Brain Ticklers

RESULTS FROM SPRING 2012

Perfect

* Couillard, J. Gregory	IL	A	'89
* Kimsey, David B.	AL	A	'71
* Prince, Lawrence R.	CT	B	'91
* Riedesel, Jeremy M.	OH	B	'96
* Rowland, R. Wilson	MD	B	'51
* Schmidt, V. Hugo	WA	B	'51
* Slegel, Timothy J.	PA	A	'80
* Snelling, William E.	GA	A	'79
* Spong, Robert N.	UT	A	'58
* Strong, Michael D.	PA	A	'84
* Zapor, Richard A.	CA	E	'84

Other

* Alexander, Jay A.	IL	Γ	'86
* Bernacki, Stephen E.	MA	A	'70
* Bohdan, Timothy E.	IN	Γ	'85
* Colello, Marc C.	NY	I	'97
* Doniger, Kenneth J.	CA	A	'77
Handley, Vernon K.	GA	A	'86
Johnson, Roger W.	MN	A	'79
Jones, Donlan F.	CA	Z	'52
* Jones, John F.	WI	A	'59
Lalinsky, Mark A.	MI	Γ	'77
* Lui, Huiliang	CA	A	'07
* Marks, Lawrence B.	NY	I	'81
Marks, Benjamin	Member's son		
Mercer, Robert	Non-member		
* Rasbold, J. Charles	OH	A	'83
Rentz, Peter E.	IN	A	'55
* Rubin, James D.	MI	Γ	'82
Sauer, Daniel M.	MI	B	'05
Sauer, Jon	Member's brother		
* Sentman, Mark H.	CA	E	'86
Sentman, Andrew	Member's son		
Sentman, Michale	Member's daughter		
* Stein, Gary M.	FL	Δ	'04
* Stribling, Jeffrey R.	CA	A	'92
* Summerfield, Steven L.	MO	Γ	'85
* Thaller, David B.	MA	B	'93
Vegeais, James A.	IL	A	'86
* Voellinger, Edward J.	Non-member		
von Laven, Kurt A.	CA	Γ	'12

* Denotes correct bonus solution

SPRING REVIEW

The most difficult regular problems were No. 2, about sequences, and No. 3, about the smallest solution to an equation. Only about half of the entries had correct solutions to these Ticklers. The Bonus problem, about arranging integers around a circle, was easier than usual with about 80% of the entries having correct solutions.

SUMMER SOLUTIONS

Readers' entries for the Summer problems will be acknowledged in the Winter '13 BENT. Meanwhile, here are the answers:

1 The problem asks for the exact movements of Paul, Quentin, Ron, Sam and Ted (P, Q, R, S and T), given their truthful statements, the train schedule, and the mileage and biking rate between the train stations (A, B, C, D, and E). From their exact movements, one deduces that S and T left their bikes at the Bingchester station (B) and T defaced the Moaning Lisa. The time table only allows each suspect to travel from their starting station, through B, to their destination station. Each suspect could travel to or from B by bike or train, so that there are four possibilities for their mode of transportation (TT, TB, BT, or BB). In P's case, he had to travel from A to B by train, then B to E by bike as no other means could put him at E by 9:52. Similarly, Quentin traveled from E to C by bike only, as no other means could put him in C by 9:58. Ron traveled from A to D by train as he left his bicycle at A. Sam traveled from C to B by bike, then B to D by train as no other means could put him in D at 10:03. Ted traveled from E to B by bike, then B to A by train as no other means could put him in E at 8:59. Therefore, their exact movements were:

- *Paul: (at A 9:14; dep. A 9:15 by T; arr. B 9:23; dep. B 9:24 by B, arr. E 9:52),*
- *Quentin: (at E 9:01; dep. E 9:01 by B; arr. B 9:29; dep. B 9:29 by B, arr. C 9:57),*
- *Ron: (at A 9:14; dep. A 9:15 by T; arr. B 9:23; dep. B 9:25 by T, arr. D 9:56),*
- *Sam: (at C 8:56; dep. C 8:56 by B; arr. B 9:24; dep. B 9:25 by T, arr. E 9:56),*
- *Ted: (at E 8:59; dep. E 8:59 by B; arr. B 9:27; dep. B 9:30 by T, arr. A 9:38).*

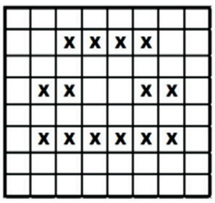
This means that Sam and Ted left their bikes at B and that Ted defaced the Moaning Lisa.

