 0-0.

B won all 3 games, with only 4 goals, so 2 of its games were 1-0, and they scored 2 goals in their third. D scored no goals, and drew one game, which must have been 0-0, with either A or C. D must have lost the other two games by scores of 0-1 and 0-2. A conceded 3 goals in their draw with C, and none to D, so B must have scored 2 goals against A. Therefore, B must have beat C 1-0 and D 1-0. A, which is known to have scored at least 6 goals, put in 3 against C, and no more than one against B, so must have scored a minimum of 2 against D. We conclude that A beat D 2-0, scored once in their loss to B, and that D drew with C 0-0.

4 The smallest set of 10 consecutive primes in arithmetic progression (for N=0 to 9) is produced by: **199 + 210N**. Number theory tells us that the difference between consecutive terms in an arithmetic sequence of primes is the product of the prime numbers that are less than the required number of terms in the sequence. This can be observed by inspecting the first few sets of M consecutive primes:

Terms	Formula	Primes
3	3+ 2*N*	3, 5, 7
5	5+ 6*N*	5, 11, 17, 23, 29
6	7+30*N*	7, 37, 67, 97, 127, 157

For the above three sequences of length 3, 5, and 6, the difference between terms is 2, 2x3=6 and 2x3x5=30 respectively. Since we asked for 10 primes in the arithmetic progression, the difference must be a multiple of 2x3x5x7=210. Using a table of primes or a spreadsheet, it is not too hard to find that the first term is 199.

5 BEAVER + TIGER = RABBIT is **251453 + 60753 = 312206**. From the leading digits, R=B+1, and from the least significant, T = 2R (mod 10). From these two equations, B cannot be 0, 4 (that would imply T=0), 8 (causing a conflict with T at 8) or 9 (which implies R=0). This leaves six possible (B,R,T) triples: (1,2,3), (2,3,6), (3,4,8), (5,6,2), (6,7,4) and (7,8,6). To determine potential E values, observe that E+T >= 9 to provide a carry to the most significant digit. Fixing E implies I, because I=(2E)+(carry of 2R). Considering all possible E values, eliminate E that forces conflicts with E or I. Finally, observe that A=B-I or A=B-I-1 and E+T=A or E+T+1=A. This leaves only one possibility for (B,R,T,E,I,A)=(2,3,6,5,0,1). Pick V and G from the remaining digits such that (V+G+1) mod 10=B or V+G=11. V and G must be 4 and 7, so we pick G=7 for the largest TIGER.

Bonus This problem is known as Tower of Hanoi with cyclic moves only. Let f(*n*) be the number of moves to move an *n*-stack one peg, and g(*n*) be the number of moves to move an *n*-stack two pegs. Then, to move an n-stack 1 peg, you first have to move an n-1 stack 2 pegs, then move the n-th disk 1 peg and then move the n-1 stack 2 pegs. Thus, $f(n) = g(n-1) + 1 + g(n-1) = 2g(n-1) + 1$. To move an n-stack 2 pegs, you first have to move an n-1 stack 2 pegs, then move the n-th disk 1 peg, then move the n-1 stack 1 peg, then move the n-th disk 1 peg and finally move the n-1 stack 2 pegs. So, $g(n) = g(n-1) + 1 + f(n-1) + 1 + g(n-1) = 2g(n-1) + f(n-1) + 2$. Substituting for $f(n-1)$ gives $g(n) = 2g(n-1) + 2g(n-2) + 3$. We also have $g(n) = f(n-2) + 1 + f(n-1) + 1 = f(n-1) + f(n-2) + 2$. Substituting this into the equation for $f(n)$ gives $f(n) = 2f(n-1) + 2f(n-2) + 3$.

n	1	2	3	4	5	6	7	8	9
f(n)	1	5	15	43	119	327	895	2447	6687
g(n)	2	7	21	59	163	447	1223	3343	9135

The non-recursive formulas are most easily found through internet search at oeis.org, sequences A005665 and A005666.

$$f(n) = (\sqrt{3}/6)\ [\ (1+\sqrt{3})^{n+1} - (1-\sqrt{3})^{n+1}] - 1$$

$$g(n) = (\sqrt{3}/12)\ [\ (1+\sqrt{3})^{n+2} - (1-\sqrt{3}^{n+2}] - 1$$

Computer Bonus Without loss of generality, one can estimate the expected value of the number of calls to get a Bingo on a randomly selected card using a Monte Carlo technique, i.e., by simulating multiple sequences of randomly drawn balls and filling in a single fixed card. Our estimates calculate that, on average, **41.37** balls are required to get a Bingo.

