Brain Ticklers

because he works at the Reliable Data Dump. Finally, using premise 3, Christopf does not suffer from mixed dominance because his mother is, or has been, a shaman and he suffers from shingles.

2) It takes seven pours to divide the eight pints in the jug into two equal parts. The state space is small enough to explore all unique alternatives in the decision tree.

3) The final league table is:

<table>
<thead>
<tr>
<th></th>
<th>W</th>
<th>L</th>
<th>D</th>
<th>GF</th>
<th>GA</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>C</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>D</td>
<td>3</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

B defeats A 2-1, defeats C 1-0, and defeats D 2-0, draws with C 3-3. C draws with D 0-0.

B won all 3 games, with only 4 goals, so 2 of its games were 1-0, and they scored 2 goals in their third. D scored no goals, and drew one game, which must have been 0-0, with either A or C. D must have lost the other two games by scores of 0-1 and 0-2. A conceded 3 goals in their draw with C, and none to D, so B must have scored 2 goals against A. Therefore, B must have beat C 1-0 and D 1-0. A, which is known to have scored at least 6 goals, put in 3 against C, and no more than one against B, so must have scored a minimum of 2 against D. We conclude that A beat D 2-0, scored once in their loss to B, and that D drew with C 0-0.

4) The smallest set of 10 consecutive primes in an arithmetic sequence of primes is the product of the prime numbers that are less than the required number of terms in the sequence. This can be observed by inspecting the first few sets of M consecutive primes:

<table>
<thead>
<tr>
<th>Terms</th>
<th>Formula</th>
<th>Primes</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3 + 2N</td>
<td>3, 5, 7</td>
</tr>
<tr>
<td>5</td>
<td>5 + 6N</td>
<td>5, 11, 17, 23, 29</td>
</tr>
<tr>
<td>6</td>
<td>7 + 30N</td>
<td>7, 37, 67, 97, 127, 157</td>
</tr>
</tbody>
</table>

For the above three sequences of length 3, 5, and 6, the difference between terms is 2, 2x3=6 and 2x3x5=30 respectively. Since we asked for 10 primes in the arithmetic progression, the difference must be a multiple of 2x3x5x7=210. Using a table of primes or a spreadsheet, it is not too hard to find that the first term is 190.

5) BEAVER + TIGER + RABBIT is 251453 + 60753 = 312206. From the leading digits, R=B+1, and from the least significant, T = 2R (mod 10). From these two equations, B cannot be 0, 4 (that would imply T=0), 8 (causing a conflict with T at 8 or 9 (which implies R=0). This leaves six possible (B,R,T) triples: (1,2,3), (2,3,6), (3,4,8), (5,6,2), (6,7,4) and (7,8,6). To determine potential E values, observe that E+T >= 9 to provide a carry to the most significant digit. Fixing E implies I, because I=2E+(carry of 2R). Considering all possible E values, eliminate E that forces conflicts with E or I. Finally, observe that A=B-I or A=B-I+1 and E+T=A or E+T+1=A. This leaves only one possibility for (B,R,T,E,I,A)=(2,3,6,5,0,1). Pick V and G from the remaining digits such that (V+G+1) mod 10=B or V+G=11. V and G must be 4 and 7, so we pick G=7 for the largest TI.

This problem is known as Tower of Hanoi with cyclic moves only. Let f(n) be the number of moves to move an n-stack one peg, and g(n) be the number of moves to move an n-stack two pegs. Then,
to move an \( n \)-stack 1 peg, you first have to move an \( n \)-stack 2 pegs, then move the \( n \)-th disk 1 peg and then move the \( n \)-1 stack 2 pegs. Thus, \( f(n) = g(n-1) + 1 + g(n-1) + 1 + g(n-1) = 2g(n-1) + 1 \). To move an \( n \)-stack 2 pegs, you first have to move an \( n \)-1 stack 2 pegs, then move the \( n \)-th disk 1 peg, then move the \( n \)-1 stack 1 peg, then move the \( n \)-th disk 1 peg and finally move the \( n \)-1 stack 2 pegs. So, \( g(n) = g(n-1) + 1 + f(n-1) + 1 + g(n-1) = 2g(n-1) + f(n-1) + 2 \). Substituting for \( f(n-1) \) gives \( g(n) = 2g(n-1) + 2(g(n-2) + 2) \). We also have \( g(n) = f(n-2) + 1 + f(n-1) + 1 = f(n-1) + f(n-2) + 2 \). Substituting this into the equation for \( f(n) \) gives \( f(n) = 2f(n-1) + 2f(n-2) + 3 \).

The non-recursive formulas are most easily found through internet search at oeis.org, sequences A005665 and A005666.

\[
f(n) = \left( \begin{array}{c} 3/16 \\ \end{array} \right) \left( \begin{array}{c} 1+3^n \\ \end{array} \right) - 1 \\
g(n) = \left( \begin{array}{c} 3/12 \end{array} \right) \left( \begin{array}{c} 1+3^n \\ \end{array} \right) - 1
\]

**Computer Bonus** Without loss of generality, one can estimate the expected value of the number of calls to get a Bingo on a randomly selected card using a Monte Carlo technique, i.e., by simulating multiple sequences of randomly drawn balls and filling in a single fixed card. Our estimates calculate that, on average, 41.37 balls are required to get a Bingo.