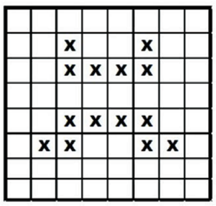
2 The cryptic multiplication ABCDEF = BCDEFA x M decodes as 923076 = 230769 x 4. Rewrite the original cryptic as 100,000 x A + BCDEF = (10 x BCDEF + A)M, which simplifies to (100,000 - M)A = BCDEF(10 x M - 1). Thus, (100,000 - M)A must be divisible by (10 x M - 1). Letting M = 2, 3, 4, etc., and trying 99,998A/19; 99,997A/29; 99,996A/39; etc., we find that only M = 4 and M = 5 are possibilities. For M = 5, 99,995A is divisible by 49 if A = 7, which gives 99,995/7 = 14285 for BCDEF, but this requires that both M and F be 5. For M = 4, we have 99,996/39 = 2564. A must be greater than 3 to give a 5-digit number and cannot be 4 (because M = 4). Trying A = 5, 6, ..., 9, we find that only 9 gives a solution without a duplicated value, so BCDEF = 9 x 2564 = 23076.

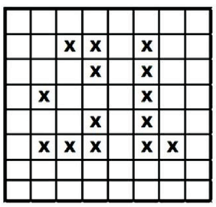
3 The precise time that the minute and hour hands of an analog watch coincide with the second hand within half a second of an hourly mark is 9:49:05.4545. The minute hand crosses over the hour hand 11 times in each 12 hour cycle or every 12/11 = 1.090909 hour = 1:05.454545=1:05:27.272727. The position of the second hand at the time the minute hand crosses the hour hand can be found by multiplying 27.272727 by 1, 2, 3, ..., 10 and subtracting multiples of 60 seconds. This gives 27.2727; 54.5454; 21.8181; 49.0909; 16.3636; 43.6363; 10.9090; 38.1818; 5.4545; 32.7272 for the position of the second hand when the hour and minute hands coincide. As can be seen, there is only one case in which the second hand is within half a second of an hourly mark. It occurs at 9(1:05.454545) = 9:49:05.4545… .

4 The minimum number of people in a group to have at least a 50% chance that three or more have a common birthday or two or more separate pairs share different common birthdays is 36. This probability is $1 - P_A(N) - P_B(N)$, where $P_A(N)$ is the probability that none of the N people in the group share a common birthday and $P_B(N)$ is the probability that exactly two people have the same birthday. $P_A(N) = (364/365)(363/365)(362/365)\ldots((365-(N-1))/365)$. To see this, choose one person. The probability that a second person has a different birthday is 364/365, as there are 364 days that don't match. The probability that a third person's birthday matches neither of the first two birthdays is 363/365, and so forth for the rest of the group, so the probability of at least one common birthday is the complement of $P_A(N)$, or $1 - P_A(N)$. $P_B(N)$ is calculated in a similar fashion and equals $C(N, 2)(1/365)(364/365)(363/365)\ldots((365-(N-2))/365)$. $C(N, 2)$ is the number of ways of selecting 2 people from a group of N, that is, the number of possible pairs; 1/365 is the probability that a pair have a common birthday; 364/365 is the probability that a third person in the group does not have that birthday, etc. The desired answer is $1 - P_A(N) - P_B(N)$. Trying successive values of N, at 36 people, $P_A(N) + P_B(N)$ drops below 0.5 for the first time. A spreadsheet finds the answer quickly.

5 The minimum number of knights that can be placed on a standard chess board such that every square is attacked is 14. There are three basic solutions, as shown. All other solutions are rotations or reflections of these three basic solutions. It is straightforward to find the symmetrical solutions by trial and error. We used a computer program to prove that a solution did not exist for 13 or fewer knights.