NEW FALL PROBLEMS

1 At exactly noon, Bob left the Red Lion at Upper Darby and set off by foot on the trail to the Purple Cow in Lower Merion. At exactly noon, Carl left the Purple Cow and set off by bicycle on the same trail for the Red Lion. When they met, Bob had covered 4 miles. After a 10 minute chat, Carl gave Bob the bike and walked on to the Red Lion, where he drank a bottle of beer in 3 minutes and set off again for the Purple Cow. Bob, as soon as Carl left him, cycled on to the Purple Cow, drank a bottle of beer in 3 minutes, and cycled off again toward the Red Lion. This time they met 7 miles from the Purple Cow and discussed football for 10 minutes. Then Bob relinquished the bike and walked on to the Red Lion, and Carl cycled on to the Purple Cow. Each drank another bottle of beer, in the same time as before, and off our heroes went again with Carl still riding and Bob afoot. They collided with each other 2 miles from the Red Lion. If they each maintain their own individual walking and riding rates for the entire episode, how far is the Red Lion from the Purple Cow?

—A Tantalizer by Martin Hollis in *New Scientist*

2 Ann collects stamps. She has half as many from Canada as from Japan; one tenth as many from France as from Denmark; one fifth as many from Libya as from Egypt; five times as many from Haiti as from France; two fewer from Libya than from Canada; three times as many from India as from Canada; half as many from Australia as from Korea; one fewer from Korea than from India; four times as many from Brazil as from France; and twice as many from Guatemala as from Libya. Ann has a total of 303 stamps. How many Canadian stamps does Ann have, and how many French stamps?

—*Logic Puzzles to Bend Your Brain* by Kurt Smith

3 Consider all possible cryptic additions with only two addends, where the addends may be strings of any length. An example of such a cryptic is: ABCA + DCDCD = ABCDE. The solution to this cryptic is: 5275 + 47474 = 52746. We are concerned with cryptics that have a unique, that is only one, solution in base 10. For such a cryptic, define its N-value as the number resulting from deleting the + and = signs from its solution. For example, the N-value of the above cryptic is 52,754,747,452,746. Cryptics, such as A + B = C, with more than one solution, have no N-value. Of all possible N-values, what are the four smallest?

—An Enigma by Keith Austin in *New Scientist*

4 What is the probability that a positive integer, N, chosen at random, will have no repeated prime divisors? That is, if p is a prime divisor of N, then p^2 is not a divisor of N?

—*A Biography of the World's Most Mysterious Number* by Alfred S. Posamentier and Ingmar Lehmann

5 A young Tau Bate is contemplating her financial future. Confident in her engineering skills, she assumes that her salary will increase 5% annually for an indefinite time. On the advice of a financial planner, she intends to invest 10% of her total income (salary plus dividends) at the end of each year in a respected mutual fund. The fund pays out a flat annual dividend of 10% at the end of each year. The principle is to remain invested indefinitely. For year 1, her income is simply her starting salary S, and she invests 10% of that. In year 2, her total income is 1.05 S plus 10% of her accrued investment, which is 0.1 S. At the end of the year, she adds 10% of her year 2 income to her investment. Find closed formulas for her total income for year N, and for the total amount invested at the beginning of that year.

—Adapted from George Boole, 1860

Bonus Find positive integers a, b, c, and d which simultaneously solve the following two equations:

$$a^2 + b^3 = c^4$$

$$a^4 + b^6 = d^7$$

Find the solution with the smallest value of d. Note that zero is not a positive integer.

—Allan Gottlieb's Puzzle Corner in *Technology Review*

Computer Bonus In a children's game called Beetle, the object is to draw a beetle based on the results of throwing a die. Each face of the die allows adding a different part to the beetle: 1-body, 2-head, 3-eye (two needed), 4-feeler (two needed), 5-leg (six needed), 6-tail. You must have a body before you can add any other parts, and you must have a

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