**NEW FALL PROBLEMS**

1. At exactly noon, Bob left the Red Lion at Upper Darby and set off by foot on the trail to the Purple Cow in Lower Merion. At exactly noon, Carl left the Purple Cow and set off by bicycle on the same trail for the Red Lion. When they met, Bob had covered 4 miles. After a 10 minute chat, Carl gave Bob the bike and walked on to the Red Lion, where he drank a bottle of beer in 3 minutes and set off again for the Purple Cow. Bob, as soon as Carl left him, cycled on to the Purple Cow, drank a bottle of beer in 3 minutes, and cycled off again toward the Red Lion. This time they met 7 miles from the Purple Cow and discussed football for 10 minutes. Then Bob relinquished the bike and walked on to the Red Lion, and Carl cycled on to the Purple Cow. Each drank another bottle of beer, in the same time as before, and off our heroes went again with Carl still riding and Bob afoot. They collided with each other 2 miles from the Red Lion. If they each maintain their own individual walking and riding rates for the entire episode, how far is the Red Lion from the Purple Cow?

—A Tantalizer by Martin Hollis in New Scientist

2. Ann collects stamps. She has half as many from Canada as from Japan; one tenth as many from France as from Denmark; one fifth as many from Libya as from Egypt; five times as many from Haiti as from France; two fewer from Libya than from Canada; three times as many from India as from Canada; half as many from Australia as from Korea; one fewer from Korea than from India; four times as many from Brazil as from France; and twice as many from Guatemala as from Libya. Ann has a total of 303 stamps. How many Canadian stamps does Ann have, and how many French stamps?

—Logic Puzzles to Bend Your Brain by Kurt Smith

3. Consider all possible cryptic additions with only two addends, where the addends may be strings of any length. An example of such a cryptic is: ABCA + DCDCD = ABCDE. The solution to this cryptic is: 5275 + 47474 = 52746. We are concerned with cryptics that have a unique, that is only one, solution in base 10. For such a cryptic, define its \( N \)-value as the number resulting from deleting the + and = signs from its solution. For example, the \( N \)-value of the above cryptic is 52,754,747,452,746. Cryptics, such as \( A + B = C \), with more than one solution, have no \( N \)-value. Of all possible \( N \)-values, what are the four smallest?

—An Enigma by Keith Austin in New Scientist

4. What is the probability that a positive integer, \( N \), chosen at random, will have no repeated prime divisors? That is, if \( p \) is a prime divisor of \( N \), then \( p^2 \) is not a divisor of \( N \)?

—A Biography of the World’s Most Mysterious Number by Alfred S. Posamentier and Ingmar Lehmann

5. A young Tau Bate is contemplating her financial future. Confident in her engineering skills, she assumes that her salary will increase 5% annually for an indefinite time. On the advice of a financial planner, she intends to invest 10% of her total income (salary plus dividends) at the end of each year in a respected mutual fund. The fund pays out a flat annual dividend of 10% at the end of each year. The principle is to remain invested indefinitely. For year 1, her income is simply her starting salary S, and she invests 10% of that. In year 2, her total income is 1.05 S plus 10% of her accrued investment, which is 0.1 S. At the end of the year, she adds 10% of her year 2 income to her investment. Find closed formulas for her total income for year \( N \), and for the total amount invested at the beginning of that year.

—Adapted from George Boole, 1860

**Bonus** Find positive integers \( a, b, c, \) and \( d \) which simultaneously solve the following two equations:

\[
a^2 + b^3 = c^4 \\
a^4 + b^6 = d^7
\]

Find the solution with the smallest value of \( d \). Note that zero is not a positive integer.

—Allan Gottlieb’s Puzzle Corner in Technology Review

**Computer Bonus** In a children’s game called Beetle, the object is to draw a beetle based on the results of throwing a die. Each face of the die allows adding a different part to the beetle: 1-body, 2-head, 3-eye (two needed), 4-feeder (two needed), 5-leg (six needed), 6-tail. You must have a body before you can add any other parts, and you must have a...
Brain Ticklers
(Continued from page 57)
head before you can add eyes or feelers. Calculate the exact expected number of die tosses required to complete the beetle.

—World’s Greatest Dice Games

Send your answers to any or all of the Fall Brain Ticklers to Curt Gomulinski, Tau Beta Pi, P. O. Box 2697, Knoxville, TN 37901-2697 or email to BrainTicklers@tbp.org as plain text only (no HTML, no attachments). The cutoff date for entries to the Fall column is the appearance of the Winter BENT in early January. The method of solution is not necessary, unless you think it will be of interest to the judges. We welcome any interesting problems that might be suitable for the column. The Computer Bonus is not graded. Curt will forward your entries to the judges who are H.G. McIlvried, III, PA Λ ’53, F.J. Tydeman, CA Δ ’73, D.A. Dechman, TX Α ’57, and the columnist for this issue.

—J.C. Rasbold, OH Α ’83

Send us your witty caption for this photo from our archives, and if it is judged one of the best, you will win a TBP t-shirt.

It shows Robert C. “Red” Matthews, Illinois Alpha 1902, retired Secretary-Treasurer aged 90, with unidentified friends at the 1969 Convention in Houston, TX. Email entries to tbp@tbp.org or mail them to HQ by November 1.

• We received thirty great entries for the Summer Caption Contest. Thank you to all who participated. Here are the five winning captions for the photo below:

“The fine civil engineering men of Illinois Alpha pilot testing the Illinois DOT latest idea for highway construction worker protection barriers.”—Solange C. Dao P.E., FL Α ’95

“When they told us Tau Beta Pi would be more fun than a barrel of monkeys, this isn’t quite what I expected.”—Matthew R. Gargani, PA Λ ’14

“Patent No. 677606 – An Apparatus for the Prevention of Wedgies”—Keith A. Lyskowski, NY Α ’90

“I told you not to let engineers join our fraternity. See what happens when they plan a keg party!”—Frank P. Tuhy, NJ Α ’65

“Can’t we just have a secret handshake!”—Richard A. Suhar P.E., OH Α ’83

Congratulations to the winners!

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