Bonus. When the pin constraining a 100 cm long uniform rod leaning against a frictionless wall with an initial horizontal angle of 30 degrees is removed, the horizontal velocity of the center of the rod as it passes the point where the rod initially touched the frictionless floor is 36.9 cm/s. Let y be the height of the upper end of the rod and x the horizontal position of the lower end of the rod. Conservation of energy equates the loss in gravitational potential energy to the gain in kinetic energy. The potential energy is $100mg(25 - y/2)$; where m is the mass/length and g is 980 cm/sec². Note that the vertical and horizontal velocities vary from point to point along the length of the rod. At a point b cm from the lower end of the rod, the vertical velocity is $(b/100)(dy/dt)$ and the horizontal velocity is $[(100-b)/100](dx/dt)$ when the rod is touching the wall. To find the total kinetic energy, you have to integrate over the rod's length. The increase in kinetic energy is $\Delta K.E.=\int_0^{100}(m/2)[(b/100)^2(dy/dt)^2 + ((100-b)/100)^2(dx/dt)^2]db = (100m/6)[(dy/dt)^2+(dx/dt)^2]$. Since $x^2+y^2 = 100^2$ and $2x(dx/dt) + 2y(dy/dt) = 0$, we get $(dy/dt)^2 = (x/y)^2(dx/dt)^2$ and $\Delta K.E.=(100m/6)(100^2/y^2)(dx/dt)^2$. Equating the potential energy loss to the kinetic energy gain yields $(dx/dt)^2=3g/100^2(50y^2- y^3)=0.294(50y^2-y^3)$. During the fall, the center of gravity of the rod accelerates horizontally in reaction to the normal force exerted by the wall. The horizontal acceleration decreases smoothly to zero at the moment that the rod leaves contact with the wall. The lower end of the rod, that is moving at twice the horizontal velocity of the center of the rod, stops accelerating at this time. The horizontal velocity of the center of the rod is constant after the rod leaves the wall. It equals $(1/2)\ dx/dt$ when $d^2x/dt^2=0$. Taking the derivative of $(dx/dt)^2=0.294(50y^2-y^3)$ yields $2(dx/dt)d^2x/dt^2=0.294(100y-3y^2)(dy/dt)$. Thus, $d^2x/dt^2=0$ when $y = 100/3$. So, $dx/dt = 73.8$ cm/sec, and the center of the rod is moving at 36.9 cm/sec as it passes the point where the end of the rod initially touched the floor.

Computer Bonus. The number of ways that 14 married couples can be seated in chairs numbered consecutively from 1 to 28 (meaning rotations and reflections count as different arrangements) about a round table in such a manner that there is always one man between two women and no man is next to his own wife is 1,905,270,127,543,015,833,600. Break the problem into two parts. First place the 14 women; then place the men. There are 14! ways to place the women into the even-numbered chairs and 14! ways to place them into the odd numbered chairs. Pick one of those assignments. A recursive computer program counts 10,927,434,464 ways to place the men for each specific arrangement of women. So, the total number of ways to seat the 14 couples is to 2(14!)(10,927,434,464) = 1,905,270,127,543,015,833,600.

NEW FALL PROBLEMS

1 Two small rockets, one with a mass of 2 kg and the other with a mass of 26 kg, are traveling through space with no forces acting on them. The velocity of each is a positive integral number of meters per second such that the difference in their kinetic energies is 1 J. Each rocket then fires a short burst of its ion engine, after which each again has a velocity of an integral number of meters, and the difference in their kinetic energies is still 1 J. What are the smallest positive velocities which the two rockets can have before and after they fire their engines? Assume that the masses of the rockets do not change when firing the engines.

—Adapted from
Theodore J. Stadnik, *OH Λ '05*

2 Find a six-digit number with the following properties: a) the first (leftmost) digit is four more than the second digit, b) the third digit is one less than the second digit, c) the product of the first two digits equals the two-digit number formed by the third and fourth digits, d) the fifth digit equals the sum the first and third digits, and e) the sixth digit equals the sum of the fourth and fifth digits.

—**Mark A. Noblett,** P.E., *NY N '70*

3 A band of pirates has stolen a small chest containing nine identical bottles of gold dust that each weigh exactly the same. They decide to wait until the next day to divide the loot. During the night, one pirate surreptitiously opens one of the bottles and removes a small amount of gold. Later he starts to worry that, if he gets caught, he will have to walk the plank, so he sneaks back, grabs a bottle at random and replaces the gold he took. Using only a simple two-pan balance scale, what is the minimum number of weighings
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BRAIN TICKLERS

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required to assure either that the bottles all have the same weight or to determine the light and heavy bottles?

—*You'd Better Be Really Smart Brain Bafflers* by Tim Sole and Rod Marshall

4 Alice and Beth each toss n fair coins. What is the probability that they throw the same number of heads?

—adapted from *Problems in Probability Theory, Mathematical Statistics and Theory of Random Functions* by A.A. Sveshnikov

5 A Tau Bate is standing 5 meters away from a long, tall, vertical wall. She holds a garden hose in her hand. The nozzle is 1 meter above the ground and water emerges from the nozzle at a velocity of 10 m/s. Using only her wrist to vary the horizontal and vertical angles at which water leaves the nozzle, what is the maximum area of the wall that she can wet down? Use 9.81 m/s^2 for the acceleration due to gravity and report the answer to 3 significant digits.

—**John L. Bradshaw,** *PA A '82*

Bonus. A plane takes off from an airport on the equator and heads due northeast. If it always maintains a true northeast heading, what is the exact latitude at which it will cross its starting longitude for the first time? Assume that the earth is a perfect sphere and that the plane can be refueled while in flight.

—Technology Review

Computer Bonus. This cryptogram is in honor of Jim Froula and his many years of service to Tau Beta Pi; he was named Secretary-Treasurer in 1982 and served as Executive Director until his retirement in 2011.

J·FROULA = RETIRE·S

Of course, we want Jim to enjoy maximum REST. The symbol · stands for multiplication. The problem is in base 12, and all the usual rules apply.

—**Howard G. McIlvried, III,** *PA Γ '53*

Send your answers to any or all of the Brain Ticklers to: **Curt Gomulinski, Tau Beta Pi, P.O. Box 2697, Knoxville, TN 37901-2697** or email plain text only to: *BrainTicklers@tbp.org*. The cutoff date for entries to the Fall column is the appearance of the Winter Bent around mid-January. The method of solution is not necessary. We also welcome any interesting new problems that may be suitable for use in the column. The Computer Bonus is not graded. Curt will forward your entries to the judges, who are: **H.G. McIlvried III,** *PA Γ '53;* **F. J. Tydeman,** *CA Δ '73;* **D.A. Dechman,** *TX A '57*, and the columnist for this issue,

—**J.L. Bradshaw,** *PA A '82.*

EXECUTIVE COUNCIL

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for Saturday night were approved, and an invitation to Ray A. Rothrock, *TX Δ '77*, to be the keynote speaker was extended. A proposal from the Advisor Recruitment and Development Committee to invite additional advisors to Convention was approved. A plan to offer professional development sessions on Thursday was accepted.

The Council approved a series of amendments to be reviewed by committees at the 2012 Convention. Amendments include clarifying the appointment and terms of Association officials, removing requirements for written documents in certain situations, changing a graduate student eligibility requirement to be "enrolled" instead of "in residence," clarifying the role of alumnus chapters in amendment ratification, removing paper catalog card stipulations, updating reporting requirements, clarifying chapter record-keeping, allowing THE BENT subscription price to be set by the Executive Council with Convention review, updating out-of-date language, and correcting incorrect internal references.

The Council reviewed the performance of Executive Director Gomulinski and established goals for the current fiscal year.

LETTERS TO THE EDITOR

(Continued from page 6)

have otherwise overlooked had I not had the honor of hearing Dr. Feisel present it in person. Thankfully Lyle's Laws will continue to be available at the Tau Beta Pi website or perhaps even in book form one day. Many thanks to Dr. Feisel for his extraordinary words of wisdom.

Gregory M. Wilkins, Ph.D., IL A '92

Scholars

•I am greatly honored to have received a Tau Beta Pi Scholarship! This not only aids in paying for college tuition, but is also an incredible recognition. Being a Tau Bate has been an outstanding experience! I devote my time to the Arizona Alpha Chapter as vice president and look forward to serving as president next year.

Ina A. Kundu, AZ A '13

•I would like to express my immense gratitude to Tau Beta Pi for this scholarship. It is a greatly appreciated financial relief. I am very proud to be a member of Tau Beta Pi and look forward to furthering my education in the field of computer engineering.

Ethan C. Grefe, WI A '13

• Thank you for honoring me with a TBΠ scholarship. It is with deep and humble gratitude that I accept this award. The scholarship has brightened my future two semesters considerably, and has brought joy and relief to my parents.

Anjali K. Bains, NY A '13

•I would like to thank the Tau Beta Pi Association and Board for the honor of awarding me a TBΠ Scholarship, and to all those who donate to make these scholarships possible. I am extremely grateful, and this will help me to continue my studies.

Breana K. Pabst, MT A '13

• My wife and I would like to express our gratitude for this generous scholarship. It will be such a blessing to us in the upcoming year.

Isaiah J. Davies, UT A '13